

University of New Mexico



42

Uniform Single Valued Neutrosophic Graphs

S. Broumi¹, A. Dey², A. Bakali³, M. Talea¹, F. Smarandache⁴ L. H. Son⁵, D. Koley⁶

¹ Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco.

E-mail: broumisaid78@gmail.com

²Saroj Mohan Institute of Technology, West Bengal, India, E-mail:arindam84nit@gmail.com ³ Ecole Royale Navale, Casablanca, Morocco, e-mail: assiabakali@yahoo.fr

Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA, E-mail: fsmarandache@gmail.com

⁵Vietnam National University, Hanoi, Vietnam, E-mail: sonlh@vnu.edu.vn

⁶Techno India Hooghly Campus, West Bengal, India, E-mail:dipakkoley973@gmail.com

Abstract. In this paper, we propose a new concept named the uniform single valued neutrosophic graph. An illustrative example and some properties are examined. Next, we develop an algorithmic approach for computing the complement of the single va-

lued neutrosophic graph. A numerical example is demonstrated for computing the complement of single valued neutrosophic graphs and uniform single valued neutrosophic graph.

Keywords: Single valued neutrosophic sets; Uniform single valued neutrosophic graph; Complement operators

1 Introduction

In 1965, Zadeh [7] originally introduced the concept of fuzzy set(FSs) which is characterized by a membership degree in [0, 1] for each element in the dataset. It may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the truth- membership degree because there is some kind of hesitation degree. On the basis of fuzzy sets, Atanassov [4] added a non-membership in the definition of intuitionistic fuzzy sets (IFSs) and later Smarandache [2] introduced the neutrosophic sets (NSs) with the appearance of the truthmembership degree (T), the falsehood-membership degree (F), and the indeterminacy degree (I). Wang et al. [3] proposed various set theoretical operators and linked to single valued neutrosophic sets The concept of neutrosophic sets have been successfully applied to many fields [16].

Fuzzy graph has been studied extensively in the past years [5,8,9]. Later on, Smarandache [1] proposed neutrosophic graphs in some special types such as neutrosophic offgraph, neutrosophic bipolar/tripolar/ multipolar graph. Presently, works on neutrosophic vertex-edge graphs and neutrosophic edge graphs are progressing rapidly. Broumi et al.[13] introduced certain types of single valued neutrosophic graphs (in short SVNG) such as strong single valued neutrosophic graph, constant single valued neutrosophic graph, complete single valued neutrosophic graph with their properties and examples. Neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph were introduced in [15]. The necessary and sufficient condition for a single valued neutrosophic graph to be an isolated single valued neutrosophic graph has been presented in [10]. Other extensions of the neutrosophic graph have been described in [11,12, 14].

Up to now, to the best of our knowledge, there has been no study on the uniform single valued neutrosophic graph. Thus, we propose in this paper a new concept named the uniform single valued neutrosophic graph. An illustrative example and some properties are examined. Next, we develop an algorithmic approach for computing the complement of the single valued neutrosophic graph.

The remainder of this paper is organized as follows. In Section 2, we present the basic definitions. In section 3, we introduce the concept of uniform single valued neutrosophic graph and investigate its properties. Section 4 introduces an algorithm for computing the complement of single valued neutrosophic graphs. A numerical example is presented in Section 5. Finally, Section 6 outlines the conclusion of this paper and suggests several directions for future research.

2 Preliminaries

In this section, we have present the basic definitions of fuzzy sets, neutrosophic sets, single valued neutrosophic sets, fuzzy graphs, uniform fuzzy graphs, complement of single valued neutrosophic graph which will be useful to our main work in the next sections.

Definition 1[1]. Let X be the universe of discourse and its elements denoted by x. In fuzzy theory, a fuzzy set

S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L. H. Son, D. Koley. Uniform Single Valued Neutrosophic Graphs

A of universe X is defined by the function $T_A(x)$, called the membership function of set A. $T_A: X \rightarrow [0, 1]$ (1)

For any element x of universe
$$X, T_A(x)$$
 equals the degree, between 0 and 1, to which x is an element of set A, This degree represents the membership value or degree of membership of element x in set A.

Definition 2[1]. Let X be a space of points and let $x \in X$. A **neutrosophic set** A in X is characterized by a truth membership function T, an indeterminacy membership function I, and a falsehood membership function F which are real standard or nonstandard subsets of]-0,1+[, and T, I, F: X→]-0,1+[. The neutrosophic set can be represented as,

$$A = \{ (x, T_A(x), I_A(x), F_A(x)) : x \in X \}$$
(4)

There is no restriction on the sum of T, I, F, So

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$$
. (5)

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$. Thus it is necessary to take the interval [0,1] instead of $]^{-}0,1^{+}[$. For practical applications, it is difficult to apply $]^{-}0,1^{+}[$ in the real life applications such as engineering and scientific problems.

