AUT Journal of Modeling and Simulation

AUT J. Model. Simul., 49(1)(2017)85-94
DOI: 10.22060/miscj.2016.827

Uncertainty Measurement for Ultrasonic Sensor Fusion Using Generalized Aggregated Uncertainty Measure 1

A. Mohammad-Shahri* and M. Khodabandeh

Department of Electrical Engineering, Iran University of Science & Technology, Tehran, Iran

ABSTRACT: In this paper, target differentiation based on the pattern of data which are obtained by a set of two ultrasonic sensors is considered. A neural network based target classifier is applied to these data to categorize the data of each sensor. Then the results are fused together by Dempster–Shafer theory (DST) and Dezert–Smarandache theory (DSmT) to make a final decision. The Generalized Aggregated Uncertainty measure named GAU1, as an extension to the Aggregated Uncertainty (AU), is used to evaluate DSmT. Then the GAU1 and AU as the uncertainty measures are applied to the obtained results of the decision makers to evaluate DSmT and DST accordingly. The introduced configuration for decision making has enough flexibility and robustness to use as a distributed sensor network.

Review History:

Received: 9 July 2016
Revised: 23 September 2016
Accepted: 24 September 2016
Available Online: 1 October 2016

Keywords:

Target classification
DST
DSmT
ultrasonic sensor
uncertainty measure

1- Introduction

Detection of standard landmarks or some objects that are similar to pre-known targets helps solve the problems of map building and localization by an autonomous mobile robot. Algorithms to be applied for localizing the pose of target surfaces are different depending on their types. That is, the type of target surface should be classified before the localization process.

Different sensors provide different kinds of information which should be fused together in order to obtain a complete picture of the real world. More specifically, multi-sensor data fusion aims to overcome the limitations of individual sensors and produce an accurate, robust and reliable estimate of the world state based on multi-sensory information [1]. Information often contains uncertainties which are usually related to physical constraints, detection algorithms, and the transmitting channel of the sensors [2].

Sonar is a very useful and cost-effective mode of sensing for mobile robots [3]. The multi-ultrasonic sensor based mobile robot has been widely discussed by many researchers [3-11]. Two main issues, map building, and autonomous navigations are discussed in most of papers and theses. The feature based map building methods [4, 5] attempt to model the geometric features of the environment according to the sensor responses. In [8], measurement scheme is proposed which uses only two sets of ultrasonic sensors to determine the location and the type of target surface. This study concentrates on target differentiation based on the pattern of data which are obtained by a set of two ultrasonic sensors, including two transmitters and two corresponding receivers.

Neural networks have been employed efficiently as pattern classifiers in numerous applications [3, 6, 7]. Homg proposed an effort to apply the several multi-class classifiers that are the maximum likelihood classifier, the radial basis function neural network, the fuzzy support vector machine and the error correcting output codes method to classify the ultrasonic supraspinatus images [9]. Neural networks have been used to process amplitude and TOF information of a set of ultrasonic sensors in order to reliably handle the target classification problem [3]. Similarly, echo signal amplitude, TOF information and the differences of these data are used based on neural networks to do classification task [12]. A similar task is performed in using target classification by employing TOF information of the sensors. After acquiring the data of sensors, the classification of different targets by using neural networks would be done for outcomes of each sensor. Afterward, the results are fused together to make a final decision. This configuration for target classification with sensor fusion has sufficient flexibility and robustness to be used as distributed sensor networks.

The evidence theory, also known as Dempster–Shafer theory [13], is one of the most popular frameworks to deal with uncertain information. This theory is often presented as a generalization of probability theory, where the additivity axiom is excluded. The evidence theory allows each subset of the universe to have a non-null confidence and not only the singletons as in the probability theory [14]. In the evidence theory, the singletons as in the probability theory have non-null confidence. The theory has some limitations in high conflict problems. In [15], alternative combination rules have been proposed to resolve the appeared conflicts of evidence.

The Dezert–Smarandache Theory (DSmT) is a theory of plausible and paradoxical reasoning proposed by Dezert and Smarandache in recent years [16-19]. It can be considered as an extension of the classical Dempster–Shafer theory (DST) [13] but with fundamental differences. DSmT allows formally combining any types of independent sources of information represented in terms of belief functions, while it is mainly

Corresponding author, E-mail: shahri@iust.ac.ir

focused on the fusion of uncertain, highly conflicting and imprecise sources of evidence. There are some successful applications of DSmT in target type tracking [20] and robot map building [9,21]. Also, a DSmT–AHP based multi-criteria decision making is proposed in [22]. Some other applications of DSmT in classification problem are reported such as [23]. The new advances and applications of DSmT for information fusion are collected in [24].

As uncertain information often exists on all levels of fusion process [2], it is important to have an uncertainty evaluation after sensor fusion for a better decision making. Moreover, when sensor measurements are used in a sensor fusion framework to fuse the data, uncertainty analysis within this framework would be a useful tool to help final decision-making if an appropriate way exists to measure all kinds of uncertainties. In 1928 and 1948, Hartley [25] and Shannon [26], respectively established the field of information theory and developed information entropy as a measure of redundancy. Hartley measure and Shannon entropy have been used in the possibilities and probabilities frameworks, respectively. Based on these approaches, information or preferably uncertainty-based information can be quantified by different general measures commonly called measures of uncertainty [27].

Several theories have been developed to deal with uncertainty such as probability theory, fuzzy sets theory, possibility theory, evidence theory, and rough sets theory. Instead of opponents, they should rather be seen as complementary, each of them being designed for dealing with different types of uncertainty. Three main types of uncertainty have been identified by Klir and Yuan [28]: fuzziness, conflict, and non-specificity, the latter two are unified under the term ambiguity.

Different measures of ambiguity often called measures of total uncertainty have been proposed [29-35]. Among them, Maeda et al. [36] followed by Harmanec and Klir [37] proposed a measure of aggregated uncertainty named AU. This measure is defined in the framework of the evidential theory that aggregates the non-specificity and conflict. It has been proved that this measure satisfies the five requirements defined by Klir and Harmanec [37,38]. Bronevich and Klir , within a broad range of theories of imprecise probabilities, have formalized the notion of a total aggregated measure of uncertainty and various dis-aggregations into measures of non-specificity and conflict [39]. As another uncertainty measure, Jousselme et al. [14] introduced a new measure of aggregated uncertainty, named AM for Ambiguity Measure that aims at eliminating the shortcomings of AU such as computing complexity. By AM, an alternative for measuring ambiguity in Dempster–Shafer theory is offered. But actually, their proposed measure, AM, is not, in a general sense, sub-additive. Klir and Lewis [40] showed this by a specific counterexample which clearly demonstrates that their assumption in the last step of the proof is incorrect and that AM indeed violates sub-additivity.

