Stochastic Functions of Blueshift vs. Redshift

藍移與紅移隨機函數研驗

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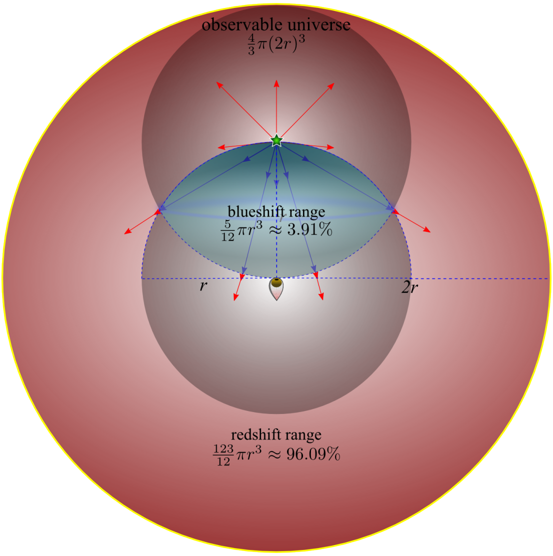
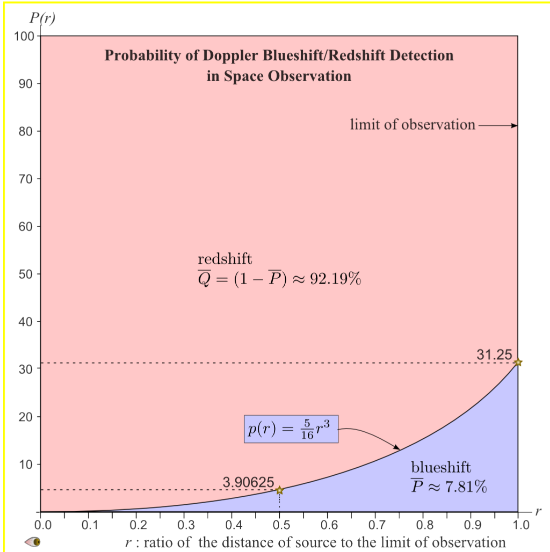
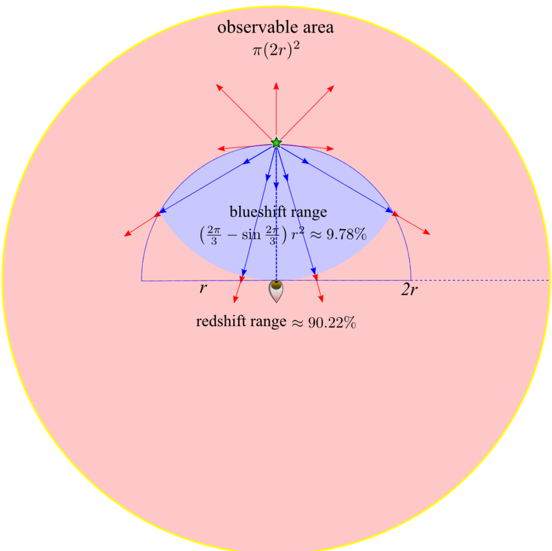


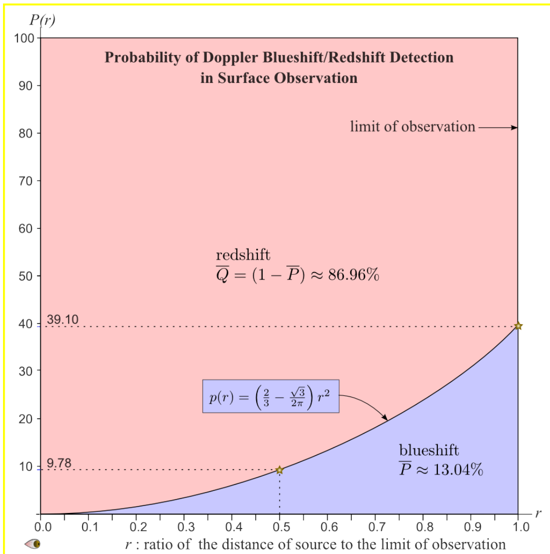
Blueshift vs. Redshift in Space



Blueshift vs. Redshift Distribution in Space



Surface Blueshift vs.Redshift



Surface Blueshift vs.Redshift Distribution

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Abstract

Viewing the random motions of objects, an observer might think it is 50-50 chances that an object would move toward or away. It might be intuitive, however, it is far from the truth. This study derives the probability functions of Doppler blueshift and redshift effect of signal detection.

The fact is, Doppler redshift detection is highly dominating in space, surface, and linear observation. Under the conditions of no quality loss of radiation over distance, and the observer has perfect vision; It is more than 92% probability of detecting redshift, in three-dimensional observation, 87% surface, and 75% linear. In cosmic observation, only 7.81% of the observers in the universe will detect blueshift of radiations from any object, on average. The remaining 92.19% of the observers in the universe will detect redshift. It it universal for all observers, aliens or Earthlings at all locations of the universe.

