

What went wrong with the “interpretation” of Quantum Theory?

Robert H. McEachern

This slideshow is a preliminary account of how the misinterpretation of quantum theory originated.

What's the Matter - with Physics?

With the Theory (the computational model)?

With the **Interpretation** of the theory (the physical model)?

With the **Scientific Method**?

Why the confusion concerning the things (matter) that exist?

Are they particles? Or waves? Or wave-particle dualities?

Is the Double Slit Experiment - “the *only* mystery?”

Is Bell's Inequality Theorem & experiments - a new mystery?

What's the ultimate source of all the uncertainty?

The Heisenberg Uncertainty Principle, or its (mis)interpretation?

The underlying Math: Fourier Analysis & Information Theory

“Could you really persuade, if we don’t listen?”

Plato, “The Republic”, book 1 327c

“The trouble with people is not that they don’t know
but that they know so much that ain’t so.”

Josh Billings (Henry Wheeler Shaw)

“A new scientific truth does not triumph by convincing
its opponents and making them see the light, but rather
because its opponents eventually die, and a new
generation grows up that is familiar with it.”

Max Planck

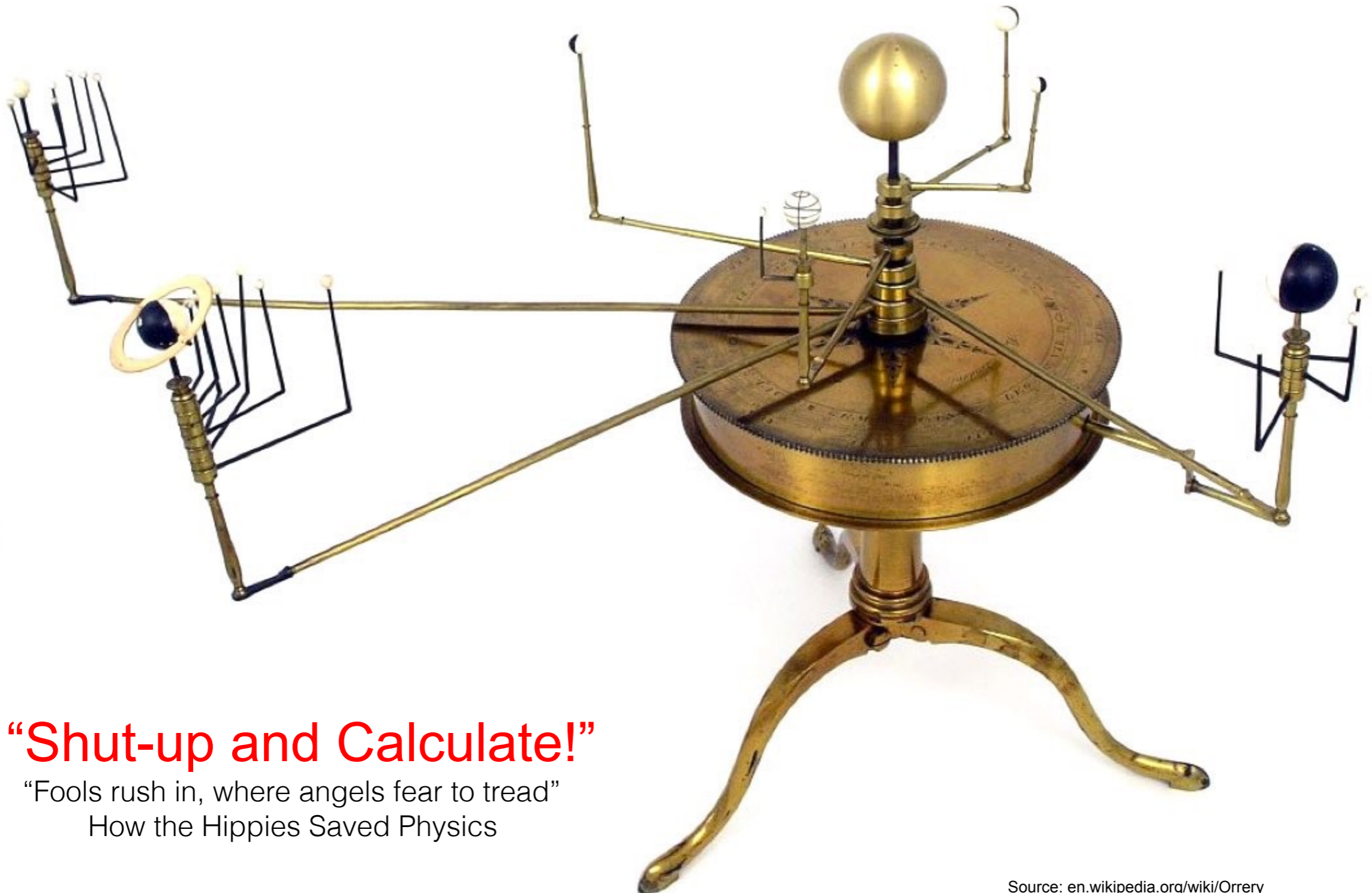
“Claude (Shannon) told me this story. He may have been kidding, but it illustrates both his sense of humor and his delightfully self deprecating nature, and it certainly could be true. The story is that Claude was in the middle of giving a lecture to mathematicians in Princeton, when the door in the back of the room opens, and in walks Albert Einstein. Einstein stands listening for a few minutes, whispers something in the ear of someone in the back of the room, and leaves. At the end of the lecture, Claude hurries to the back of the room to find the person that Einstein had whispered too, to find out what the great man had to say about his work. The answer: Einstein had asked directions to the men’s room.”

