THE INFINITY OF TWIN PRIMES

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DEFINITION 1

\[ 6x \pm 1 \] are twin primes where \( x = 6nm \pm (n \pm m) \) has no solution for positive integers \( x, n, \) and \( m \).

DEFINITION 2

Given \( n \) and \( m \) are interchangeable,

all solutions for \( x \) are \( x \mod (6n + 1) \pm n = 0 \equiv (x \pm n) \mod (6n + 1) = 0 \), for all \( n \leq \sqrt{x}/6 \).

There are four results for each \( x, n \): \( (x \pm n) \mod (6n + 1) \).

DEFINITION 3

The distribution of \( x \) values with no solution is bounded by

\[
D(x) \geq x \prod_{n=1}^{\left\lceil \sqrt{x} \right\rceil} \frac{6n-3}{6n+1}
\]

such that \( 0 < \frac{D(x)}{x} \leq 1 \) for all \( x \).

DEFINITION 4

The average distribution of \( x \) values with no solution in a range defined by \( 6(n - 1)^2 \leq x \leq 6n^2 \), where \( n > 1 \), grows by a minimum rate of

\[
\left[ \frac{6(2n-1)}{6(2n-1)-1} \times \frac{6n-3}{6n+1} \right] > 1
\]

\( \therefore \) there will tend to exist more \( x \) values with no solution in subsequent ranges of \( \langle x \rangle \), where \( |\langle x \rangle| = 6(2n-1) \) and \( 6(n - 1)^2 \leq x \leq 6n^2 \), as \( n \) increases.

\( \therefore \) there will exist twin primes to infinity.