In spite of efficiency of AU measure, this uncertainty measure and its associated algorithm for computing, presented by Harmanec [38] are devoted for DST framework and cannot be applied to DSmT. Vatsa et al. used DSmT to fuse fingerprint information [41] and consequently, based on the pignistic probability (BetP) and likelihood ratio test, a decision is made to accept or reject [42]. They proposed a contextual unification framework to dynamically select the most appropriate evidence-theoretic fusion algorithm for a given scenario.

Two generalized AU measures named GAU1 and GAU2 have been introduced by the authors [43]. It is proved that the new measures have enough efficiency to evaluate the DSmT based results.

In order to evaluate the new measure GAU1 in a target classification problem, an experimental setup based on ultrasonic sensors is configured. Neural networks are used as data level in the first level of fusion. Neural networks are trained by acquired data of the set of ultrasonic sensors and then outputs are used by the decision maker based on a thresholding and statistical algorithm to perform the differentiation task. Finally, the obtained results of the DSmT based decision maker are evaluated by the uncertainty measure GAU1. The other uncertainty measure, i.e. GAU2 has been previously examined by the authors for the target differentiation problem in [44].

This paper is organized as in the following: the sensor fusion frameworks, DST and DSmT considered in uncertainty analysis are reviewed in section 2. Section 3 is devoted to a discussion of the AU measure which is the most important uncertainty measure for DST framework presented till now. Moreover, the Generalized AU measure, GAU1, for DSmT is represented in this section. In section 4, experimental studies are carried out on uncertainty measurement for a target classification problem. Finally, some concluding remarks are presented in section 5.

2- Evidential Reasoning Frameworks

2- 1- Dempster–Shafer Theory

In this theory, $\Theta=\{\theta_1,\theta_2,\ldots,\theta_n\}$, assumed to be the frame of discernment of the fusion problem under consideration having n exhaustive and exclusive elementary hypotheses θ_i [13]. This corresponds to Shafer's model of the problem.

The DST framework power set 2^Θ is defined as the set of all composite propositions built from elements of Θ with $\cup$ operator such that:

1- $\varnothing,\theta_1,\theta_2,\ldots,\theta_n\in 2^\Theta$
2- if $A,B\in 2^\Theta$ then $A\cup B\in 2^\Theta$
3- No other elements belong to 2^Θ, except those obtained by using rules 1 or 2.

In Shafer's model, a basic belief assignment (bba) $m(.):2^\Theta\rightarrow[0,1]$ associated to a given body of evidence A is defined by:

$$m(\varnothing)=0,\ \sum_{A\in 2^\Theta} m(A)=1,\quad 0\le m(A)\le 1 \tag{1}$$

Shafer defines the belief and plausibility functions of $A\subseteq\Theta$ as

$$Bel(A)=\sum_{B\in 2^\Theta, B\subseteq A} m(B) \tag{2}$$

$$Pl(A)=\sum_{B\in 2^\Theta, B\cap A\neq\varnothing} m(B)=1-Bel(\bar{A}) \tag{3}$$

where $\bar{A}$ denotes the complement of the proposition A in Θ.

Dempster's rule of combination is defined by:

$$m(\varnothing)=0,$$

$$m(A)=\frac{\sum_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y=A}} m_1(X)m_2(Y)}{1-\sum_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y=\varnothing}} m_1(X)m_2(Y)},\forall(A\neq\varnothing)\in 2^{\Theta} \tag{4}$$

m(.) is a proper basic belief assignment if and only if the denominator in equation (4) is non-zero. The degree of conflict between two sources is defined by [13]:

$$k_{12}=\sum_{\substack{X,Y\in 2^{\Theta}\\ X\cap Y=\varnothing}} m_1(X)m_2(Y) \tag{5}$$

2- 2- Dezert–Smarandache Theory

Dezert–Smarandache Theory is a theory of plausible and paradoxical reasoning [17-19,24]. The development of DSmT arises from the necessity to overcome the inherent limitations of Dempster–Shafer Theory (DST) [13] which are closely related to the acceptance of Shafer's model for the fusion problem under consideration. The foundation of DSmT is based on the definition of the Dedekind's lattice D^{Θ} also called hyper-power set of the frame Θ in the sequel. In the DSmT framework, Θ is firstly considered as only a set $\{\theta_1,\theta_2,\ldots,\theta_n\}$ of n exhaustive elements in closed world assumption without introducing other constraints, such as exclusivity or non-existential constraints. This corresponds to the free DSm model on which the classic DSm rule of combination is based. DSmT starts with the notion of free DSm model, denoted $M^f(\Theta)$.

Depending on the intrinsic nature of the elements of the fusion problem under consideration, it can, however,, happen that the free model does not fit the reality. These integrity constraints are then explicitly and formally introduced into the free DSm model $M^f(\Theta)$ in order to adapt it properly to fit as close as possible with the reality and permit to construct a hybrid DSm model $M(\Theta)$ on which the combination will be efficiently performed. DSmT has been presented to manage as efficiently and precisely as possible imprecise, uncertain and potentially high conflicting sources of evidence while keeping in mind the possible dynamicity of the information fusion problematic.

The Dedekind's lattice also called in the DSmT framework hyper-power set D^{Θ} is defined as the set of all composite propositions built from elements of Θ with $\cup$ and $\cap$ operators such that:

1- $\varnothing,\theta_1,\theta_2,\ldots,\theta_n\in D^{\Theta}$
2- if $A,B\in D^{\Theta}$ then $A\cup B\in D^{\Theta}$,$A\cap B\in D^{\Theta}$
3- No other elements belong to D^{Θ}, except those obtained by using rules 1 or 2.