Arthur Lewbel

That is about as much interest as any physicist had in Shannon’s Information Theory, for the first forty years it existed.

A good *Computational* model may not necessarily be a good *Physical* model

Epistemology versus Ontology



“Shut-up and Calculate!”

“Fools rush in, where angels fear to tread”
How the Hippies Saved Physics

There is a distinction between the properties of a mathematically constructed **map** describing a territory, and those of the **territory** itself



Greenland is not really three times larger than Australia

Physicists have failed to distinguish between the properties of the "reality" they are attempting to describe, and the properties of their mathematical "descriptions of that reality"; properties of descriptions of observations vs. properties of the things observed.

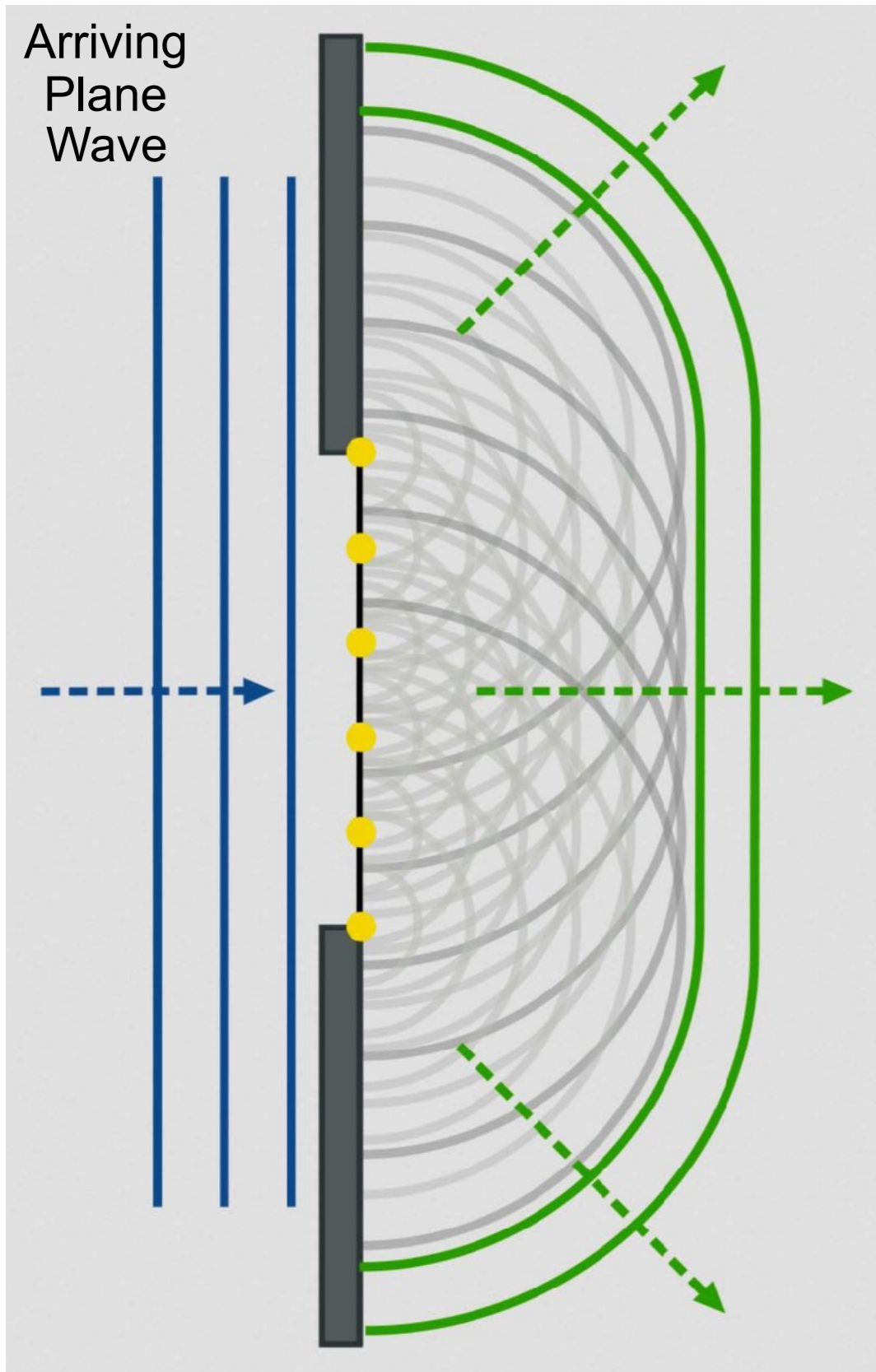
But they are two very different things.

