

Do The Real Numbers form a Set?

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Do real numbers form a set? A set is a well defined collection of objects. My question is: what is the definition of real numbers?

Real numbers create a background in which all kinds of numbers – even numbers, odd numbers ,rationals, irrationals, prime numbers et. are defined. What could be the background in which real numbers can be defined. The answer can not be the set of complex numbers because complex plane is nothing but a structure isomorphic to \mathbf{R}^2 . In other words , complex system is an extension of real number system , that is, complex number system is derived from real number system. (If someone still argue that complex number system form the background in which real numbers can be defined , I would like to go to the era when complex number system was not yet discovered)

So the question is – what is the background in which real numbers can be defined. A writing with white chalk can not be defined on a white board but only on a board of some color other than white. Another example : Our physical laws are defined in space , but what is the definition of space itself? It can not be defined , it forms a background for all the definitions but itself is undefined. Not everything in our world is definable, some objects are left undefined and definitions are ultimately given in terms of undefinables . I would like to quote here Bertrand Russel - “in a mathematical sense, a new propositional function is said to be defined when it is stated to be equivalent to (i.e.to imply and be implied by) a propositional function which has either been accepted as indefinable or has been defined in terms of indefinables.” (Principles of Mathematics)

We have different kinds of numbers- even , odd, rational, irrational which all have their specific definitions in the background of what we call real numbers . We define real numbers as union of all different kind of numbers just as we can define space as something in which all physical events take place. But these are not true definitions . True definition must be in terms of some intrinsic character of things. For example, true definition of a man is not that he is father of his sons , although it uniquely specifies the man, it does not give full picture of the man.

We observe that different kinds of numbers always exist in pairs – even-odd, rational-irrational, positive-negative integers. Hence all the definitions are relative. To define a specific character

of a set, we need the opposite character. If all the characters and anti-characters are added to form a set, we are left with no character to define the sum set. If we take the union of even and odd numbers, we get the set of natural numbers for which there is no meaning of even and odd numbers. Similarly, adding together the set of positive and negative integers we get the set of integers for which distinction between positive and negative numbers is lost. I mean being positive or negative becomes a redundant character for defining sum set of integers. Continuing the process and adding all the opposite kinds of numbers- even-odd, positive-negative, rational-irrational, we get a set with no specific character left. We call this set, *the set of real numbers* with no specific character to define it except that it is a union of all different kinds of number sets.