From a general frame Θ, a map $m(.):D^{\Theta}\to[0,1]$ associated to a given body of evidence is defined as:

$$m(\varnothing)=0,\sum_{A\in D^{\Theta}} m(A)=1, 0\leq m(A)\leq 1 \tag{6}$$

The quantity m(A) is called the generalized basic belief assignment/mass (gbba) of A. The generalized belief and plausibility functions are defined in almost the same manner as within the DST [13], i.e.

$$Bel(A)=\sum_{B\in D^{\Theta},B\subseteq A} m(B) \tag{7}$$

$$Pl(A)=\sum_{B\in D^{\Theta},B\cap A\neq\varnothing} m(B) \tag{8}$$

These definitions are compatible with the definitions of classical belief functions in the DST framework when D^{Θ} reduces to 2^{Θ} for fusion problems where Shafer's model $M^0(\Theta)$ holds.When the free DSm model $M^f(\Theta)$ holds for the fusion problem under consideration, the classic DSm rule of combination $m_{M^f(\Theta)}=m(.)=[m_1\oplus m_2](.)$ of two independent sources of evidences over the same frame with belief functions $Bel_1(.)$, $Bel_2(.)$ associated with gbba $m_1(.)$, $m_2(.)$ corresponds to the conjunctive consensus of the sources. It is given by [17-19, 24]:

$$\forall C\in D^{\Theta},$$
$$m_{M^f(\Theta)}(C)=m(C)=\sum_{\substack{A,B\in D^{\Theta}\\ A\cap B=C}} m_1(A)m_2(B) \tag{9}$$

Since D^{Θ} is closed under $\cup$ and $\cap$ set operators, this new rule of combination guarantees that m(.) is a proper generalized belief assignment, i.e. $m(.):D^{\Theta}\to[0,1]$.

3- Uncertainty Measurement

Measuring uncertainty or information means assigning a number or a value from some ordinal scale to a given model of an epistemic state. Two types of classical evidential based uncertainties, non-specificity and conflict are often measured as part of the fusion techniques such as DST fusion [38]. One of the most appropriate uncertainty measures which are developed in DST frameworks is the Aggregate Uncertainty (AU) measure. Algorithm for computing AU was originated by Harmanec [38]. The algorithm is applied to DST framework while it cannot be used for DSmT directly. To cover the problem, two Generalized Aggregate Uncertainty measures named GAU1 and GAU2 have been developed to measure uncertainty in DSmT framework [43]. In the next section, the GAU1 measure is reviewed in brief.

As already mentioned, DSmT overcomes the limitation of DST in the Shafer's model. In the DST, the frame of discernment of the fusion problem under consideration assumed to have exhaustive and exclusive elementary hypotheses but in DSmT these conditions are violated. For example, in a three-dimensional frame of discernment i.e. $\Theta=\{\theta_1,\theta_2,\theta_3\}$, similar to the classification problem mentioned in section 4, power set of DST will be:

$2^{\Theta}=\{\varnothing,\theta_1,\theta_2,\theta_3,\theta_1\cup\theta_2,\theta_1\cup\theta_3,\theta_2\cup\theta_3,\theta_1\cup\theta_2,\theta_3\}$

and in DSmT the associated set which is called hyper power set will be:

$D^{\Theta}=\{\varnothing,\theta_1,\theta_2,\theta_3,\theta_1\cup\theta_2,\theta_1\cup\theta_3,\theta_2\cup\theta_3,\theta_1\cup\theta_2\cup\theta_3,\theta_1\cap\theta_2,\theta_1\cap\theta_3,\theta_2\cap\theta_3,\theta_1\cap\theta_2\cap\theta_3,\theta_1\cap(\theta_2\cup\theta_3),\theta_2\cap(\theta_1\cup\theta_3),\theta_3\cap(\theta_1\cup\theta_2),\theta_1\cup(\theta_2\cap\theta_3),\theta_2\cup(\theta_1\cap\theta_3),\theta_3\cup(\theta_1\cap\theta_2),(\theta_1\cap\theta_2)\cup(\theta_1\cap\theta_3)\cup(\theta_2\cap\theta_3)\}$.

Figure 1 shows Shafer's model and free DSmT model for the three-dimensional framework.

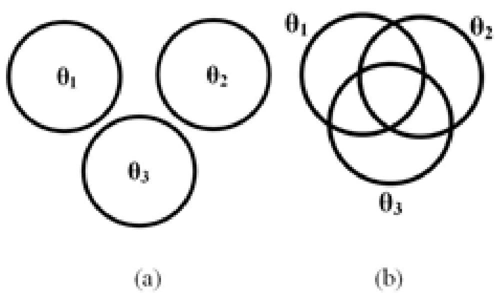

Fig. 1. Three-dimensional framework; (a) Shafer's model, (b) Free DSm model

3- 1- Generalized Aggregated Uncertainty Measure 1

Here, the idea of generalizing Aggregated Uncertainty measure in GAU1 to evaluate DSmT is explained for a 2-D problem for simplicity. In the DST framework and the algorithm of computing AU measure for a 2-D problem, such as frame of discernment $\Theta=\{A,B\}$, there are two distinct sets. The power used in DST is $\{\varnothing,A,B,A\cup B\}$ and hyper-power set in DSmT is $\{\varnothing,A,B,A\cup B,A\cap B\}$. In order to have generalized AU measure, one can apply the AU computing algorithm to DSmT by disjointing free DSm model to separated sets as shown in Figure 2. Extending the frame of discernment $\Theta=\{A,B\}$ in the computing algorithm to an extended frame $\Theta_E=\{A_E=A-B,B_E=B-A,A\cap B\}$ is the idea of the generalization. Index E is used as abbreviation of "Exclusive" and A_E,B_E are exclusive event A and exclusive event B without any community. Also one may define:

$$\theta_{E_1}=A_E,\theta_{E_2}=B_E,\theta_{E_3}=A\cap B\left(\theta_E=\theta_{E_1} or\ \theta_{E_2}\ or\ \theta_{E_3}\right)$$

and therefore:

$$2^{\Theta_E}=\{\varnothing,\theta_{E_1}=A_E,\theta_{E_2}=B_E,$$
$$\theta_{E_3}=A\cap B,\theta_{E_1}\cup\theta_{E_3}=A,$$
$$\theta_{E_2}\cup\theta_{E_3}=B,\theta_{E_1}\cup\theta_{E_2}=A\cup B\}$$

The set $2^{\Theta E}$ is used for the new power set which is obtained by extension of the frame of discernment Θ to Θ_E considering events without any community.