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1 Introduction

Even we might not be fully aware of Doppler effect in our daily life, it is everywhere in signal transmission over variable distance, *e.g.* sight and sound of moving object. Basically, any signal delivery in changing distance between the sender and the receiver. All creatures cope with Doppler effect. Ball player, predator and it's prey, mobile radio, echolocation, etc. An excellent example is fishing osprey. It has to cope with the light going through water and air in high speed, under the conditions of the environment, such as sunlight, wind, or even the noise from running boats and swimmers. Not only the location and motion but also the size of the fish, (or even selecting what kind of fish that only osprey knows). Ospreys don't have theory for Doppler effect but they are masters of coping with it.

Our perception of the universe rarely is direct physical interaction. Instead, it is the information of the physical events collected by by our sensors. It is only the description, not the source event. When continuous information is sent from different locations, and the observer is moving from place to place; It would not necessarily arrive in same order and interval initiated by the event. It is the Doppler effect of information delivery; The change in frequency of a wave (or other periodic signals) for an observer relative to the moving source. We can be fooled by what we observed and how it is delivered[1].

There is a furtherest distance an observer can reach. Basically, the limit is identical in all directions. This makes an observer right at the center of the observable universe. All information is collected centered at the observer. It creates the illusion of center of the universe to the observer. In the past, this illusion has caused significant issue in science.

Viewing the random motion, an observer might think it is 50-50 chances that an object would move toward or away. It might be intuitive, however, it is far from the truth. The fact is, redshift detection is highly dominating in space, surface, and linear observation. This study derives the probability function of blueshift vs. redshift effect of signal detection. I hope it could clarify the controversial interpretations derived from high redshift detection in cosmic surveys.

2 Creation of Doppler Effect

The vast sky provides us the range of observation farther than we can reach. However, looking for blueshift in the sky is not as easy as we think. Blueshift can only be observed when the distance between the object and the observer is shrinking. Analogous to one can only go into the woods as deep as midpoint, then it is leaving; and there is no limit of leaving the woods. This terminate and switch over nature not only sets the limit of blueshift but also increases the chance of redshift detection.

The possible region of blueshift observation in three-dimensional space observation is depicted in Figure 1.

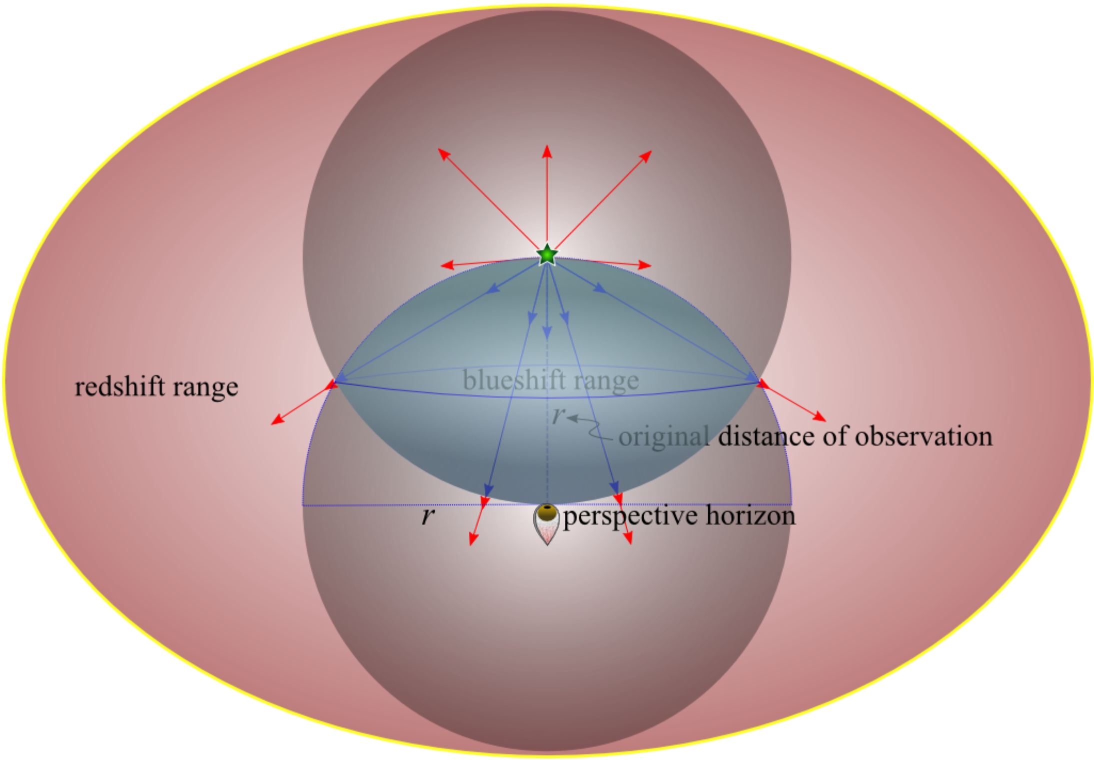

Figure 1: Limited Range of Blueshift Observation

Suppose an astronomical object (or source, show as green star), is r distance away from the observer (brown eye). If we picture two identical spheres with radius of r, one is centered at the observer, and another at the star. A disc shape of region is formed in the intersection of two spheres, shaded blue in Figure 1. Here, r is the original distance of observation, which is also the longest distance of blueshift observation. During the observation, for all the possible directions the star can go (in the perspective of the observer), blueshift can only be detected when it travels into and remains within the intersection, *i.e.* within the intersection, all subsequent distances of observation will not exceed the original.

