The Special Relativity Theory and Energy
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Abstract
Considering Special Relativity Theory, energy has been derived through the case of mass points’ collision applying the equation from the theory. But the difference of space distance and the difference of time distance are based on the different view for each frame of references. So relativistic mass and energy quantity assumed real quantity should be tried to derive directly and deductively from the same assumption for the theory.

1. Introduction
Applying the equation from the Special Relativity Theory, the mass points’ collision case has following scenario [1].
Space distance depends on the frame of reference based on the Special Relativity Theory. [2][3][4]
Time distance depends on the frame of reference based on the Special Relativity Theory.
Then mass point velocity (u) depends on the frame of reference.
Here applying momentum conservation law, mass also depends on the frame of reference. This means mass m is function of velocity u.

\[ m_u = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}} \]

But mass may be assumed real quantity and may not depend on the frame of reference. So considering the Special Relativity Theory, mass and energy should be tried to derive directly and deductively based on the same assumption for the theory and assumptions in general.

2. Derivation of energy
On the Special Relativity Theory, space and time are equivalent dimensions, and time distance t is converted to ct as a space dimension.
On the collision case, mass is changed depending on the frame of reference. But in general, mass can be changed in a frame of reference, for example, in the case of nuclear fission.
Considering above and others, deviation of energy E is
\[ \frac{dE}{dt} = \text{energy deviation on mass m change + work done on velocity u change} \]

\[ = \frac{1}{2} \frac{dm}{dt} (u^2 + c^2) + (um \frac{du}{dt} + cm \frac{dc}{dt}) \]

\[ = \frac{d}{dt} \left( \frac{1}{2} m (u^2 + c^2) \right) \]

(1)

Then total energy of mass point is

\[ E = \frac{1}{2} m (u^2 + c^2) \]

(2)

Here energy deviation based on mass change cannot be measured as work done because it changes to feat, light and various energy.

3. Close examination

Total energy of mass point is (2)

But actually we can consider about the work the mass could do in each following cases.

a) Mass is never changing \((\frac{dm}{dt} = 0)\).

\[ \frac{dE}{dt} = \frac{1}{2} \frac{dm}{dt} (u^2 + c^2) + um \frac{du}{dt} + cm \frac{dc}{dt} = um \frac{du}{dt} = \frac{1}{2} m \frac{du}{dt} \]

(because \(c\) is constant.)

Then

\[ E = \frac{1}{2} mu^2 \]

b) Mass can be changed \((\frac{dm}{dt} \neq 0)\) and \(u \neq 0\)

On (2), \( E = \frac{1}{2} m (u^2 + c^2) \)

c) Mass can be changed \((\frac{dm}{dt} \neq 0)\) and \(u = 0\)

On (2), \( E = \frac{1}{2} mc^2 \)
Reference