Definition 3.1. The measure of the Generalized Aggregated Uncertainty contained in Bel, denoted as GAU1(Bel), is defined by

$$GAU1(Bel)=max\left\{-\sum_{\theta_E\in\Theta_E}p_{\theta_E}\log_2 p_{\theta_E}\right\} \quad (10)$$

where the max{.} is taken over all $\{p_{\theta_E}\}_{\theta_E\in\Theta_E}$ such that $p_{\theta_E}\in[0,1]$ for all $\theta_E\in\Theta_E$, $\sum_{\theta_E\in\Theta_E}p_{\theta_E}=1$ and for all $A\subseteq\Theta_E$, $Bel(A)\leq\sum_{x\in A}p_x$ [43].

In this manner, the main problem of fusion still stays 2-D whereas three separated events are created such as a Shafer's model with 3 events.

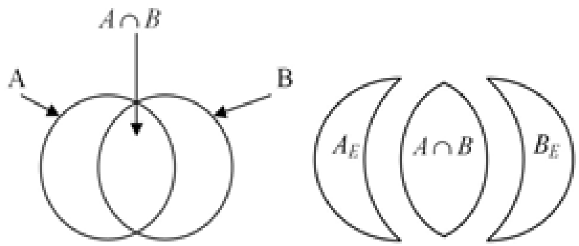

Fig. 2. Disjointing of two-dimensional framework of free DSm model to excluded sets

It must be mentioned that the refinement which is used in the presented GAU1 does not work for any frame of discernment. Clear frontiers in frames are necessary to use GAU1 as an uncertainty measure.

4- Experimental Study: Ultrasonic Sensor For Target Classification

4- 1- Experiment Setup

Ultrasonic sensors have been widely used to recognize the working environment for a mobile robot. However, because of their intrinsic problems, such as the specular reflection, the wide beam angle, and the slow propagation velocity, an excessive number of sensors are required to be integrated to achieve the various sensing goals.

In the commonly used TOF systems, an echo is produced when the transmitted pulse encounters an object. A range value= $ct_0/2$ is measured when the echo amplitude first exceeds a preset threshold level τ back at the receiver at time t_0. Here, t_0 is the TOF and c is the speed of sound in air. Speed of sound in the air at room temperature is $c=343\ m/s$.

In the experimental setup, as shown in Figure 3, two identical acoustic transmitter/receiver pairs A and B with center-to-center separation d are employed to improve the angular resolution. Each pair detects echo signals reflected from targets within its own sensitivity region. Both of the sensors can detect targets located within the joint sensitivity region which are shown in Figure 4.

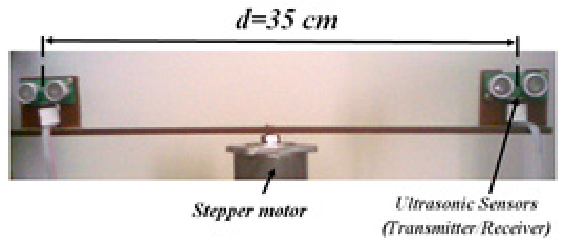

Fig. 3. Ultrasonic sensor setup

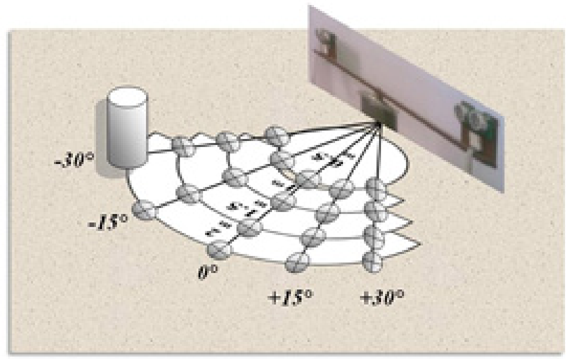

Fig. 4. Experiment setup and 20 different positions of targets

The common targets are considered to exist in a real environment of mobile robot applications such as *Plane*, *Cylinder*, and *Corner* with *90°* angle. The targets are shown in Figure 5. These artificial targets are similar to walls and corners at home as an indoor environment or natural targets, such as trees in an outdoor environment. The experiment is performed in an indoor environment. Detailed physical reflection models of these targets primitives with corresponding echo signal models are provided [45].

Two pairs of the SRF04 ultrasonic ranger are used. The SRF04 provides an echo pulse proportional to distance. Considering the measured pulse width in µSec and the relation of sound speed with the range, it is clear that; $1\mu Sec/58=1cm$. The center-to-center separation of the transducers used in the experiments is d=35cm. The entire sensing unit is mounted on a small stepper motor with the 1.8° step size which its motion is controlled by a microcontroller system. Echo signals of acquired data from the sensors are processed to calculate the distance of targets.

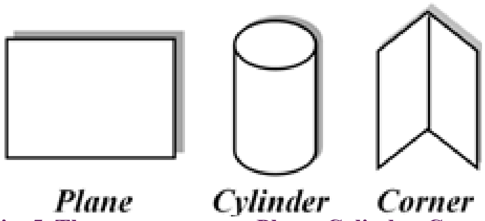

Fig. 5. Three target types; Plane, Cylinder, Corner

The targets employed in this study are cylinders with diameter 20 cm, a planar target, and a 90° corner. As shown in Figure 4, TOF data are collected at 20 sensor locations which are located at 5 different angles from φ=–30° to φ=+30° in 15° increments, and from r=0.5m to r=2m in 0.5m increments separately. To disaffect the distances of targets in target classification, the data are normalized regarding distance. Figure 6 indicates the normalized data of ultrasonic sensors for these 20 positions. In this manner, one may classify the targets regardless of the mentioned positions. Consequently, the dependency to the distance in target classification will be ignored.

4- 2- Neural Network based Target Classifier

The normalized distances of targets in the 20 positions are used to train neural networks. The network employed has one hidden, one input, and one output layer. Although there are many ways of choosing input signals to train the network, this study uses TOF signals of each sensor pair as input signals. The hidden layer comprises 50 neurons and hyperbolic tangent as nonlinear functions and linear functions at the output layer with 1 neuron. The output neuron provides a value which after applying a thresholding and statistical algorithm can be interpreted as the target type. The number of hidden layers was determined by a process known as enlarging, which starts with a relatively small number of neurons and increases the size of the hidden layer until learning occurs.