Definition 3[3]. Let X be a space of objects with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS) is characterized by truth-membership function $T_A(x)$, an indeterminate-membership function $I_A(x)$, and a falsehood-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as,

 $A = \{ (x, T_A(x), I_A(x), F_A(x)) : x \in X \} (6)$

Definition 4 [5]. *A fuzzy graph* is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ . i.e. $\sigma : V \rightarrow [0,1]$ and μ : VxV \rightarrow [0,1] such that $\mu(uv) \leq \sigma(u) \land \sigma(v)$ for all u, $v \in V$ where uv denotes the edge between u and v and $\sigma(u) \land \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.





Remark: The crisp graph $G^* = (V, E)$ is a special case of the fuzzy graph G with each vertex and edge of (V, E) having degree of membership 1 (Fig. 1).

Definition5[6,8]. The complement of a fuzzy graph $G = (\sigma, \mu)$ is a fuzzy graph $\overline{G} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) \cdot \mu(u,v)$, $\forall u, v \in V$.

Definition 6[6,8]. Let $G = (\sigma, \mu)$ be a fuzzy graph on a crisp graph $G^* = (V, E)$. Let $\sigma^* = \{x \in V \mid \sigma(x) > 0\}$. Then G is called a uniform fuzzy graph of level *k*if $\mu(x,y) = k, \forall$ $(x,y) \in (\sigma^* \times \sigma^*)$ and $\sigma(x) = k$ where *k* is positive real such that $0 < k_1 \le 1$.

Definition 7[15].Let G = (V, E) be a single valued neutrosophic graph, then the **degree of a vertex** $_i$ is defined by $d_G(x_i) = d_G(x) = (d_T(x), d_I(x), d_F(x)), d_G(x_i) =$ $(\sum_{x \neq y} T_B(x, y), \sum_{x \neq y} I_B(x, y), \sum_{x \neq y} I_B(x, y))$.

Definition 8[15].Let G = (V, E) be a single valued neutrosophic graph, then the **total degree of a vertex** x_i is defined by $td_G(x_i) = d_G(x) = (td_T(x), td_I(x), td_F(x)),$ $td_G(x_i) = (\sum_{x \neq y} T_B(x, y) + T_A(x), \sum_{x \neq y} I_B(x, y) + I_A(x), \sum_{x \neq y} I_B(x, y) + F_A(x)).$

Definition 9[13]. Let G = (V, E) be a single valued neutrosophic graph, then the complement of single valued neutrosophic graph is defined as $1. \overline{V} = V$

 $\begin{array}{l} 2.\overline{T}_{A}(x)=T_{A}(x), \ \overline{I}_{A}(x)=I_{A}(x), \ \overline{F}_{A}(x)=F_{A}(x) \ \text{for all } x\in V. \\ 3.\overline{T}_{B}(x,y)=\min\left[T_{A}(x),T(y)\right]-T_{B}(x,y) \\ \overline{I}_{B}(x,y)=\max\left[I_{A}(x),I_{A}(y)\right]-I_{B}(x,y) \text{and} \\ \overline{F}_{B}(x,y)=\max\left[F_{A}(x),F_{A}(y)\right]-F_{B}(x,y), \ \text{for all } (x,y)\in E \end{array}$

Definition 10[13]. Let G = (V, E) be a single valued neutrosophic graph. If $d_G(x_i) = (k_1, k_2, k_3)$ for all $x_i \in V$, then the single valued neutrosophic graph is called regular SVNG of degree (k_1, k_2, k_3)

Definition 11[13]. Let G = (V, E) be a single valued neutrosophic graph. If $td_G(x_i) = (k_1, k_2, k_3)$ for all $x_i \in$ V, then the single valued neutrosophic graph is called Totally regular SVNG of degree (k_1, k_2, k_3)

III. Uniform Single Valued Neutrosophic Graph

In this section, we define the concept of uniform single valued neutrosophic graphs(in short USVNGs).

Definition 8. Let G = (A, B) be a single valued neutrosophic graph where A =(T_A , I_A , F_A) is a single valued neutrosophic vertex of G and B is a single valued neutrosophic edge set of G. Let A={ $x \in V | T_A(x) > 0, I_A(x) > 0$ and $F_A(x) > 0$ }. Then G is called Uniform single valued neutrosophic graph of level (k_1, k_2, k_3) if $T_B(x,y) = k_1, I_A(x) = k_2$ and $F_B(x,y) = k_3 \forall (x,y) \in (V \times V)$ and $T_A(x) = k_1, I_A(x) = k_2$ and $F_A(x) = k_3$ where k_1, k_2 and k_3 are some positive real such that $0 < k_1, k_2, k_3 \le 1$.

Example 1. Consider an USVNG G= (A,B) on $V = \{v_1, v_1, v_3, v_4\}$ as shown in Fig.2.



Fig. 2. USVNG.

Remark: The complement of an uniform single valued neutrosophic graph is always an empty graph.