For example, all such descriptions, written in the English language, contain only 26 letters, a-z. Should one assume from this property of the description, that this must correspond to some fundamental property of the entities being described? Perhaps the universe is actually constructed from just 26 fundamental "letters" in a manner similar to that in which genetic material is constructed from the 4 "letters" of the DNA code. This assumption may seem absurd, but it is no more absurd than the one physicists have made.

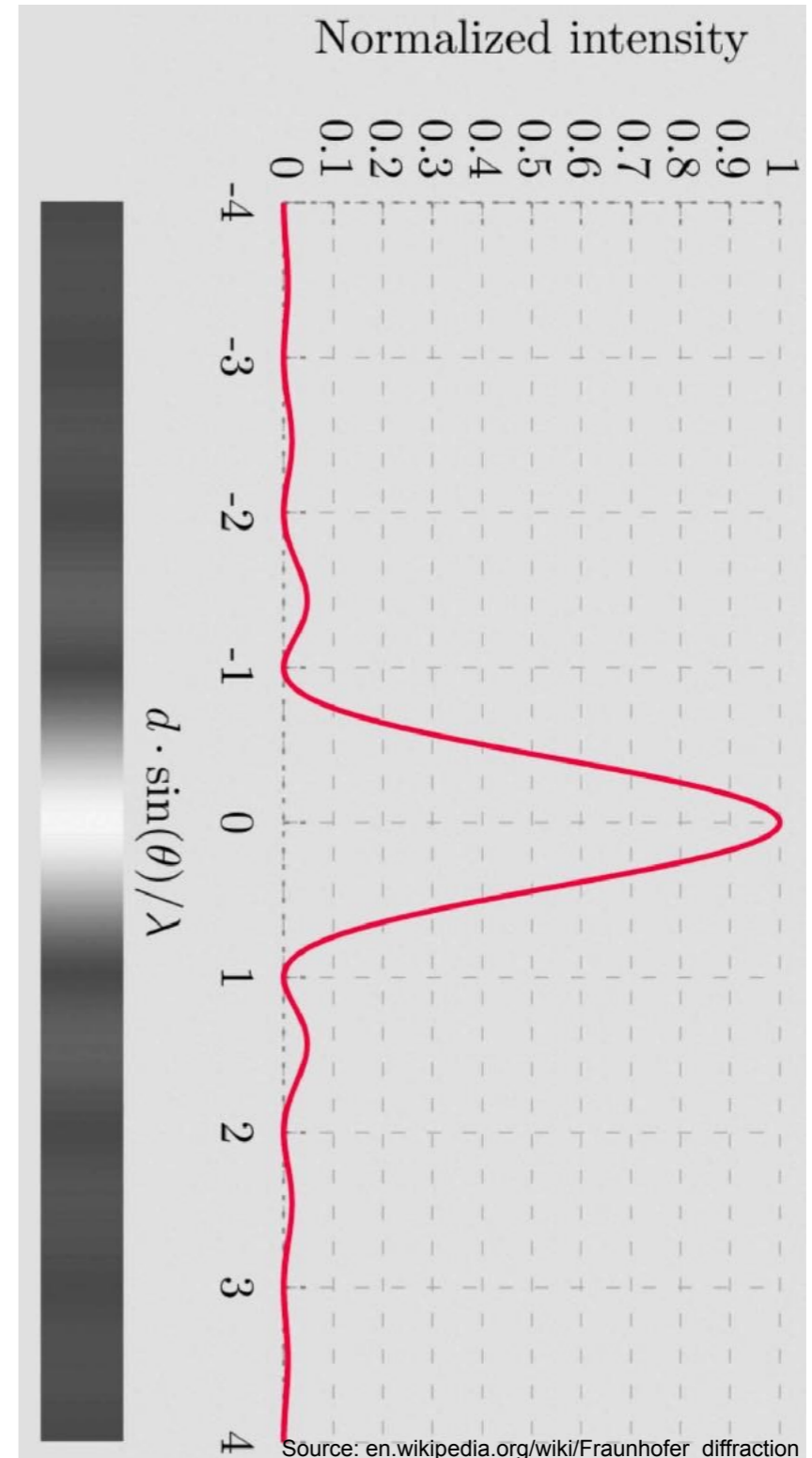
Properties like the Uncertainty Principle, Superposition and Entanglement, are all properties of the mathematical language (Fourier Analysis) being used to describe observations of the world; they are not necessarily properties of the world itself.