For each sensor, one set of data is collected for each target location for each target primitive, resulting in 60 (=4 ranges×5 angles×3 target types) sets of data. Data of all targets in 2 ranges (1m, 2m) and all 5 angles are used to train. The network is trained with these 30 sets of data using the back-propagation algorithm in multi-layer perceptron (MLP) network with a learning constant equal to 0.9, momentum constant equal to 0.5, and a sigmoid-type nonlinearity. In order to test the networks, each target primitive is placed in turn in each of the 20 locations shown in Figure 4. One set of measurements is collected for each combination of target type and angle and ranges 0.5m, 1.5m for each sensor again resulting in 30 sets of experimentally obtained data. The neural network estimates the target type from this data by considering a simple algorithm with appropriate thresholds and a frequency based statistical algorithm to distinguish the targets from probability values.

Table 1 gives the percentages of correct target type classification that are considered as the basic belief assignment of each sensor of the three targets for the case that the target object is "Plane". Accordingly, each sensor by using the trained neural network and the thresholding and statistical algorithm represents a quantity to differentiate the targets.

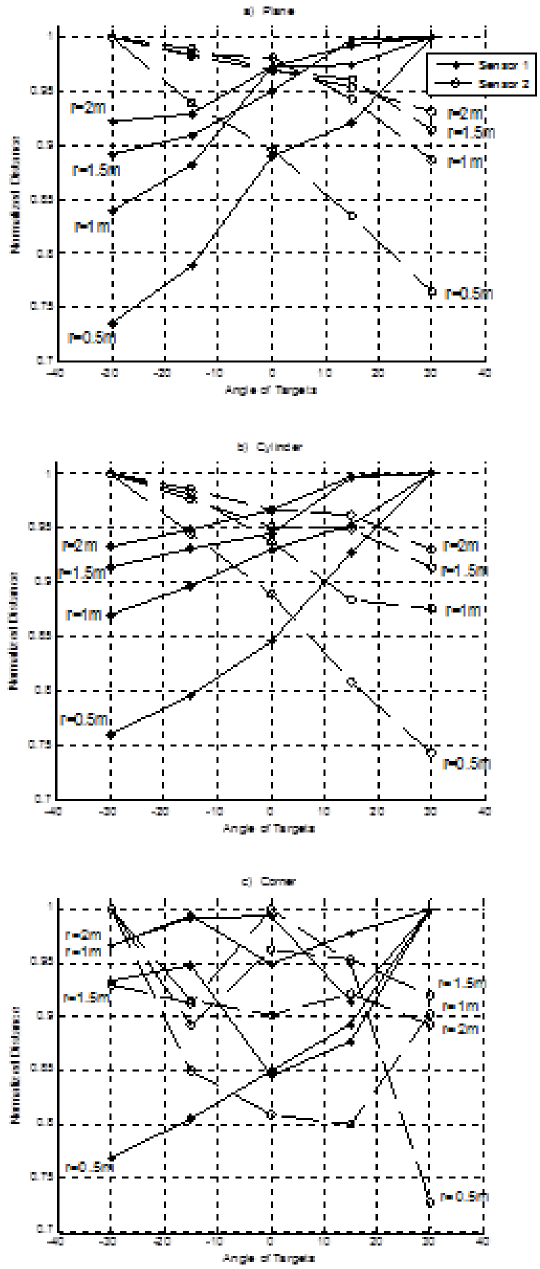

Fig. 6. Normalized data of ultrasonic sensors in 20 positions (4 ranges and 5 angles)

Table 1. Outputs of neural network based classifier as basic belief assignment; Target type: "Plane"

2^{Θ}	Sensor1	Sensor2
P	0.7333	0.5333
C_y	0	0.1333
C_o	0	0.0667
$P\cup C_y$	0.1333	0.1333
$P\cup C_o$	0.0667	0
$C_y\cup C_o$	0.0667	0.0667
Θ	0	0.0667

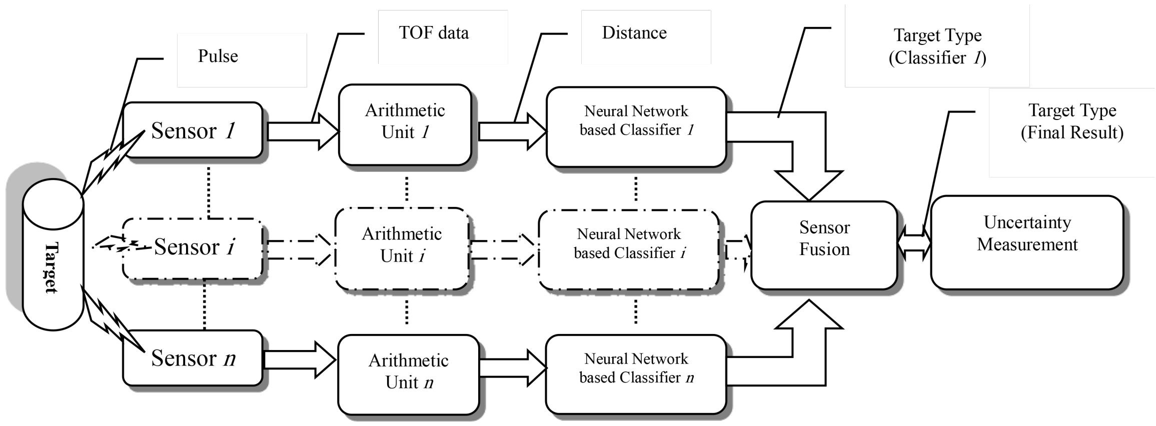

Fig. 7. Block diagram of the decision-making system

Table 2 represents results of a case that the target object is *"Corner"*. In these tables, *"P"* is used to represent *"Plane"*, *"Cy"* is for *"Cylinder"* and *"Co"* is for *"Corner"* and "Θ" is devoted to representing total ignorance i.e.$\Theta = P \cup C_y \cup C_o$.

4- 3- Sensor Fusion with Uncertainty Measurement

In this section, the mentioned sensor fusion algorithms, DST and DSmT, are applied to the results that are obtained by the neural network based classifier in order to differentiate the target types. After fusing the results of two sensors, an uncertainty measurement has been carried out according to AU measure for DST and GAU1 for DSmT.