Theorem1. If G = (A, B) is an uniform single valued neutrosophic graph of level (k_1, k_2, k_3) then G is a regular-USVNG.

Proof. Let $A=\{x \in V \mid T_A(x) > 0, I_A(x) > 0$ and $F_A(x) > 0\}$. Suppose that G is a uniform single valued neutrosophic graph. Then $T_B(x, y) = k_1, I_B(x, y) = k_2$ and $F_B(x, y) = k_3 \forall (x, y) \in \text{Eand } T_A(z) = k_1, I_A(z) = k_2$ and $F_A(z) = k_3 \forall z \in V$ for some real k_1, k_2 and k_3 where $0 < k_1, k_2, k_3 \le 1$.

Let
$$x \in V$$
. Now $d_G(x) = (d_T(x), d_I(x), d_F(x))$
 $d_G(x) = (\sum_{x \neq y} T_B(x, y), \sum_{x \neq y} I_B(x, y), \sum_{x \neq y} F_B(x, y))$
 $= (\sum_{x \neq y} k_1, \sum_{x \neq y} k_2, \sum_{x \neq y} k_3)$

=((n-1) k_1 , (n-1) k_2 , (n-1) k_3) $d_G(x)$ =((n-1) k_1 , (n-1) k_2 , (n-1) k_3) $\forall x \in V$

Therefore, G is regular uniform single valued neutrosophic graph.

Theorem 2. If G = (A, B) is a uniform single valued neutrosophic graph of level (k_1, k_2, k_3) then G is a totally regular- USVNG.

Proof. Let $A = \{x \in V \mid T_A(x) > 0, I_A(x) > 0$ and $F_A(x) > 0\}$. Suppose that G is a uniform single valued neutrosophic graph. Then $T_B(x, y) = k_1$, $I_B(x, y) = k_2$ and $F_B(x,y) = k_3 \forall (x,y) \in \text{Eand } T_A(z) = k_1$, $I_A(z) = k_2$ and $F_A(z) = k_3 \forall z \in V$ for some real k_1 , k_2 and k_3 where $0 < k_1, k_2, k_3 \le 1$.Let $x \in V$. Now, $td_G(x) = (d_T(x) + T_A(x), d_I(x) + I_A(x), d_F(x) + F_A(x)) \\ td_G(x) = (\sum_{x \neq y} T_B(x, y) + T_A(x), \sum_{x \neq y} I_B(x, y) \\ + I_A(x), \sum_{x \neq y} F_B(x, y) + F_A(x))$

 $+ I_{A}(x), \sum_{x \neq y} F_{B}(x, y) + F_{A}(x)$ =(($\sum_{x \neq y} k_{1}$) + $k_{1}, (\sum_{x \neq y} k_{2}$) + $k_{2}, (\sum_{x \neq y} k_{3}) + k_{3}$) =((n-1) $k_{1}+k_{1}, (n-1) k_{2}+k_{2}, (n-1) k_{3}+k_{3}$) t $d_{G}(x)$ =($nk_{1}, n k_{2}, n k_{3}$) $\forall x \in V.$

Therefore, G is totally-regular uniform single valued neutrosophic graph.

Theorem 3. If G = (A, B) is a uniform single valued neutrosophic graph of level (k_1, k_2, k_3) on $G^*=(V, E)$, then the order of G is O(G)= (nk_1, nk_2, nk_3) .

Proof: Let $A=\{x \in V \mid T_A(x) > 0, I_A(x) > 0$ and $F_A(x) > 0\}$. Suppose that G is a uniform single valued neutrosophic graph. Then $T_B(x, y) = k_1$, $I_B(x, y) = k_2$ and $F_B(x,y) = k_3 \forall (x,y) \in \text{Eand } T_A(z) = k_1$, $I_A(z) = k_2$ and $F_A(z) = k_3 \forall z \in V$ for some real k_1 , k_2 and k_3 where $0 < k_1, k_2, k_3 \le 1$.Let $x \in V$. Now $O(G)=(O_T(G), O_I(G), O_F(G))$

$$O(G) = \left(\sum_{x \in V} T_A(x), \sum_{x \in V} I_A(x), \sum_{x \in V} f_A(x)\right)$$

= $\left(\sum_{x \in V} k_1, \sum_{x \in V} k_2, \sum_{x \in V} k_3\right)$
Then, $O(G) = (nk_1, n k_2, n k_3).$

$$= (\sum_{x \in V} k_1, \sum_{x \in V} k_2, \sum_{x \in V} k_3)$$

Then, $O(G) = (nk_1, n k_2, n k_3).$

Theorem 4.The uniform single valued neutrosophic graph is a generalization of uniform fuzzy graph.

Proof: Straightforward.