Consequently, it is inevitable that these properties will appear in all such descriptions of reality, regardless of whether or not they are properties of the entities being described, just as it is inevitable that the 26 letters appear in all such descriptions written in English.

Huygen's Principle

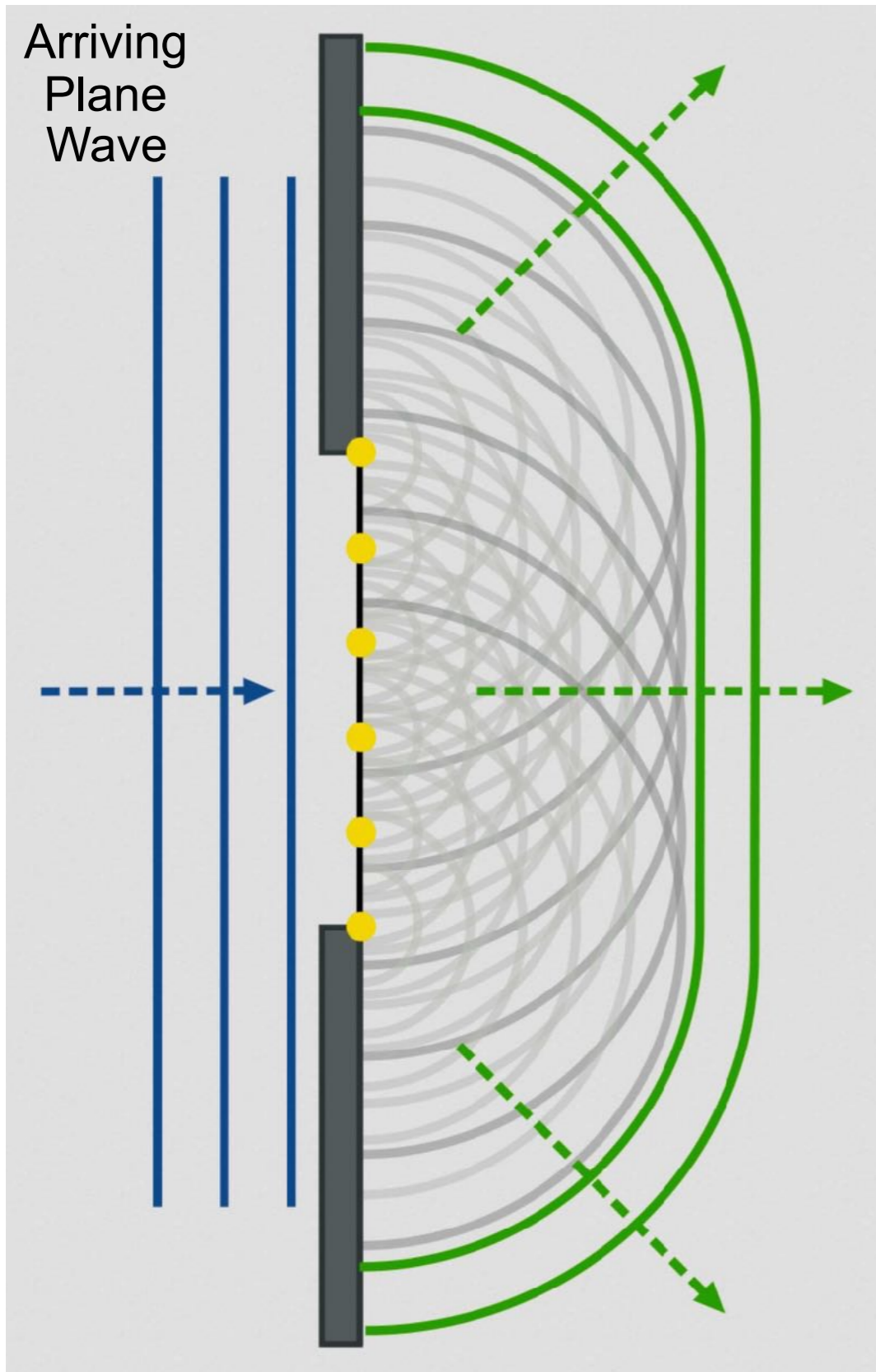


Source: en.wikipedia.org/wiki/Huygens-Fresnel_principle

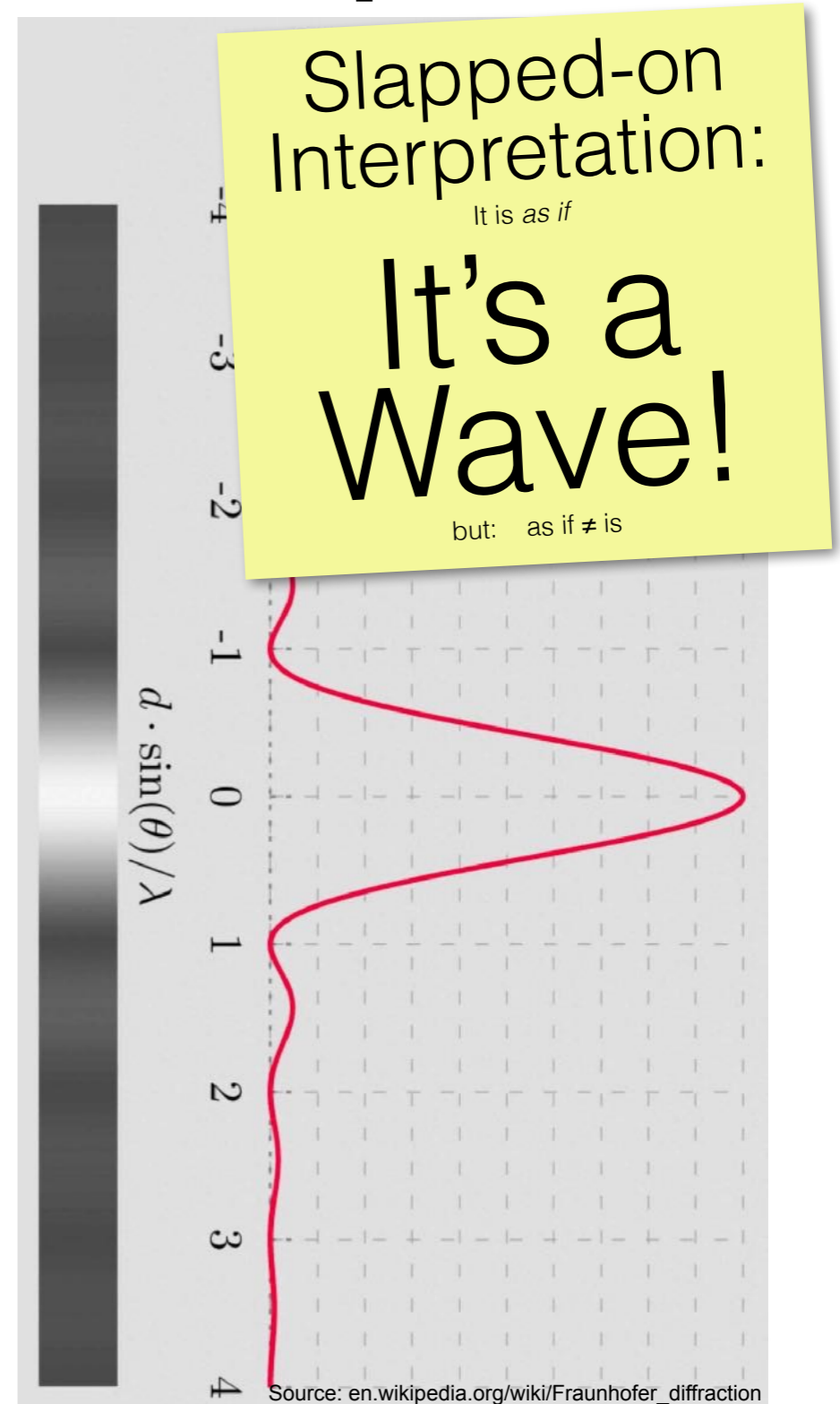


Expanding circular waves **interfere** with each other, creating a **diffraction** pattern