The volume of the disc will shrink exponentially with the decreasing distance of the observation, however, it will stay the same shape as long as the distance of observation is decreasing. It will disappear when the star travels out of the intersection. In other words, unless the object collides with the observer, it will pass and redshift will be observed afterward. This means that blueshift observation has termination, and it will switch to redshift.

On the other hand, redshift will be detected in all directions outside of the blueshift range, and it can continue beyond the limit of the observation. It will never switch to blueshift unless the relative trajectory of the object is curved. Then, the object is not departing.

3 Mathematical Model of Blueshift Detection in Space

The total volume of the blueshift range is two equal size spherical caps, as shown in Figure 2.

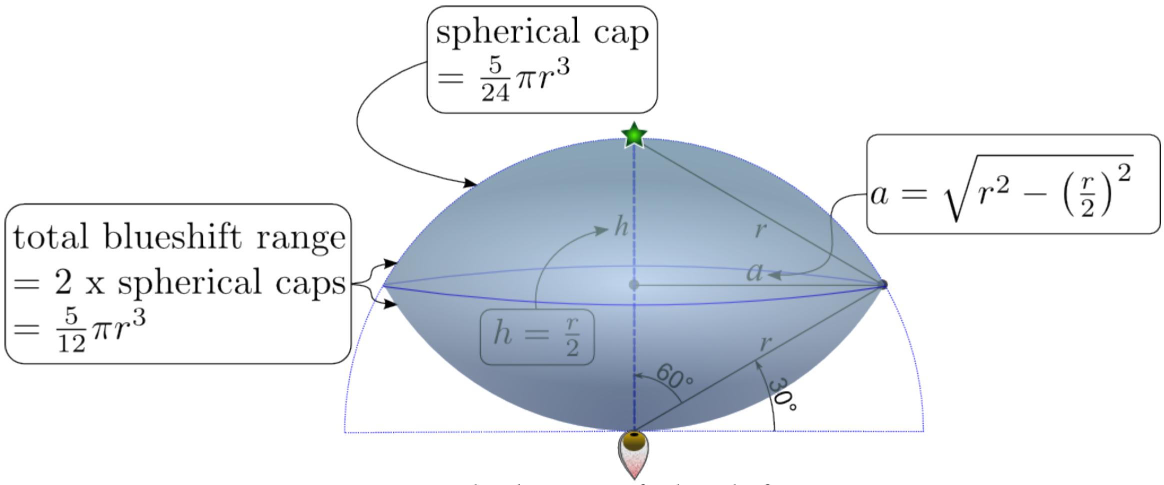

Figure 2: Calculations of Blueshift Range

Using the following formula to calculate the volume of the spherical cap:

$$\frac{\pi h}{6}(3a^2 + h^2)$$

$$\begin{aligned} \textit{height:of the cap (h)} &= \frac{r}{2} \\ \textit{radius of the base of the cap (a)} &= \sqrt{r^2 - \left(\frac{r}{2}\right)^2} \end{aligned}$$

Step by step calculations:

$$\begin{aligned}
\frac{\pi h}{6}(3a^2+h^2) &= \frac{\pi \frac{r}{2}}{6}\left(3\left(\sqrt{r^2-\left(\frac{r}{2}\right)^2}\right)^2+\left(\frac{r}{2}\right)^2\right) \\
&= \frac{\pi r}{12}\left(3r^2-3\left(\frac{r}{2}\right)^2+\frac{r^2}{4}\right) \\
&= \frac{\pi r}{12}\left(\frac{12r^2}{4}-\frac{3r^2}{4}+\frac{r^2}{4}\right) \\
&= \frac{\pi r}{12}\left(\frac{10r^2}{4}\right) \\
&= \frac{10}{48}\pi r^3 \\
&= \frac{5}{24}\pi r^3 \\
\textit{Alternatively,} & \\
h &= \frac{r}{2} \\
a &= \sin 60^\circ r = \frac{\sqrt{3}}{2}r \\
a^2 &= \frac{3}{4}r^2 \\
\frac{\pi h}{6}(3a^2+h^2) &= \frac{\pi \frac{r}{2}}{6}\left(3\left(\frac{3}{4}r^2\right)+\left(\frac{r}{2}\right)^2\right) \\
&= \frac{\pi r}{12}\left(\frac{9r^2}{4}+\frac{r^2}{4}\right) \\
&= \frac{\pi r}{12}\left(\frac{10r^2}{4}\right) \\
&= \frac{10}{48}\pi r^3 \\
&= \frac{5}{24}\pi r^3
\end{aligned}$$

Hence, the total volume of the blueshift range is twice of the spherical cap:

$$2\left(\frac{5}{24}\pi r^3\right) = \frac{5}{12}\pi r^3$$

4 Probability Function of Blueshift Detection

There is limit of how far an observer can reach. It is considered identical in all directions of observation. Despite the observer is unlikely at the center of the universe, however, observation creates the illusion of centering observer. All data collected will be centered at the observer. As a result, an observer is always located at the center of the observable universe.

Suppose, on linear average, the object is expected to be located at halfway between the observer and the edge of the observable universe. Then, the distance from the observer to the edge of the observable universe (radius of the observable universe, or limit of observation) is $2r$, as the depicted in Figure 3.