4- 3- 1- Decision-Making System

Figure 7 shows a block diagram of the decision-making system. After gathering the TOF data of a target by ultrasonic sensors, some computations are performed by an arithmetic unit to present normalized distances for classifier unit. In this study, a neural network based classifier is used and the target type is the output of each classifier unit. The results of the classifiers are fused together in a sensor fusion unit and its final results are evaluated by an uncertainty measurement unit.

By this configuration, it is possible to extend the sensory system while there is no need to modify or train the neural networks again. In other words, if a neural network is trained by sufficiently rich data, it also can be used for classification of additional sensory systems. Rich data can be collected when the target objects are placed in several different angles and distances from the sensors setup. As shown in Figure 7, there could be several paths, including sensors, arithmetic units, and neural network based classifiers blocks before the fusion process. Because of this configuration, in the case of any failure in a path, there would be no problem in total decision making. It means that this configuration for target classification with sensor fusion has flexibility and robustness to some extent.

Table 2. Outputs of neural network based classifier as basic belief assignment; Target type: "Corner"

2^Θ	Sensor1	Sensor2
P	0	0
C_y	0.1333	0
C_o	0.8	1
$P \cup C_y$	0.0667	0

4- 3- 2- DST as Decision Maker and AU Measure

Table 1 shows the results of neural network based classifier for the "Plane" target. By applying DST according to equation (4), the following results are represented as final decision:

$$m_{DST}(P) = 0.8698, \quad m_{DST}(P \cup Cy) = 0.0355$$
$$m_{DST}(Cy) = 0.0592, \quad m_{DST}(P \cup Co) = 0.0059$$
$$m_{DST}(Co) = 0.0178, \quad m_{DST}(Cy \cup Co) = 0.0118$$
$$m_{DST}(\Theta) = 0$$

As it is clear, in a supposed Shafer's model, by applying DST more certain decisions have been obtained in comparison to the results of individual sensors. According to Table 2, when the target is *"Corner"* and DST is applied, final decision is: $m_{DST}(Co)=1$.

Now, for uncertainty measurement in DST fusion, the AU measure, and its computing algorithm is applied. Firstly, the uncertainty involved in the decision results of Sensors1&2 are computed by AU measure.

Table 3. Uncertainty measurement for sensor 1&2 and DST results using AU measure

Results	Sensor1	Sensor2	Sensor1+Sensor2	DST
Target: Plane	1.1035	1.2730	2.3765	0.6680
Target: Corner	0.7210	0	0.7210	0

4- 3- 3- DSmT as decision maker and GAU1 measure

In this section, DSm model is supposed for the previously mentioned study and DSmT fusion rule is applied to the results of sensors. For uncertainty measurement, the GAU1 is used for DSmT results. There are clear frontiers in the experiment used to target differentiation by ultrasonic sensors. Thus, GAU1 is applicable to evaluate uncertainty involved in DSmT results.

As already stated, when DSmT is used rather than DST, hyper-power set and free DSm model should be considered instead of power set and Shafer's model respectively. As a result, the number of events to be decided is more than a number of events in DST. These results show the capability of DSmT to overcome continuous problems. Also, more exactly decisions can be made by DSmT, particularly in conflict problems.

Table 4 illustrates the results of ultrasonic sensor fusion by DSmT according to equation (9) when the target is *"Plane"*.

These results show that the decisions are distributed to a larger set of sub-events. To measure uncertainty in DSmT results, the GAU1 measure is utilized. In a three-dimensional problem in GAU1, there are seven distinct sub-events as: $\theta_1=P_E, \theta_2=Cy_E, \theta_3=Co_E, \theta_4=(P\cap Cy)_E, \theta_5=(P\cap Co)_E, \theta_6=(Cy\cap Co)_E, \theta_7=P\cap Cy\cap Co$ where *"E"* is used to show exclusivity. For example P_E means exclusive P.

DSmT fusion showed its capabilities in continuous problems as well as problems with non-exclusive events rather than DST. Basically, DST has not enough efficiency to deal with problems with such models. On the other hand, advantages of using DSmT fusion should be studied in uncertainty point of view as well. To investigate the uncertainty improvement in the results of DSmT fusion, uncertainties in the results of each sensor have to be considered. The uncertainties in sensors1&2 are computed in section 4.3.2 by using AU measure. However, to have a meaningful comparison, these computations should be repeated by considering the DSm model and GAU1 measure.

As it is clear, DSmT fusion reduces the amount of uncertainty in final decisions. Uncertainty in DSmT fusion results is less than the sum of uncertainties in Sensors1&2 and even less than the uncertainty of each sensor.

For the further study, it is supposed in the second experiment that the target is *"Corner"*. Table 5 demonstrates the DSmT fusion results for *"Corner"* target. These results are more cautious results in comparison to the DST results that are presented in Table 2. Therefore, the final decisions are suitable in continuous problems and problems with non-exclusive propositions.

Table 4. Decisions made by DSmT-based fusion; Target type: "Plane"

D^Θ	m_{DSmT}
P	0.6444
Cy	0.0267
Co	0.0089
P∪Cy	0.0267
P∪Co	0.0044
Cy∪Co	0.0089
P∩Cy	0.0978
P∩Co	0.0489
Cy∩Co	0
P∩(Cy∪Co)	0.0844
Cy∩(P∪Co)	0.0089
Co∩(P∪Cy)	0.0089
P∪(Cy∩Co)	0.0089
Cy∪(P∩Co)	0.0178
Co∪(P∩Cy)	0.0044
P∩Cy∩Co	0
(P∩Cy)∪(P∩Co)∪(Cy∩Co)	0
Θ	0

For *"Corner"* target such as *"Plane"* target, uncertainty in DSmT fusion results are less than the sum of uncertainties in Sensors1&2 and even less than the uncertainty of each sensor.

Therefore, it can be concluded that DSmT has improved the results in uncertainty point of view.

It should be noted that the comparison in the uncertainty value between AU and GAU1 measures is not correct, because of the different models and frameworks which are used in their fusion theorems. As the hyper-power set has higher dimension than power set, measured uncertainty in GAU1 is more than AU when the final decision of different sensors are similar for an unknown target. However, in the case of conflict measurements, DSmT must be used instead of DST. Also, these experiments demonstrate that DSmT presents a smooth decision, especially in continuous models. Since AU is presented for DST and cannot be applied to the DSmT results, GAU1 is applicable as uncertainty measure for DSmT fusion results. Moreover, this study shows the efficiency of DSmT to improve the final results in uncertainty point of view.