Huygen's Principle

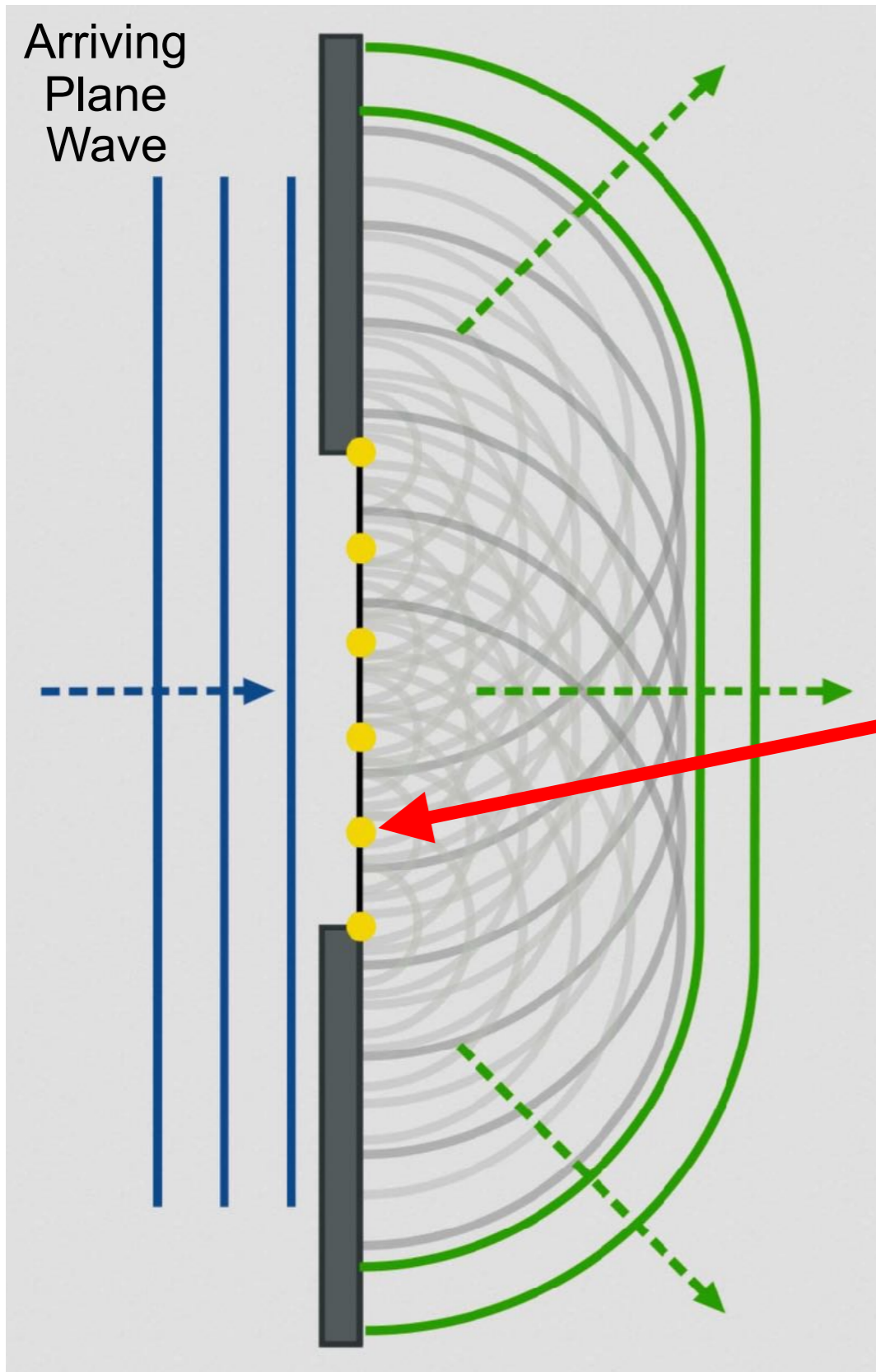


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