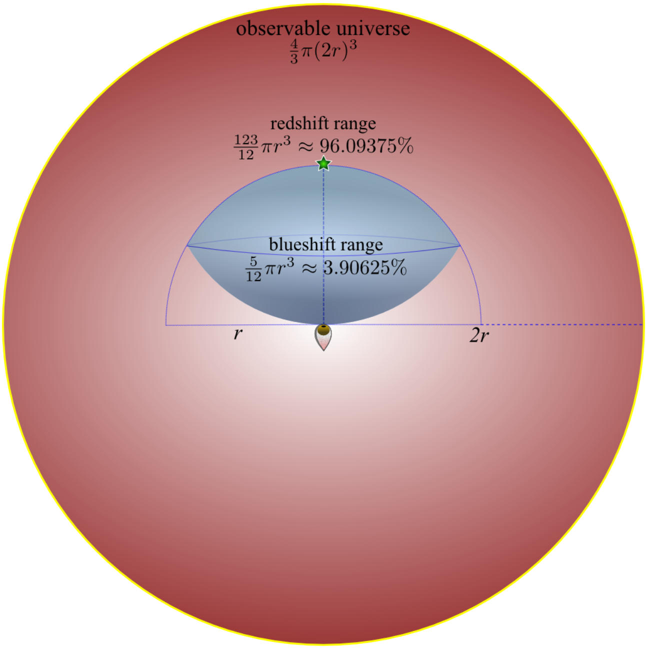

Figure 3: Linear Expected Probability of Blueshift Detection

Then, the size of the observable universe is $\frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$. Comparing the volume of the blueshift range to the total volume of the observable universe, we have:

$$\begin{aligned} &= \frac{5}{12}\pi r^3 \div \frac{32}{3}\pi r^3 \\ &= \frac{5}{128} \\ &= 3.90625\% \end{aligned}$$

This is the linear expected probability of blueshift will be detected, *i.e.* the object is expected to be halfway to the limit of the observation on average. Then, we can calculate the size of the redshift range,

$$\frac{32}{3}\pi r^3 - \frac{5}{12}\pi r^3 = \frac{123}{12}\pi r^3$$

and the linear expected probability of redshift detection,

$$\begin{aligned}
&= \frac{123}{12}\pi r^3 \div \frac{32}{3}\pi r^3 3 \\
&= 1 - \frac{5}{128} \\
&= \frac{123}{128} \\
&= 96.09375\%
\end{aligned}$$

Standardized the equation by setting the radius of the observable universe (or limit of the observation) to one, then the total volume of observable universe is $\frac{4}{3}\pi$. The probability function of the blueshift detection for any object at the location (r) within the limit of observation [0, 1] can be obtained with the following probability density function:

$$\begin{aligned}
p(r) &= \frac{5}{12}\pi r^3 \div \frac{4}{3}\pi \\
&= \frac{5}{16} r^3 = 0.3125 r^3 \\
& \quad \textit{where } (0 \le r \le 1)
\end{aligned}$$

Here r is the ratio of the distance of observation and the limit of observation:

$$r = \frac{\textit{distance of observation}}{\textit{limit of observation}}$$

With this standardized probability function, we can calculate the probability of blueshift and redshift detection per distance. For example: if we can detect the Doppler effect up to 20 billion light years (bly), and like to know the probability of blueshift detection of an object located at 4 bly away. Then from the equation $p(r) = \frac{5}{16} r^3$, we have:

$$\begin{aligned}
p\left(\frac{4}{20}\right) &= \frac{5}{16}\left(\frac{4}{20}\right)^3 \\
&= 0.0025 = 0.25\%
\end{aligned}$$

And the probability of redshift detection is:

$$q\left(\frac{4}{20}\right) = 1 - p\left(\frac{4}{20}\right) = 99.75\%$$

5 Probability Distribution of Blueshift Detection

The probability distribution of Doppler blueshift detection in space is shown in Figure 4.

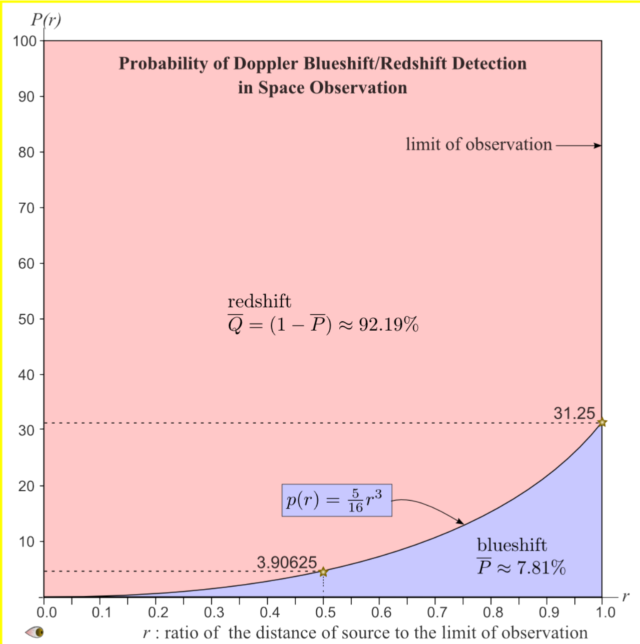

Figure 4: Probability Distribution of Blueshift Detection in Space

From Figure 4, we can see the probability of blueshift detection is within the range of [0, 31.25]%, and the majority is located from father than half of the observable universe; and redshift within the range of [68.75, 100]%. And we have the weighted population mean of blueshift detection:

$$\int_0^1 \frac{5}{16} r^3 \mathrm{d}r$$

Computing the integral of probability function yields the expected population mean of blueshift detection, $\overline{P}$:

$$\overline{P} = \sum_{r=0}^{1} \frac{5}{16} r^3 \Delta r \approx 7.81\%$$

And the expected population mean of redshift detection: $\overline{Q}$:

$$\overline{Q} = 1 - \overline{P} \approx 92.19\%.$$

A table of accumulated probability distribution of blueshift detection is listed in Section 11. Referring to the table, we can calculate the probability of blueshift detection within a range of observation.

