NEUTROSOPHIC TOPOLOGIES IN CRISP APPROXIMATION SPACES

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Abstract: In this paper we introduce a new type of neutrosophic topology and we investigate the topological structures neutrosophic rough sets. Further we examine the reflexivity and transitivity of neutrosophic rough sets and obtain some of its properties.

Keywords: Approximation Operators, Neutrosophic Rough Topological spaces, Neutrosophic Rough sets Rough sets .

Introduction: Topology is a branch of mathematics, which has application not only every other branch of mathematics but also in many real time problems. The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approach is based on the fuzzy set notion proposed by L. Zadeh [12] and intuitionistic fuzzy set by Attanossov [4]. Rough set theory presents still another attempt to this problem. A rough set, first described by Zdzisław I. Pawlak [9], is a formal approximation of a crisp set in terms of a pair of sets which give the upper and Rough sets have been proposed for a very wide variety of applications. In particular, the rough set approach seems to be important for Artificial Intelligence and cognitive sciences, especially in machine learning, knowledge discovery, data mining, expert systems, approximate reasoning and pattern recognition. Also fuzzy sets and rough sets are combained and new hybrids are proposed. [5], [6]. One direction of rough theory [1] is to investigate the relationship between rough approximation operators and topological structures of rough sets.

Neutrosophic Logic has been proposed by Florentine Smarandache [10, 11] which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors. Neutrosophic topology [7] is also one among its developments

In this paper define neutrosophic rough sets neutrosophic topological spaces in the sense of Lowen [8] and its structures and examine the relationship between neutrosophic rough sets and neutrosophic topological spaces.

Preliminaries:

Definition2.1[10]:

A neutrosophic set A on the universe of discourse X is defined as

A= $\{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$

where $T, I, F: X \to [0, 1]$ and

$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2[1]:

A neutrosophic relation R is a neutrosophic set$R = \{< x, y >, T_R(x, y), I_R(x, y), F_R(x, y) / x, y \in U\}$ $T_R: U \times U \to [0,1]$,$I_R: U \times U \to [0,1]$, $F_R: U \times U \to [0,1]$ and satisfies $0 \leq T_R(x, y) + I_R(x, y) + F_R(x, y) \leq 3$,

for all (x,y) $\in U \times U$.

Definition 2.3.[3]

Let U be a nonempty universe of discourse which may be infinite. A subset R ∈ P(U × U) is referred to as a (crisp) binary relation on U. The relation R is referred to as reflexive if for all x ∈ U, (x, x) ∈ R; R is referred to as symmetric if for all x, y ∈ U, (x, y) ∈ R implies (y, x) ∈ R; R is referred to as transitive if for all x, y, z ∈ U, (x, y) ∈ R and (y, z) ∈ R imply (x, z) ∈ R; R is referred to as a similarity relation if R is reflexive and symmetric; R is referred to as a preorder if R is reflexive and transitive; and R is referred to as an equivalence relation if R is reflexive, symmetric and transitive.

Definition 2.4.[1]

For an arbitrary crisp relation R on U, we can define a set-valued mapping Rs : U → P(U) by: $R_S(x) = \{y \in U | (x, y) \in R\}$, x ∈ U.

$R_S(x)$ is called the successor neighborhood of x with respect to R.

Definition 2.6.[2] A neutrosophic set A is contained in another neutrosophic set B. (i.e

A ⊆ B ⇔ $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$,∀ x∈ X.

Definition 2.7: [2]

The complement of a neutrosophic set (F,A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \rceil A)$ where

$T_{F^c}(x) = F_F(x)$, $I_{F^c}(x) = 1 - I_F(x)$, $F_{F^c}(x) = T_F(x)$.

Definition 2.8: [2]

Let U be a non empty set, and

$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$

are neutrosophic sets. Then

$A \,\breve{\cup}\, B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$

$A \,\tilde{\cap}\, B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$.

3.Neutrosophic Rough sets in Crisp Approximation Space.

Definition 3.1:

Let (U, R) be a crisp approximation space. For A ∈ N(U), the upper and lower approximations of A w.r.t. (U, R), denoted by $\overline{R}(A)$ and $\underline{R}$ (A), respectively, are defined as follows: we define the upper and lower approximation with respect to (U, R), denoted by $\overline{R}$ and $\underline{R}$ respectively.

$\overline{R}(A) = \{< x, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x) >/x \in U\}$

$\underline{R}(A) = \{< x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) > x \in U\}$

$T_{\overline{R}(A)}(x) = \bigvee_{y \in R_S(x)} [T_A(y)]$ $I_{\overline{R}(A)}(x) = \bigvee_{y \in R_S(x)} [I_A(y)]$

$F_{\overline{R}(A)}(x) = \bigwedge_{y \in R_{S(}(x)} [F_A(y)]$

$T_{\underline{R}(A)}(x) = \bigwedge_{y \in R_{S(}(x)} [T_A(y)]$

$I_{\underline{R}(A)}(x) = \bigwedge_{y \in R_{S(}(x)} [I_A(y)]$

$F_{\underline{R}(A)}(x) = \bigvee_{y \in R_S(x)} [F_A(y)]$

A neutrosophic rough set is the approximation of a neutrosophic set with respect to a crisp approximation space.