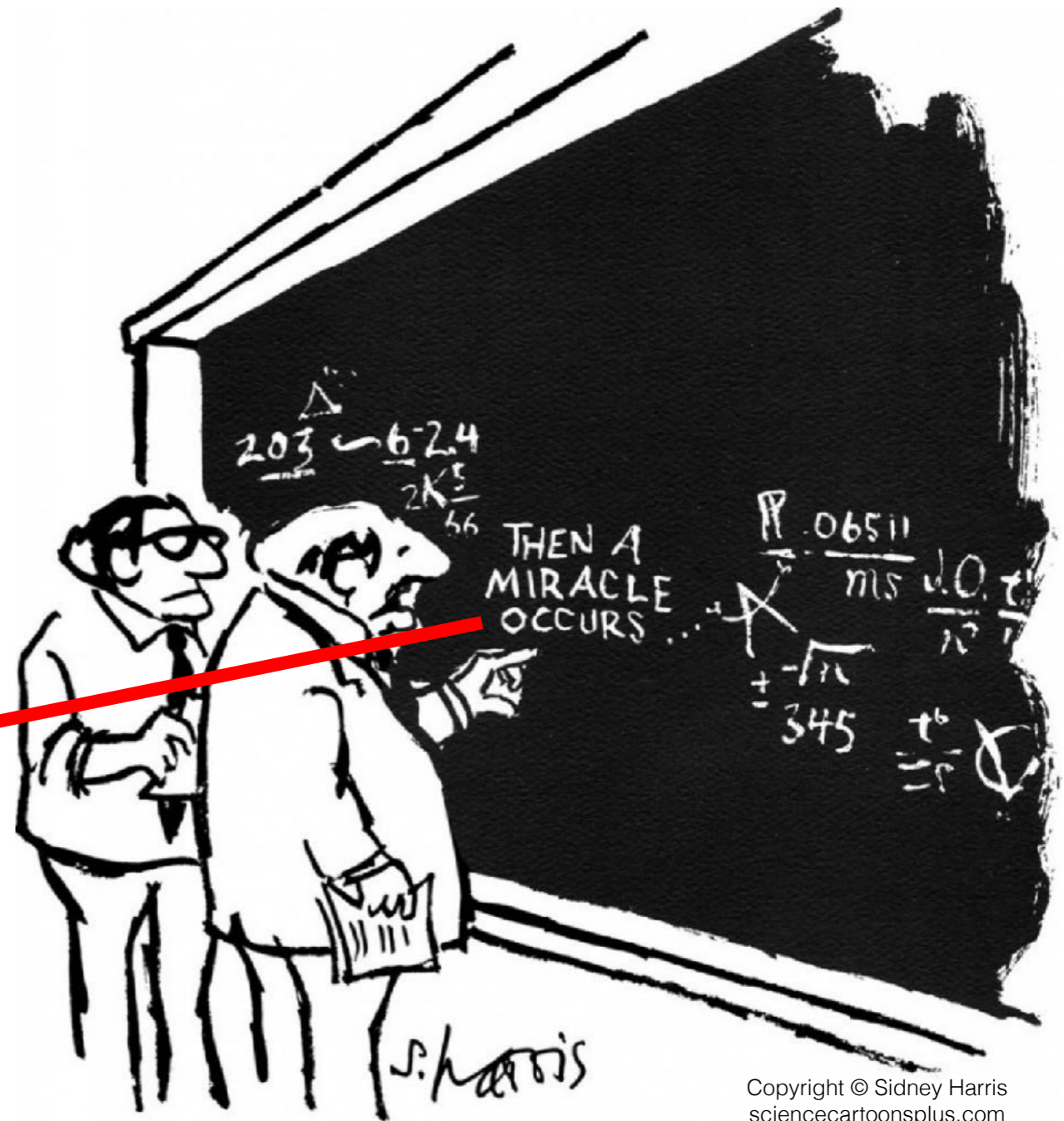


Expanding circular waves **interfere** with each other, creating a **diffraction** pattern