For example, the expected Doppler detection of objects located in near half of the observable universe:

$$\begin{aligned} blueshift: P(r \leq 0.5) &= 0.48867201\%, \\ redshift: Q(r \leq 0.5) &= 1 - P(r \leq 0.5) \\ &= 99.51132799\%. \end{aligned}$$

And, the expected probability of Doppler detection of objects located in far half of the observable universe:

$$\begin{aligned} blueshift: P(0.5 \leq r \leq 1) &= P(r \leq 1) - P(r \leq 0.5) \\ &= (7.81406258 - 0.48867201)\% \\ &= 7.32539057\%, \\ redshift: P(0.5 \leq r \leq 1) &= 1 - 7.32539057\% \\ &= 92.67460943\%. \end{aligned}$$

6 Underlying Conditions

The function obtained here is the probability of a object coming toward (blueshift) or moving away (redshift) from the observer's prospective, *i.e.* the physical displacement of the object in reference to the observer. The information obtained from the observation is centered at the observer.

It is assumed that the distance of observation is always changing, due to the spiral trajectory of observer gives very little chance of fixed distance of observation, and next to zero in long run. That is, only blueshift or redshift will be detected.
Above all, it is under the conditions that,

- there is no attenuation of radiation over distance,
- observer remains fixed (or the action of the observer does not alter the information obtained from the observation), and
- the observer has perfect vision.

However, none of which is true in heterogeneous universe.

7 Probability Function in Surface Observation

In a two dimensional surface case, the area of the blueshift range is shown in Figure 5.

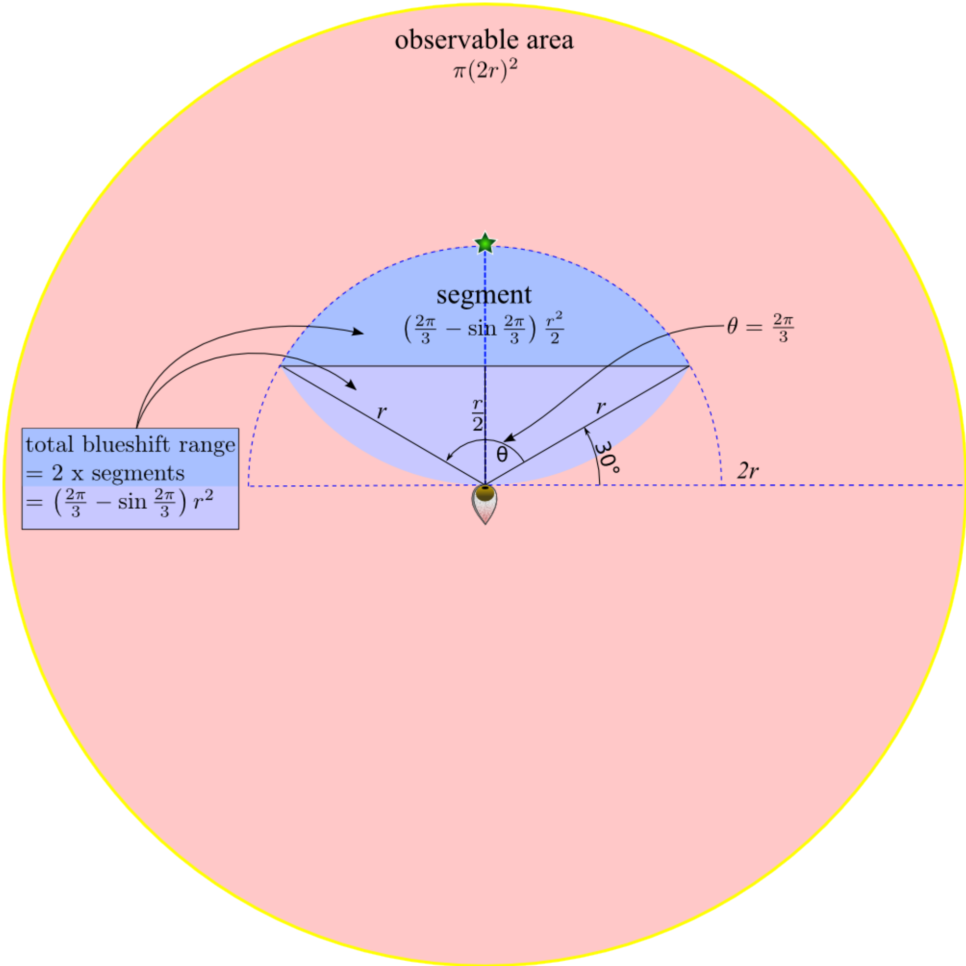

Figure 5: Calculations of Blueshift Range in Surface Observation

From the formula of calculating the area of segment:

$$\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)\frac{r^2}{2}$$

and the blueshift range is twice,

$$\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)r^2$$

Standardize the equation by setting the radius of the observable area to one, the total observable area becomes (π). The probability of the blueshift detection for any object at the location (r) can be obtained with the following probability density function:

$$
\begin{aligned}
p(r) &= \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)\frac{r^2}{\pi} \\
&= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\frac{r^2}{\pi} \\
&= \left(\frac{2}{3} - \frac{\sqrt{3}}{2\pi}\right)r^2 \\
&\approx 0.391r^2 \\
& \quad \textit{where } (0 \le r \le 1)
\end{aligned}
$$

The probability distribution of blueshift observation on surface is shown in Figure 6.