S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L. H. Son, D. Koley. Uniform Single Valued Neutrosophic Graphs

IV. Computing Complement of Single Valued Neutrosophic Graph

In this section, we present in the last paper, a peudocode of an algorithm computing the complement of single valued neutrosophic graph. This algorithm has the ability of computing the complement of fuzzy graphs, strong intuitionistic fuzzy graphs, uniform fuzzy graphs and also uniform single valued neutrosophic graphs.

The following flowchart demonstrates the algorithm to compute the complement operator is presented in Fig.3V.Numerical Example

In this section, we present an example to compute the complements of the uniform single valued neutrosophic graph. Consider a graph in Fig.4.



Fig. 4.A uniform single valued neutrosophic graph

Using the above pseudo code, the output result for the complement of a uniform single valued neutrosophic graph is in Fig. 5.

C:\	C:V)ocume	nts and Setti	ngs\said\E	Bureau\code	e of si	ngle va	alued i	ne	- C
Ple	ase	entei	• no of ve	rtex:4						
Ple	ase	entei	• (T.I.F)m	enbershi	p values	of v	ertex	:0.4	0.2	0.3
P1e	ase	entei	• (T.I.F)m	enbershi	p values	of v	ertex	:0.4	0.2	0.3
P1e	ase	entei	• (T,I,F)m	enbershi	p values	of v	ertex	:0.4	0.2	0.3
P1e	ase	entei	• (T,I,F)m	enbershi	p values	of v	ertex	:0.4	0.2	0.3
Ple	ase	entei	• the edge	s (x to	y):1 2					
Ple	ase	entei	• (T,I,FÌm	embershi	ip values	of e	dge : Ø	1.4 0.	.2 Ø.	.3
Ple	ase	entei	• the edge	s (x to	y):2 3					
Ple	ase	entei	• (T,I,F)m	embershi	p values	of e	dge : Ø	1.4 0	.2 Ø.	.3
Ple	ase	entei	• the edge	s (x to	y):3 4					
Ple	ase	entei	• (T,I,F)m	embershi	lp values	of e	dge : Ø	1.4 0.	.20.	.3
Ple	ase	entei	• the edge	s (x to	y):4_1					_
Ple	ase	entei	• (T,I,F)m	embershi	p values	of e	dge : Ø	1.4 0	.20.	.3
Ple	ase	entei	• the edge	s (x to	y):4_2					
Ple	ase	entei	• (T,I,F)m	embershi	p values	of e	dge :	0.4 (0.2 0	1. 3
Ple	ase	entei	• the edge	s (x to	y):1_3					_
Ple	ase	entei	• (T,I,F)m	embershi	p_values	ofe	dge : Ø	1.4 0	.20	.3
The	co	mpleme	nt of Sin	gle_valu	led_neutro	osoph	ic gr	aphs	is:	
1 -	• 2	edge r	nembership	value=	0.000000	0.00	0000	0.000	1000	
1 -	- 3	edge n	nembership	value=	0.000000	0.00	เดกกก	0.00	1000	
1 -	• 4	edge r	nembership	value=	0.000000	0.00	0000	0.00	1000	
2 -	• 1	edge n	nembership	value=	0.000000	0.00	เดกดด	0.00	1000	
2 -	• 3	edge r	nembership	value=	0.000000	0.00	0000	0.000	1000	
2 -	• 4	edge r	nembership	value=	0.000000	0.00	0000	0.000	1000	
3 -	• 1	edge n	nembership	value=	0.000000	0.00	0000	0.000	1000	
3 -	- 2	edge r	nembership	value=	0.000000	0.00	ииии	0.000	1000	
3 -	• 4	edge n	nembership	value=	0.000000	0.00	เขยยย	0.00	1000	

Fig. 5. The outputs

Example 2 Consider a fuzzy graph as shown in Fig.6



Fig. 6.Fuzzy graph

Using the above pseudo code, the output result for the complement of fuzzy graph is as follows:

Please enter no of vertex 4 Please enter (T,F) values of vertex:0.1 0 Please enter (T,F) values of vertex:0.3 0 Please enter (T,F) values of vertex:0.1 0 Please enter (T,F) values of vertex:0.4 0 Please enter the edges $\langle x \text{ to } y \rangle 1 2$ Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.1 0 Please enter the edges $\langle x \text{ to } y \rangle 3 4$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 2$ Please enter the edges $\langle x \text{ to } y \rangle 4 2$ Please enter the edges $\langle x \text{ to } y \rangle 4 2$ Please enter the edges $\langle x \text{ to } y \rangle 4 2$ Please enter the edges $\langle x \text{ to } y \rangle 4 3$ Please enter the edges $\langle x \text{ to } y \rangle 4 3$ Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0.4 0 edges membership values of complement of IFG are 1 - 2 edge membership value 0.000000 0.000000 1 - 4 edge membership value 0.000000 0.000000 2 - 1 edge membership value 0.000000 0.000000 2 - 1 edge membership value 0.000000 0.000000 3 - 1 edge membership value 0.000000 0.000000 3 - 2 edge membership value 0.000000 0.000000 3 - 4 edge membership value 0.000000 0.000000	🔤 C:\Documents and Settings\said\Burea	au\dey article 🗕 🗖
Please enter (T,F) values of vertex:0.1 0 Please enter (T,F) values of vertex:0.3 0 Please enter (T,F) values of vertex:0.1 0 Please enter (T,F) values of vertex:0.4 0 Please enter the edges (x to y)1 2 Please enter the edges (x to y)2 3 Please enter the edges (x to y)3 4 Please enter the edges (x to y)3 4 Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 2 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 2 Please enter the edges (x to y)4 2 Please enter the edges (x to y)4 3 Please enter (T,F) values of edge:0.3 0 Please enter the edges (x to y)1 3 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value 0.000000 0.000000 1 - 4 edge membership value 0.000000 0.000000 2 - 1 edge membership value 0.000000 0.000000 2 - 4 edge membership value 0.000000 0.000000 3 - 1 edge membership value 0.000000 0.000000 3 - 2 edge membership value 0.000000 0.000000 3 - 4 edge membership value 0.000000 0.000000	Please enter no of vertex 4	
Please enter (T,F) values of vertex:0.3 0 Please enter (T,F) values of vertex:0.1 0 Please enter (T,F) values of vertex:0.4 0 Please enter the edges $\langle x \text{ to } y \rangle 1 2$ Please enter the edges $\langle x \text{ to } y \rangle 2 3$ Please enter the edges $\langle x \text{ to } y \rangle 2 3$ Please enter the edges $\langle x \text{ to } y \rangle 3 4$ Please enter the edges $\langle x \text{ to } y \rangle 3 4$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 1$ Please enter the edges $\langle x \text{ to } y \rangle 4 2$ Please enter the edges $\langle x \text{ to } y \rangle 4 3$ Please enter the edges $\langle x \text{ to } y \rangle 4 3$ Please enter the edges $\langle x \text{ to } y \rangle 1 3$ Please enter $\langle T,F \rangle$ values of edge:0.0 edges membership values of complement of IFG are 1 - 2 edge membership value = 0.000000 0.0000000 1 - 4 edge membership value = 0.000000 0.0000000 2 - 1 edge membership value = 0.000000 0.0000000 2 - 1 edge membership value = 0.000000 0.0000000 3 - 1 edge membership value = 0.000000 0.0000000 3 - 2 edge membership value = 0.000000 0.0000000 3 - 2 edge membership value = 0.000000 0.0000000000000000000000000	Please enter (T,F) values of ver	tex:0.1 0
Please enter (T,F) values of vertex:0.1 0 Please enter (T,F) values of vertex:0.4 0 Please enter the edges (x to y)1 2 Please enter the edges (x to y)2 3 Please enter the edges (x to y)3 4 Please enter the edges (x to y)3 4 Please enter the edges (x to y)3 4 Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0 0 edges membership values of edge:0 0 edges membership values of edge:0 0 edges membership value 0.000000 0.000000 1 - 2 edge membership value 0.100000 0.000000 2 - 1 edge membership value 0.000000 0.000000 2 - 1 edge membership value 0.000000 0.000000 3 - 1 edge membership value 0.000000 0.000000 3 - 2 edge membership value 0.000000 0.000000	Please enter (T,F) values of ver	tex:0.3 0
Please enter (T,F) values of vertex:0.4 0 Please enter the edges (x to y)1 2 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)3 4 Please enter the edges (x to y)3 4 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 2 Please enter the edges (x to y)3 4 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 2 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0.0 edges membership values of complement of IFG are 1 - 2 edge membership value 0.000000 0.000000 1 - 4 edge membership value 0.000000 0.000000 2 - 1 edge membership value 0.000000 0.000000 2 - 1 edge membership value 0.000000 0.000000 3 - 1 edge membership value 0.000000 0.000000 3 - 2 edge membership value 0.000000 0.000000	Please enter (T,F) values of ver	tex:0.1 0
Please enter the edges (x to y)1 2 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)2 3 Please enter the edges (x to y)3 4 Please enter the edges (x to y)3 4 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.3 0 Please enter the edges (x to y)1 3 Please enter (T,F) values of edge:0 0 edges membership values of edge:0 0 edges membership value 0.000000 0.000000 1 - 3 edge membership value 0.100000 0.000000 2 - 1 edge membership value 0.000000 0.000000 2 - 4 edge membership value 0.000000 0.000000 3 - 1 edge membership value 0.000000 0.000000 3 - 2 edge membership value 0.000000 0.000000	Please enter (T,F) values of ver	tex:0.4 0
Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)2 3 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.3 0 Please enter the edges (x to y)4 3 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.100000 0.000000 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 4 - 2 edge membership value= 0.000000 0.0000000 3 - 4 edge membership value= 0.000000 0.0000000 3 - 4 edge membership value= 0.000000 0.0000000 3 - 4 edge membership value= 0.000000 0.0000000	Please enter the edges (x to y)1	2
Please enter the edges (x to y)2 3 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)3 4 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 2 Please enter the edges (x to y)4 2 Please enter the edges (x to y)4 3 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0.4 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000	Please enter (T,F) values of edg	e:0.1 0
Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)3 4 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 2 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.000000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Please enter the edges (x to y)2	3
Please enter the edges (x to y)3 4 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 1 Please enter the edges (x to y)4 2 Please enter the edges (x to y)4 2 Please enter (T,F) values of edge:0.3 0 Please enter the edges (x to y)1 3 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value 0.000000 0.000000 1 - 3 edge membership value 0.100000 0.000000 2 - 1 edge membership value 0.000000 0.000000 2 - 4 edge membership value 0.000000 0.000000 3 - 1 edge membership value 0.000000 0.000000 3 - 2 edge membership value 0.000000 0.000000 3 - 4 edge membership value 0.000000 0.000000	Please enter (T,F) values of edg	e:0.1 0
Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.1 0 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0 0 edges enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Please enter the edges (x to y)3	4
Please enter the edges (x to y)4 1 Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 2 Please enter the edges (x to y)1 3 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Please enter (T,F) values of edg	e:0.1 0
Please enter (T,F) values of edge:0.1 0 Please enter the edges (x to y)4 2 Please enter (T,F) values of edge:0.3 0 Please enter (T,F) values of edge:0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.100000 0.000000 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Please enter the edges (x to y)4	1
Please enter the edges (x to y)4 2 Please enter (T,F) values of edge:0.3 0 Please enter the edges (x to y)1 3 Please enter (T,F) values of complement of IFG are edges membership values of complement of IFG are 1 - 2 edge membership value = 0.000000 0.000000 1 - 3 edge membership value = 0.000000 0.000000 2 - 1 edge membership value = 0.000000 0.000000 2 - 3 edge membership value = 0.000000 0.000000 2 - 4 edge membership value = 0.000000 0.000000 3 - 1 edge membership value = 0.100000 0.000000 3 - 2 edge membership value = 0.000000 0.000000 3 - 4 edge membership value = 0.000000 0.000000	Please enter (T,F) values of edg	e:0.1 0
Please enter (T,F) values of edge:0.3 0 Please enter the edges (x to y)1 3 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.000000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 2 edge membership value= 0.000000 0.000000	Please enter the edges (x to y)4	2
Please enter the edges (x to y)1 3 Please enter (T,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.100000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Please enter (T,F) values of edg	e:0.3 0
Please enter (1,F) values of edge:0 0 edges membership values of complement of IFG are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.100000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.00000000000000000000000000	Please enter the edges $(x to y)$ 1	3
edges membership values of complement of 1% are 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.100000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Please enter (I,F) values of edg	e:00
 1 - 2 edge membership value= 0.000000 0.000000 1 - 3 edge membership value= 0.100000 0.000000 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000 	edges membership values of compl	ement of IFG are:
 1 - 3 edge membership value= 0.100000 0.000000 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000 	1 – 2 edge membership value= 0.0	00000 0.000000
 1 - 4 edge membership value= 0.000000 0.000000 2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.100000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000 	1 - 3 edge membership value= 0.1	00000 0.000000
2 - 1 edge membership value= 0.000000 0.000000 2 - 3 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	1 – 4 edge membership value= 0.0	00000 0.000000
2 - 3 edge membership value= 0.000000 0.000000 2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	2 – 1 edge membership value= 0.0	00000 0.000000
2 - 4 edge membership value= 0.000000 0.000000 3 - 1 edge membership value= 0.100000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	Z - 3 edge membership value= 0.0	00000 0.000000
5 - 1 edge membership value= 0.000000 0.000000 3 - 2 edge membership value= 0.000000 0.000000 3 - 4 edge membership value= 0.000000 0.000000	z – 4 eage membership value= 0.0	00000 0.000000
3 - 4 edge membership value= 0.000000 0.0000000	5 - 1 euge membership value= 0.1	00000 0.000000
5 – 4 eage membership value= 0.0000000 0.0000000	5 - 2 euge membership value= 0.0	
	5 - 4 eage membership value= 0.0	00000 0.000000