Theorem 3.2.

Let (U, R) be a crisp approximation space, then the upper and lower rough neutrosophic approximation operators satisfy the following properties:

∀A, B, A_j ∈ N(U), j ∈ J, J is an index set, ∀ (α, β, γ) ∈ [0,1],

(L1) $\underline{R}$ (∼ A) =∼ $\overline{R}$ (A),

(U1) $\overline{R}$ (∼ A) =∼ $\underline{R}$ (A);

(L2) $\underline{R}$ (U) = U

(U2) $\overline{R}$ (∅) = ∅;

(L3) $\underline{R}$ (∩ $_{j∈J}$ A_j) = ∩ $_{j∈J}$ $\underline{R}$ (A_j),

(U3) $\underline{R}$ (∪ $_{j∈J}$ A_j) = ∪ $_{j∈J}$ $\underline{R}$ (A_j),

(L4) $\underline{R}$ (A ∪ B) ⊇ $\underline{R}$ (A) ∪ $\underline{R}$ (B),

(U4) $\overline{R}$ (A ∩ B) ⊆ $\overline{R}$ (A) ∩ $\overline{R}$ (B);

(L5) A ⊆ B ⇒ $\underline{R}$ (A) ⊆ $\underline{R}$ (B),

(U5) A ⊆ B ⇒ $\overline{R}$ (A) ⊆ $\overline{R}$ (B)

Theorem 3.3.

Let R be a neutrosophic relation on U and $\overline{R}$ and $\underline{R}$ the lower and upper approximation operators induced by (U, R). Then

(1)R is reflexive ⇔

R1) $\underline{R}$ (A)⊆ A , ∀ A ∈ N(U) ,

R2) A ⊆ $\overline{R}$ (A), ∀ A ∈ N(U).

(2)R is symmetric ⇔

S1) $T_{\overline{R}\,(1_x)}$(y) = $T_{\overline{R}\,(1_y)}$(x), ∀ (x,y)∈ U ×U,

S2) $I_{\overline{R}\,(1_x)}$(y) = $I_{\overline{R}\,(1_y)}$(x), ∀ (x,y) ∈ U ×U,

S3) $F_{\overline{R}\,(1_x)}$(y) = $F_{\overline{R}\,(1_x)}$(y), ∀ (x,y)∈ U ×U,

S4) $T_{\underline{R}(1_{U-\{x\}})}$(y) = $T_{\underline{R}(1_{U-\{y\}})}$(x), ∀ (x,y)∈ U ×U,

S5) $I_{\underline{R}(1_{U-\{x\}})}$(y) = $I_{\underline{R}(1_{U-\{y\}})}$(x), ∀ (x,y)∈ U ×U,

S6) $F_{\underline{R}(1_{U-\{x\}})}$(y) = $F_{\underline{R}(1_{U-\{y\}})}$(x), ∀ (x,y)∈ U ×U.

(3) R is transitive ⇔

T 1) $\underline{R}$ (A) ⊆ $\underline{R}$ ($\underline{R}$ (A)) ∀ A ∈ N(U)

T2) $\overline{R}$ ($\overline{R}$ (A)) ⊆ $\overline{R}$ (A), ∀ A ∈ N(U)

4. Neutrosophic Topological Space.

Definition 4.1:

A neutrosophic topology in the sense of Lowen [8] on a nonempty set U is a family τ of neutrosophic sets in U satisfying the following axioms:

(i) $\overline{\overline{\alpha, \beta, \gamma}} \in \tau$ for all $(\alpha, \beta, \gamma) \in$ [0,1]

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii) $\bigcup_{i \in J} A_i \in \tau$ for a family $\{A_i / i \in J\} \subseteq \tau$,

Where J is an index set.

In this case the pair (U, τ) is called a neutrosophic topological space and each neutrosophic set A in τ is referred to as neutrosophic open set in (U, τ). The complement of neutrosophic open set in the neutrosophic topological space (U, τ) is called a neutrosophic closed set in (U, τ).

Definition 4.2:

Let (U, τ) be a neutrosophic topological space and A ∈ N(U). Then the neutrosophic interior and neutrosophic closure of A are, respectively, defined as follows:

int(A) = ∪{G | G is a N open set and G ⊆ A}, cl(A) = ∩{K | K is a N closed set and A ⊆ K}, and int and cl : N(U) → N(U) are, respectively, called the neutrosophic interior operator and the neutrosophic closure operator of τ, and sometimes in order to distinguish, we denote them by int_τ and cl_τ .