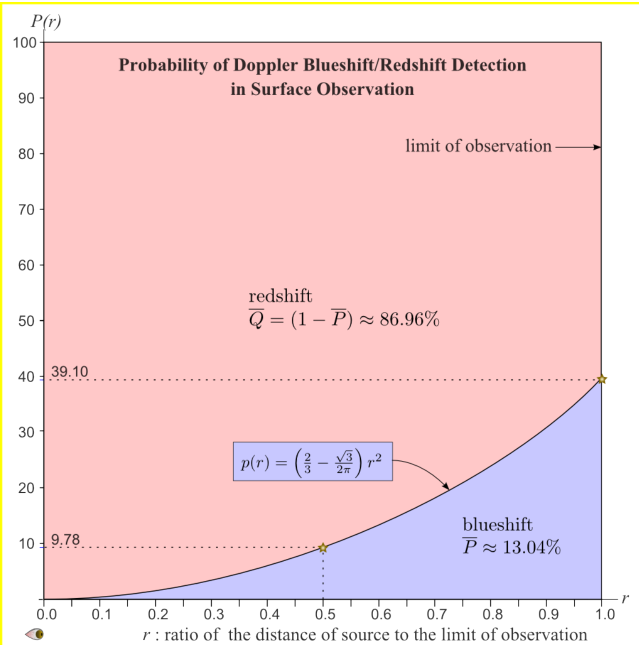

Figure 6: Probability Distribution of Blueshift Detection in Surface Observation

The population mean of blueshift detection in surface observation:

$$\int_0^1 \left(\frac{2}{3} - \frac{\sqrt{3}}{2\pi}\right)r^2 \mathrm{d}r$$

Computing the integral yields the expected population mean of blueshift detection, $\overline{P}$:

$$\overline{P} = \sum_{r=0}^{1} \left(\frac{2}{3} - \frac{\sqrt{3}}{2\pi}\right)r^2 \Delta r \approx 13.04\%$$

And the expected population mean of redshift detection $\overline{Q}$:

$$\overline{Q} = (1 - \overline{P}) \approx 86.96\%$$

Here we can see the minimum probability of blueshift is 0.0%, and the maximum 39.1% of observable area; and redshift is in the range of [60.9, 100]%.

8 Probability Function in Linear Observation

The assumption of 50:50 redshift to blueshift posts a problem even in linear observation. Let say all objects in west (left) side of an observer has equal chance to go eastward or westward in linear path. The westward action will go left toward the west limit of observation. The eastward action will go right, pass the observer, and continue to the east limit of observation as depicted in Figure 7.

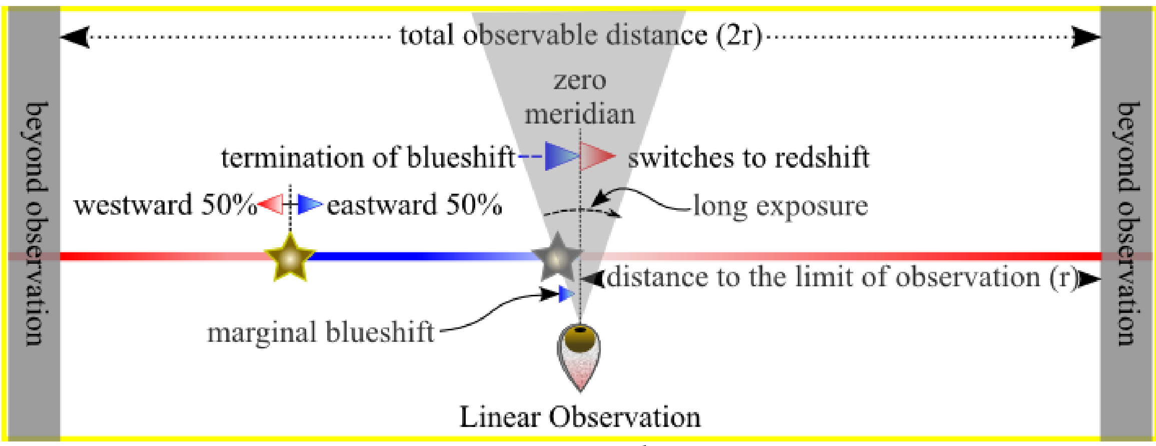

Figure 7: Linear Observation

The westbound action shows all redshift through out. However, the eastbound action shows blueshift for a duration, then switches to redshift. In long term, the observer will have chance of 1:2 blueshift to redshift, or possible $\frac{1}{3}$ of blueshift observation simply by count.