Table 5. Decisions made by DSmT-based fusion; Target type: "Corner"

D^Θ	m_{DSmT}
P	0
Cy	0
Co	0.8
P∪Cy	0
P∪Co	0
Cy∪Co	0
P∩Cy	0
P∩Co	0
Cy∩Co	0.1333
P∩(Cy∪Co)	0
Cy∩(P∪Co)	0
Co∩(P∪Cy)	0.0667
P∪(Cy∩Co)	0
Cy∪(P∩Co)	0
Co∪(P∩Cy)	0
P∩Cy∩Co	0
(P∩Cy)∪(P∩Co)∪(Cy∩Co)	0
Θ	0

Table 6. Uncertainty measurement for sensor 1&2 and DSmT results using GAU1 measure

Results	Sensor1	Sensor2	Sensor1+Sensor2	DSmT
Target: Plane	2.64	2.7335	5.3735	2.3933
Target: Corner	2.7028	1.9056	4.6084	1.9056

5- Conclusions

This study focused on the uncertainty evaluation problem in decision-making systems. An experimental setup of ultrasonic sensors is established in this research to evaluate sensor fusion theories and their associated uncertainty measures. Total decision maker system is composed of parallel paths with some blocks such as sensors, arithmetic units, classifiers, sensor fusion and uncertainty measurement blocks. This configuration for target classification with sensor fusion has sufficient flexibility and robustness to use as distributed sensor network. A common neural network based classifier is used for each sensor path to get the classification results of the sensors.

DST and DSmT as efficient evidential reasoning theorems have been used in sensor fusion block. To evaluate the performance of DST in uncertainty point of view, the AU measure has been employed. Uncertainty evaluation of the fusion results obtained by DST illustrates that uncertainty involved in the final results are less than or equal to the sum of uncertainties in the sensors. However, DST has inherent limitations which are closely related to the acceptance of Shafer's model for the fusion problem under consideration. DSmT and its associated uncertainty measure, GAU1, are applied to the results of sensors. The final decision in the presented configuration has uncertainty less than each sensor's measurement. In the fusion processes, based on the frameworks of discernment more accurate results are made in final decisions by DSmT rather than DST. DSmT produces more precise and smoother decisions because of the free model which is applied. This makes the DSmT more applicable for continuous problems.

According to the presented results, the following conclusions may be drawn; DST and DSmT as decision makers are applied for target classification and they have presented appropriate results, especially the experiment shows the capability of DSmT for continuous models. GAU1 as an uncertainty measure for DSmT particularly in conflict problems is applied to evaluate DSmT results. A parallel configuration to have a practical sensor network is employed.

GAU1 is a suitable uncertainty measure for DSmT but its application is limited to the problems with the frame of discernment with the clear frontier. This deficiency could be considered as a future work. As a suggestion, GAU1 can be approximately applied to the problems with continuous borders.

The following suggestions might be considered as further studies;

- Employing other classification methods instead of using the neural networks, thresholding, and statistical algorithm to make decisions about sensory data
- Utilizing ultrasonic echo signal amplitudes as acquired data in addition to TOF data
- Looking for an uncertainty measure with less complexity than AU and GAU1 in computation, which satisfies the requirements of uncertainty measures
- Developing an uncertainty interval using the lower limit (i.e. *GAU1(Bel)*) and the upper limit by defining a plausibility function based uncertainty measure to help the final decision making according to the size of the uncertainty interval.

References

[1] H. Hu, J.Q. Gan, Sensors and data fusion algorithms in mobile robotics University of Essex, Colchester CO4 3SQ, United Kingdom, 10 January 2005.

[2] P. Pong, S. Challa, Empirical analysis of generalized uncertainty measures with Dempster-Shafer fusion, in: 10th Int. Conf. on Information Fusion, Quebec, Que, 9-12 July, 2007.

[3] B. Barshan, B. Ayrulu, S.W. Utete, Neural network-based target differentiation using sonar for robotics applications, IEEE Transactions on Robotics and Automation, 16(4) (2000) 435-442.

[4] K.J. Kyriakopoulos, A. Curran, Ultrasonic navigation for a wheeled non-holonomic vehicle, Journal of Intelligent and Robotic Systems, 12(3) (1995) 239-258.

[5] G. Oriolo, G. Ulivi, M. Vendittelli, Real-time map building and navigation for autonomous robots in unknown environments, IEEE Transactions on Systems, Man and Cybernetics-Part B: (Cybernetics), 28(3) (1998) 316-333.

[6] S. Watanabe, M. Yoneyama, An ultrasonic visual sensor for three-dimensional object recognition using neural networks, IEEE Transactions on Robotics and Automation, 8(2) (1992) 240-249.

[7] M.H. Horng, Performance evaluation of multiple classifications of the ultrasonic supraspinatus images by using ML, RBFNN and SVM classifiers, Expert Systems with Applications, 37 (2010) 4146-4155.

[8] Y. Han, H. Hahn, Localization and classification of target surfaces using two pairs of ultrasonic sensors, Robotics and Autonomous Systems, 33 (2000) 31-41.

[9] X. Li, X. Huang, M. Wang, Robot map building from sonar sensors and DSmT, Information & Security (I&S), 20 (2006) 104-121.

[10] B. Ayrulu, B. Barshan, Reliability measure assignment to sonar for robust target differentiation, Pattern Recognition, 35 (2002) 1403-1419.

[11] Y. Han, M. Han, H. Cha, M. Hong, H. Hahn, Tracking of a moving object using ultrasonic sensors based on a virtual ultrasonic image, Robotics and Autonomous Systems, 36 (2009) 11-19.

[12] M. Khodabandeh, M. Analoui, A. Mohammad-Shahri, Target differentiation using sonar data for robot applications; neural network approach, in: International Conference on Control, Automation and Systems, COEX, Seoul, KOREA, October 17-20, 2007, pp. 1958-1961.

[13] G. Shafer, A mathematical theory of evidence, Princeton Univ. Press, Princeton, NJ, 1976.