It can be shown that cl(A) is a neutrosophic closed set and int(A) is a neutrosophic open set in (U, τ), and

i) A is an N open set in (U, τ) iff int(A) = A,

ii) A is an N closed set in (U, τ) iff cl(A) = A. Moreover,

(iii) cl(∼ A) =∼ int(A) ∀A ∈ N(U),

(iv) int(∼ A) =∼ cl(A) ∀A ∈ N(U)

It can be verified that the neutrosophic closure operator satisfies following **properties:**

(C1) cl($\overline{\overline{\alpha, \beta, \gamma}}$) = $\overline{\overline{\alpha, \beta, \gamma}}$ ∀(α, β, γ) ∈[0, 1].

(C2) cl(A ∪ B) = cl(A) ∪ cl(B) ∀A, B ∈ N(U).

(C3) cl(cl(A)) = cl(A) ∀A ∈ N(U).

(C4) A ⊆ cl(A) ∀A ∈ N(U).

Properties (C1)–(C4) are called the neutrosophic closure axioms.

properties:

(I1) int($\overline{\overline{\alpha,\beta,\gamma}}$) = $\overline{\overline{\alpha,\beta,\gamma}}$ ∀(α, β, γ) ∈ [0,1].

(I2) int(A ∩ B)= int(A) ∩ int(B) ∀A, B ∈ N(U).

(I3) int(int(A)) = int(A) ∀A ∈ N(U).

(I4) int(A) ⊆ A ∀A ∈ N(U).

Definition 4.3.

A mapping cl : N(U) → N(U) is referred to as a neutrosophic closure operator if it satisfies axioms (C1)-(C4).

Definition 4.4.

A mapping int : N(U) → N(U) is referred to as a neutrosophic interior operator if it satisfies axioms (I1)-(I4).

It is easy to show that a neutrosophic interior operator determines a neutrosophic topology,

τ_{Int} = {A ∈ N(U)|int(A) = A}.

So, the neutrosophic open sets are the fixed points of int. Dually, from a neutrosophic closure we can obtain a neutrosophic topology on U by setting

τ_{Cl} = {A ∈ N(U)|cl(∼ A) =∼ A}.

Definition 4.5.

A neutrosophic topology τ on U is called a neutrosophic Alexandrov topology if the intersection of arbitrarily many neutrosophic open sets is still open, or equivalently, the union of arbitrarily many neutrosophic closed sets is still closed. A neutrosophic topological space (U, τ) is said to be an neutrosophic Alexandrov space if τ is a neutrosophic Alexandrov topology on U. A neutrosophic topology τ on U is called a neutrosophic clopen topology if, for every A ∈ N(U), A is IF open in (U, τ) if and only if A is neutrosophic closed in (U, τ). A topological space (U, τ) is said to be a neutrosophic clopen space if τ is a neutrosophic clopen topology on U.

5. Neutrosophic Topologies of Neutrosophic Rough Sets.

In this section we discuss the relationship between neutrosophic topological spaces and rough neutrosophic sets. Throughout this section we always assume that U is a nonempty universe of discourse, R a crisp binary relation on U, and $\overline{R}$ and $\underline{R}$ the rough neutrosophic approximation operators. Denote

τ_R = {A ∈ N(U)| $\underline{R}$ (A) = A}.

The next theorem shows that any reflexive binary relation determines neutrosophic Alexandrov topology.

Theorem 5.1.

If R is a reflexive crisp binary relation on U, then τ_R is a neutrosophic Alexandrov topology on U.

Theorem 5.2

Assume that R is a crisp binary relation on U. Then the following statements are equivalent:

(1) R is a preorder, i.e., R is a reflexive and transitive relation;

(2) The upper rough neutrosophic approximation operator $\overline{R}$: N(U) → N(U) is an neutrosophic closure operator;

(3) The lower rough neutrosophic approximation operator $\underline{R}$: N(U) → N(U) is an neutrosophic interior operator.

Lemma 5.3.

If R is a symmetric crisp binary relation on U, then for all A, B ∈ N(U),

$\overline{R}$ (A) ⊆ B ⇐⇒ A ⊆ $\underline{R}$ (B).

Theorem 5.4. Let R be a similarity crisp binary relation on U, and $\overline{R}$ and $\underline{R}$ the rough neutrosophic approximation operators. Then $\overline{R}$ and $\underline{R}$ satisfy property for A ∈ N(U),

$\underline{R}$ (A) = A ⇐⇒ A = $\overline{R}$ (A) ⇐⇒ $\underline{R}$ (∼ A) =∼ A ⇐⇒∼ A = $\overline{R}$ (∼ A).

The next theorem shows that a neutrosophic topological space induced from a reflexive and symmetric crisp approximation space is a neutrosophic clopen topological space.

Theorem 5.5.

Let R be a similarity crisp binary relation on U, and and $\overline{R}$ and $\underline{R}$ the rough neutrosophic approximation operators. Then τ_R is a neutrosophic clopen topology on U.

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