Example 3 Consider an uniform intuitionistic fuzzy graph as shown in Fig.7



Using the above pseudo code, the output result for the complement of uniform intuitionistic fuzzy graph is as follows

C:	•	C:V	Docum	ents and S	ettings\s	aidV	Bureau\dey	article	- 🗆
P]	lea	ase	ente	r no of	vertex	4			
P]	lea	ase	ente	er (T,F)	values	of	vertex: (0.3 0.6	
P]	lea	ase	ente	r (T,F)	values	of	vertex: (0.3 0.6	
P]	lea	ase	ente	r (T,F)	values	of	vertex:	0.3 0.6	
P]	lea	ase	ente	r (T,F)	values	of	vertex: (0.3 0.6	
P]	lea	ase	ente	r the ed	lges (x	to	y)1 2		
P]	lea	ase	ente	r (T,F)	values	of	edge: 0.3	30.6	
P]	lea	ase	ente	r the ed	lges (x	to	y)Ž 3		
P]	lea	ase	ente	r (T,F)	values	of	edge: 0.3	30.6	
P]	lea	ase	ente	r the ed	lges (x	to	y)3 4		
P]	lea	ase	ente	r (T,F)	values	of	edge: 0.3	30.6	
P 1	lea	ase	ente	r the ed	lges (x	to	y)1 3		
P]	lea	ase	ente	r (T,F)	values	of	edge: 0.3	30.6	
P 1	lea	ase	ente	r the ed	lges (x	to	y)2 4		
P]	lea	ase	ente	r (T,F)	values	of	edge: 0.3	30.6	
P]	lea	ase	ente	r the ed	lges (x	to	y)4 1		
P 1	lea	ase	ente	r (T,F)	values	of	edge: 0.:	30.6	
ed	lge	es	membe	ership va	alues o:	f co	omplement	of IFG	are:
1		2	edge	membersl	hip val	ue=	0.000000	0.0000	10
1		3	edge	membersl	hip val	ue=	0.00000	0.0000	10
1		4	edge	membersl	hip val	ue =	0.00000	0.0000	0
2		1	edge	membersl	hip val	ue =	0.00000	0.0000	10
2		3	edge	membersl	hip val	ue=	0.00000	0.0000	10
2		4	edge	membersl	hip val	ue =	0.00000	0.00000	0
3		1	edge	membersl	hip val	ue =	0.000000	0.0000	0
3		2	edge	membersl	hip val	ue =	0.000000	0.0000	0
3		4	edge	membersl	hip val	ue =	0.00000	0.00000	0

VI. CONCLUSION

In this paper, we propose a new uniform single valued neutrosophic graph and an algorithm for computing its complement. Some theorems of the uniform single valued neutrosophic graph have been examined. The algorithm in this research also enables us to compute the complement of uniform single valued neutrosophic graph. In the future, we plan to extended this algorithm for computing the complement of others variants of single valued neutrosophic graphs.

ACKNOWLEDGMENT

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the paper

REFERENCES

- F. Smarandache, Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset: Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics, Pons Editions Brussels, 2016,170p
- [2] F. Smarandache, Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998; http://fs.gallup.unm.edu/eBookneutrosophics6.pdf (last edition online).
- [3] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, Single valued Neutrosophic Sets, Multisspace and Multistructure 4, 2010, pp. 410-413.

- [4] K. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, 1986, pp. 87-96.
- [5] K. R. S. Narayan and M. S. Sunitha, Some Remarks on Complement of Fuzzy Graphs, IOSR Journal of Mathematics (IOSR-JM), Volume 7, Issue 4 ,2013, pp. 75-77.
- [6] K. Bhanu Chander, T. V. Pradeep Kumar, A Note on Uniform Fuzzy Graphs and the Relationship between Regularity and Uniformity of a Fuzzy Graph International Journal of Innovative Research in Science, Engineering and Technology,2017,pp.2425-2431.
- [7] L. Zadeh, Fuzzy sets. Inform and Control, 8, 1965, pp.338-353.
- [8] M, Vijaya, Complement properties on normal product of strong fuzzy graphs, International Journal of Science and Research, Vol. 4, Issue7, 2015, pp.749-751.
- [9] M.Akram, B.Davvaz, Strong intuitionstic fuzzy graph, FILO-MAT, vol, 26,2012,pp.177-196.
- [10] S. Broumi, A. Bakali, M, Talea, and F, Smarandache, Isolated Single Valued Neutrosophic Graphs. Neutrosophic Sets and Systems, Vol. 11, 2016, pp.74-78.
- [11] S. Broumi, F. Smarandache, M. Talea and A. Bakali, An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. Applied Mechanics and Materials, vol.841, 2016, pp.184-191.
- [12] S. Broumi, F. Smarandache, M. Talea and A. Bakali, Decision-Making Method Based On the Interval Valued Neutrosophic Graph, Future Technologie, 2016, IEEE, pp 44-50.
- [13] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "Single Valued Neutrosophic Graphs," Journal of New Theory, N 10, 2016, pp. 86-101.
- [14] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Interval Valued Neutrosophic Graphs, Critical Review, XII, 2016. pp.5-33.
- [15] S. Broumi, M. Talea, F. Smarandache and A. Bakali, Single Valued Neutrosophic Graphs: Degree, Order and Size. IEEE International Conference on Fuzzy Systems (FUZZ), 2016, pp. 2444-2451
- [16] More information on http://fs.gallup.unm.edu/NSS/.