[14] A.L. Jousselme, C. Liu, D. Grenier, E. Bosse, IEEE Transaction on Systems, Man and Cybernetics-Part A: Systems and Humans, 36(5) (2006) 890-903.

[15] J.P. Yang, H.Z. Huang, Y. Liu, Y.F. Li, Quantification classification algorithm of multiple sources of evidence, International Journal of Information Technology & Decision Making, 14(5) (2015) 1017-1034.

[16] J. Dezert, J.M. Tacnet, M. Batton-Hubert, F. Smarandache, Multi-criteria decision making based on DSmT-AHP, in: Advances and Applications of DSmT for Information Fusion (collected works), American Research Press (ARP), 2015.

[17] N. Abbas, Y. Chibani, A. Martin, F. Smarandache, The effective use of the DSmT for multi-class classification in: Advances and Applications of DSmT for Information Fusion (collected works), American Research Press (ARP), 2015, pp. 359.

[18] F. Smarandache, J. Dezert, Advances and applications

of DSmT for information fusion, (collected works), vol. 4, American Research Press, 2015.

[19] J. Dezert, Foundation for a new theory of plausible and paradoxical reasoning, Information Security Journal, 9 (2002) 13-57.

[20] F. Smarandache, J. Dezert, Advances and applications of DSmT for information fusion, (collected works), vol. 1, American Research Press, 2004.

[21] F. Smarandache, J. Dezert, Advances and applications of DSmT for information fusion, (collected works), vol. 2, American Research Press, 2006.

[22] F. Smarandache, J. Dezert, Advances and applications of DSmT for information fusion, (collected works), vol. 3, American Research Press, 2009.

[23] J. Dezert, A. Tchamova, F. Smarandache, P. Konstantinova, Target type tracking with PCR5 and Dempster's rules: a comparative analysis, in: 9th International Conference on Information Fusion, Florence, Italy, 10-13 July 2006.

[24] X. Huang, X. Li, M. Wang, J. Dezert, A fusion machine based on DSmT and PCR5 for robot's map reconstruction, International Journal of Information Acquisition, 3(3) (2006) 201-211.

[25] R.V.L. Hartley, Transmission of information, Bell Systems & Technology Journal, 7(535-563) (1928).

[26] C.E. Shannon, A mathematical theory of communication, Bell Systems & Technology Journal, 27 (1948) 379-423 and 623-656.

[27] G.J. Klir, M.J. Wierman, Uncertainty-based information 2nd ed. series, Studies in Fuzziness and Soft Computing 15, Physica–Verlag, Heidelberg, Germany, 1999.

[28] G.J. Klir, B. Yuan, Fuzzy sets and fuzzy logic: theory and applications, Prentice–Hall, Upper Saddle River, NJ, 1995.

[29] R.R. Yager, Entropy and specificity in a mathematical theory of evidence, International Journal of General Systems, 9(4) (1983) 249-260.

[30] G.J. Klir, A. Ramer, Uncertainty in Dempster–Shafer theory: a critical re-examination, International Journal of General Systems, 18(2) (1990) 155-166.

[31] T. George, N.R. Pal, Quantification of conflict in Dempster–Shafer framework: a new approach, International Journal of General Systems, 24(4) (1996) 407-423.

[32] N.R. Pal, J.C. Bezdek, R. Hemasinha, Uncertainty measures for evidential reasoning I: a review, International Journal of Approximate Reasoning, 7(3/4) (1992) 165-183.

[33] N.R. Pal, J.C. Bezdek, R. Hemasinha, Uncertainty measures for evidential reasoning II: a new measure of total uncertainty, International Journal of Approximate Reasoning, 8(1) (1993) 1-16.

[34] J.A. Herencia, M.T. Lamata, A generalization of entropy using Dempster–Shafer theory, International Journal of General Systems, 29(5) (2000) 719-735.

[35] A. Ramer, J. Hiller, Total uncertainty revisited, International Journal of General Systems, 26(3) (1997) 223-237.

[36] Y. Maeda, H.T. Nguyen, H. Ichihashi, Maximum entropy algorithms for uncertainty measures, International Journal of Uncertainty, Fuzziness & Knowledge Based Systems, 1(1) (1993) 69-93.

[37] D. Harmanec, G.J. Klir, Measuring total uncertainty in Dempster–Shafer theory, International Journal of General Systems, 22(4) (1994) 405-419.

[38] D. Harmanec, Uncertainty in Dempster–Shafer theory, State University of New York, New York, NY, 1996.

[39] A. Bronevich, G.J. Klir, Measures of uncertainty for imprecise probabilities: an axiomatic approach, International Journal of Approximate Reasoning, 51 (2010) 365-390.

[40] G.J. Klir, H.W. Lewis, Remarks on measuring ambiguity in the evidence theory, IEEE Transaction on Systems, Man and Cybernetics-Part A: Systems and Humans, 38(4) (2008) 995-999.

[41] M. Vatsa, R. Singh, A. Noore, M.M. Houck, Quality-augmented fusion of level-2 and level-3 fingerprint information using DSm theory, International Journal of Approximate Reasoning, 50 (2008) 51-61.

[42] M. Vatsa, R. Singh, A. Noore, Unification of evidence-theoretic fusion algorithms; a case study in level-2 and level-3 fingerprint features, IEEE Transaction on Systems, Man and Cybernetics-Part A: Systems and Humans, 39(1) (2009) 47-56.

[43] M. Khodabandeh, A. Mohammad-Shahri, Two generalizations of aggregated uncertainty measure for evaluation of Dezert-Smarandache theory, International Journal Information Technology & Decision Making (IJITDM), 11(1) (2012) 119-142.

[44] M. Khodabandeh, A. Mohammad-Shahri, Uncertainty evaluation for an ultrasonic data fusion based target differentiation problem using Generalized Aggregated Uncertainty measure 2, Measurement, 59 (2015) 139-144.

[45] B. Ayrulu, B. Barshan, Identification of target primitives with multiple decision-making sonars using evidential reasoning, International Journal of Robotic Research, 17(6) (1998) 598-623.

Please cite this article using:

A. Mohammad-Shahri and M. Khodabandeh,"Uncertainty Measurement for Ultrasonic Sensor Fusion Using Generalized Aggregated Uncertainty Measure 1", *AUT J. Model. Simul.*, 49(1)(2017)85-94.

DOI: 10.22060/miscj.2016.827