However, Doppler effect is not a simple survey of counting redcoats or bluecoats. It requires duration of observation to identify whether it is blue or red. The issue is blueshift deserts when crosses the zero meridian. Particularly in long tracking observation, *i.e.* when the object is kept at a fixed location in viewing frame for long duration of observation (long exposure and multiple exposures[1], shown as gray triangle in Figure 7), marginal blueshift radiation from eastbound object near the zero meridian (shown as gray star in Figure 7) can be washed or overridden by the subsequent redshift recorded in the same frame of exposure. Hence, the probability of blueshift observation can only be lower, when it is based on the duration of observation (exposure), as the same method used in surface and space observation earlier.

Since blueshift terminates at zero meridian, the blueshift duration is $[0, r]$, hence the average blueshift observation is $\frac{r}{2}$ based on the total observable distance of $2r$. Which means the average probability of blueshift detection is 25% ($\frac{r}{2} \div 2r$). When standardized, the probability of the blueshift detection for any object at the location (r) can be obtained with the following probability density function:

$$p(r) = \frac{r}{2}, \; where \; (0 \leq r \leq 1)$$

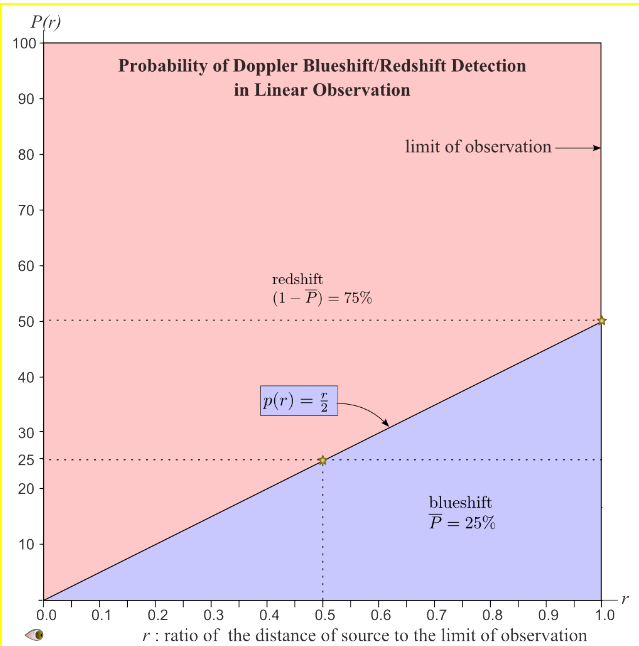

Figure 8: Probability Distribution of Blueshift Detection in Linear Observation

Figure 8 shows the expected population mean of blueshift detection in linear observation, $\overline{P} = 25\%$, with the minimum of 0% and maximum of 50%. And, redshift detection $(1 - \overline{P}) = 75\%$ within the range of $[50, 100]\%$. Redshift remains dominating, even in linear observation. The 50:50 presumption is invalid in an observation when sample can switch

[1]The Hubble Ultra Deep Field snapped 800 exposures, two exposures per orbit, averaged 21 minutes per exposure. It amounted to about 1 million seconds or 11.3 days of viewing time. The exposures were taken over four months, from Sept. 24, 2003 to Jan. 16, 2004, Hubble's Deepest View Ever.[2]

it's attributes, blueshift switching to redshift in this case.

9 Summary

Probability of Doppler Blueshift Detection at Location r, $(0 \le r \le 1)$

Blueshift Detection	Probability Function $p(r)$	Population Mean (%)	Mode (%)	Range (%)
Space	$\frac{5}{16}r^3$	7.81	31.25	0.00 – 31.25
Surface	$\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\frac{r^2}{\pi}$	13.04	39.10	0.00 – 39.10
Linear	$\frac{r}{2}$	25.00	50.00	0.00 – 50.00

Table 1: Blueshift Probability Functions

Probability of Doppler Redshift Detection at location r, $(0 \le r \le 1)$

Redshift Detection	Probability Function $1 - p(r)$	Population Mean (%)	Mode (%)	Range (%)
Space	$1 - \frac{5}{16}r^3$	92.19	100	68.75 – 100
Surface	$1 - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\frac{r^2}{\pi}$	86.96	100	60.90 – 100
Linear	$1 - \frac{r}{2}$	75.00	100	50.00 – 100

Table 2: Redshift Probability Functions

$$r = \frac{\textit{distance of observation}}{\textit{limit of observation}} = \frac{\textit{distance of object}}{\textit{observable universe}}$$

From the table, we can see the probability of redshift detection is highest in three-dimensional space. I believe it is due to more freedom for object to move around. To verify the redshift probability function in three-dimensional space, a simple cross check with redshift survey in NED's database is shown in Appendixes II, Section-10.

The truth is, Doppler blueshift detection is terminal event. It will terminate and switch to redshift at zero meridian, even there is a collision with the observer. On the other hand, redshift could continue beyond the limit of observation. This natural of Doppler effect makes redshift detection dominating in all observations. In cosmic observation, only 7.81% of the observers in the universe will detect blueshift of radiations from any object, on average. The remaining 92.19% of the observers in the universe will detect redshift. It is universal for all observers, aliens or Earthlings at all locations of the universe.

The probability function of blueshift is the risk of collision. It is also the chance to interact in dynamics. I believe it can be use to study risk vs. survival, navigation, dynamic interaction, and other applications.