Appendix

#include<stdio.h>

#include<conio.h>

#define max 100
typedef struct {
float
Truth_membership,Indterminate_membership,False_mem
bership;
}fuzzy;
fuzzy
element[max][max],compliment[max][max];//element store the membership value of vertex.Compliment store the
value of complimented graph.

int vertex;//store total number of vertex.



float vertex_membership[max][6];//store membership va-	1].Truth_membership;//store the truth-membership value				
lue of vertex.	of edge.				
void input()	element[destiny-1][origin-				
{	1].Indterminate_membership=element[origin-1][destiny-				
int i,origin,destiny;//origin & destiny store the no. of ver-	1].Indterminate_membership;//store the indterminate-				
tex.And i for iteration.	membership value of edge.				
<pre>printf("Please enter no of vertex:");</pre>	element[destiny-1][origin-				
<pre>scanf("%d",&vertex);</pre>	1].False_membership=element[origin-1][destiny-				
for(i=0;i <vertex;i++)< td=""><td>1].False_membership;//store the False-membership value</td></vertex;i++)<>	1].False_membership;//store the False-membership value				
{	of edge.				
printf("Please enter (T,I,F)menbership values of	if(element[origin-1][destiny-				
vertex:");	1].Truth_membership+element[origin-1][destiny-				
scanf("%f%f%f",&vertex_membership[i][0],&ver	1].Indterminate_membership+element[origin-1][destiny-				
<pre>tex_membership[i][1],&vertex_membership[i][2]);//store</pre>	1].False_membership>3)//store the membership value of				
the membership value of vertex	edge.				
if(vertex_membership[i][0]+vertex_membership[i	{				
][1]+vertex_membership[i][2]>=3&&(vertex_membership	<pre>printf("Error! Invalid input\n");</pre>				
[i][0]<=3&&vertex_membership[i][1]&&vertex_members	i;				
hip[i][2]))	}				
- {	}				
<pre>printf("Error Invalid input\n");</pre>	}				
i;					
}	}				
}	void output()				
for(i=0;i <vertex*(vertex-1) 2;i++)<="" td=""><td>{</td></vertex*(vertex-1)>	{				
{	int i,j;				
printf("Please enter the edges (x to y):");	float maximum,minimum,maximuma;				
scanf("%d%d",&origin,&destiny);	printf("The complement of Single valued neutro-				
	sophic graphs is:\n");				
if(origin>vertex destiny>vertex origin<=0 destin	for(i=0;i <vertex;i++)< td=""></vertex;i++)<>				
y<=0 destiny==origin)	{				
{	for(j=0;j <vertex;j++)< td=""></vertex;j++)<>				
<pre>printf("Error! Invalid input\n");</pre>	{				
i;	if(i==i)				
}	i++:				
else	if(vertex membership[i][0]>vertex membership[j][0])				
{	minimum=vertex membership[i][0]://find minimum value				
printf("Please enter (T,I,F)membership values of	between two vertex.				
edge:");	else				
scanf("%f%f%f",&element[origin-1][destiny-	minimum=vertex membership[i][0]://find minimum value				
1].Truth_membership,&element[origin-1][destiny-	between two vertex.				
1].Indterminate_membership,&element[origin-1][destiny-	if(vertex membership[i][1]>vertex membership[i][1])				
1].False_membership);//store th membership value of ed-	maximum=vertex_membership[i][1];//find maximum va-				
ge.	lue between two vertex.				
	else				
element[destiny-1][origin	maximum=vertex membership[j][1]://find maximum va-				
1].Truth_membership=element[origin-1][destiny-	lue between two vertex.				

S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L. H. Son, D. Koley. Uniform Single Valued Neutrosophic Graphs

```
if(vertex_membership[i][2]>vertex_membership[j][2])
                                                              j++;
maximuma=vertex_membership[i][2];//find maximum va-
                                                             printf("%d - %d edge membership value= %f %f %f
                                                             \n",i+1,j+1,compliment[i][j].Truth_membership,complime
lue between two vertex.
else
                                                             nt[i][j].Indterminate_membership,compliment[i][j].False_
maximuma=vertex_membership[j][2];//find maximum va-
                                                             membership);//printing complimented graph.
lue between two vertex.
                                                              }
compliment[i][j].Truth_membership=minimum-
                                                              }
element[i][j].Truth_membership;//calculating compliment.
                                                              }
compliment[i][j].Indterminate_membership=maximum-
                                                             void main()
element[i][j].Indterminate_membership;//calculating
                                                              {
                                                                      input();
compliment.
compliment[i][j].False_membership=maximuma-
                                                                      output();
element[i][j].False_membership;//calculating compliment.
                                                                      getch();
                                                                        }
}
}
       for(i=0;i<vertex-1;i++)</pre>
{
        for(j=0;j<vertex;j++)</pre>
{
        if(i==j)
                                                                         Received: July 20, 2017. Accepted: August 8, 2017.
```