10 Appendixes I - Cross-Check with Redshift Survey

The redshift Survey of NASA/IPAC Extragalactic Database (NED)[3] is used here as an example of redshift probability distribution.

Objects found in NED's list, November 2, 2014		
redshift objects ($z > 0$)	5,166,694	1.097%
blueshift objects ($z < 0$)	9,334	0.002%
marginal objects ($z = 0$)	2,939	0.001%
Total objects with redshifts	5,178,967	1.100%
Total objects without redshifts	465,814,004	98.900%
Total objects found	470,992,971	100.000%

Table 3: NASA/IPAC Extragalactic Database (NED)

If we only compare the objects with redshifts:

Objects with redshifts		
redshift objects ($z > 0$)	5,166,694	99.763%
blueshift objects ($z < 0$)	9,334	0.180%
marginal objects ($z = 0$)	2,939	0.057%
Total objects with redshifts	5,178,967	100.000%

Table 4: Redshift Objects (NED)

The spiral trajectory of observer gives very little change of fixed distance of observation, and next to zero in long run. The 2,939 ($z = 0$) objects are assumed marginal, and it will become clear in follow-up observation. The possibility can be one of the these:

1. Passing-by blueshifted objects (incoming object near the zero meridian, or perihelion, from the observe). They will show redshift in follow-up observation.

2. Redshifted objects at returning orbit (departing object at the far-end zero meridian, or aphelion, from the observe). They will show blueshift when curve back to an adequate distance in follow-up observation.

3. Inadequate detection by various causes.

However, it is expected that 92.19% of these 2,939 objects will shown redshift ($z > 0$) in follow-up observations. Despite the objects with redshifts is only a very small portion of the total objects found, the high count of redshift ($z > 0$) comparing to blueshift ($z < 0$) confirms the dominating nature of redshift.

Noted that Doppler effect is depending on the velocity not the location of the object, detected Doppler redshifts do not have to be proportional to the distance. However, NED's observation shows the positively proportional distribution of redshifts. It suggests something else more than just Doppler effect.

The very large objects without redshift, 465,814,004, is assumed inadequate detection. The distance of celestial observation can only be considered changing constantly. There will always be Doppler effect ($z \neq 0$) despite it can be beyond detection. Inadequate detection can be the result of interference of the environment and the capability of our technology of detection. Even if we ignore this very high portion (98.9%) of objects without redshifts, but, it is also expected that 92.19% of them will have ($z > 0$) when it can be obtained.

On the other hand, only 1.1% of objects with redshifts are detected, I doubt that we have sufficient information of hastening the fate of the universe and it's past.

11 Appendixes II - Accumulated Probability Distribution

Table 5: Accumulated Probability Distribution of Blueshift Detection, P(r)

r	(%)	r	(%)	r	(%)
0.00	0.00	0.01	0.00000008	0.02	0.00000128
0.03	0.00000641	0.04	0.00002000	0.05	0.00004883
0.06	0.00010125	0.07	0.00018758	0.08	0.00032000
0.09	0.00051258	0.10	0.00078125	0.11	0.00114383
0.12	0.00162000	0.13	0.00223133	0.14	0.00300125
0.15	0.00396564	0.16	0.00513281	0.17	0.00654045
0.18	0.00821949	0.19	0.01020278	0.20	0.01252502
0.21	0.01522279	0.22	0.01833455	0.23	0.02190063
0.24	0.02596323	0.25	0.03056644	0.26	0.03575621
0.27	0.04158038	0.28	0.04808864	0.29	0.05533259
0.30	0.06336567	0.31	0.07224323	0.32	0.08202246
0.33	0.09276244	0.34	0.10452414	0.35	0.11737038
0.36	0.13136587	0.37	0.14657719	0.38	0.16307280
0.39	0.18092303	0.40	0.20020009	0.41	0.22097805
0.42	0.24333287	0.43	0.26734239	0.44	0.29308631
0.45	0.32064620	0.46	0.35010554	0.47	0.38154965
0.48	0.41506573	0.49	0.45074286	0.50	0.48867201
0.51	0.52894600	0.52	0.57165955	0.53	0.61690922
0.54	0.66479348	0.55	0.71541267	0.56	0.76886897
0.57	0.82526648	0.58	0.88471116	0.59	0.94731083
0.60	1.01317520	0.61	1.08241585	0.62	1.15514624
0.63	1.23148169	0.64	1.31153942	0.65	1.39543851
0.66	1.48329991	0.67	1.57524646	0.68	1.67140285
0.69	1.77189568	0.70	1.87685339	0.71	1.98640633
0.72	2.10068668	0.73	2.21982855	0.74	2.34396787
0.75	2.47324250	0.76	2.60779212	0.77	2.74775832
0.78	2.89328456	0.79	3.04451617	0.80	3.20160035
0.81	3.36468619	0.82	3.53392464	0.83	3.70946854
0.84	3.89147259	0.85	4.08009336	0.86	4.27548933
0.87	4.47782081	0.88	4.68725002	0.89	4.90394104
0.90	5.12805982	0.91	5.35977419	0.92	5.59925386
0.93	5.84667042	0.94	6.10219731	0.95	6.36600987
0.96	6.63828530	0.97	6.91920270	0.98	7.20894300
0.99	7.50768905	1.00	7.81406258	r	1/100

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