Huygen's Principle



Source: en.wikipedia.org/wiki/Huygens-Fresnel_principle



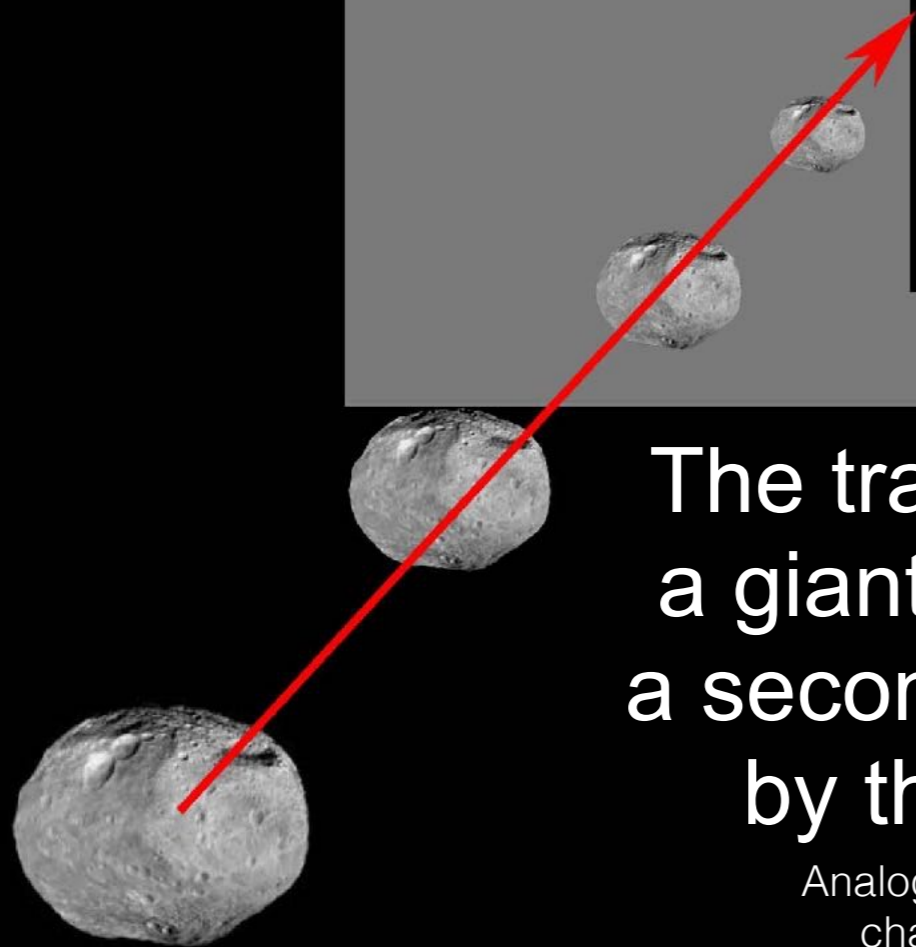
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"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

What miracle causes these circular waves to appear?

Wave Interference? or Particle Scattering?

Huge steel plate
100,000 km long



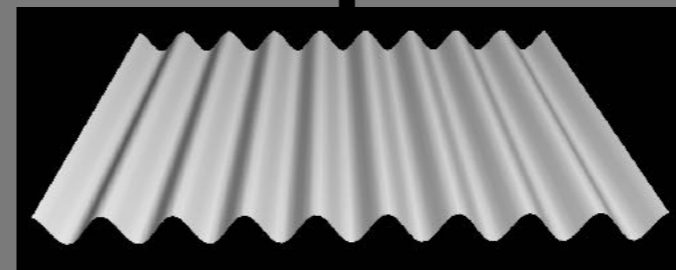
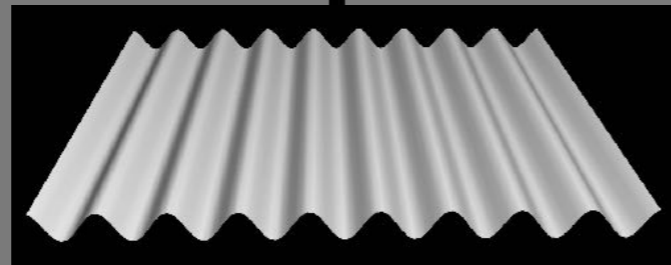
The trajectory of an Asteroid passing through a giant slit is effected by the missing mass of a second slit: the **Field** at the first slit is altered by the mere existence of the second slit:

Analogous to how your mere existence near an old TV rabbit-ear antenna changes the Field at the antenna, and consequently, the TV picture

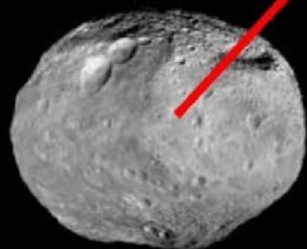
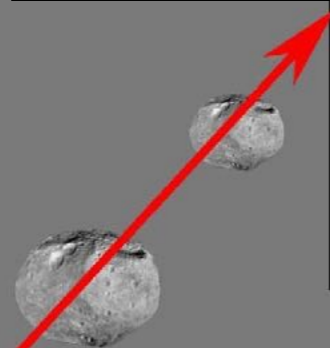
Wave Interference?

or

Particle Scattering?



Huge steel plate
100,000 km long



The trajectory of an Asteroid passing through a giant slit is effected by the missing mass of a second slit: the **Field** at the first slit is altered by the mere existence of the second slit:

This *interference* within the gravitational **Field** causes *interference* in the scattering pattern

Is the scattering **Field** smooth or rippled?

A 10 km asteroid would not "feel" a 1 meter ripple on a steel plate within each slit, as it scatters off a plate.

A 1 cm bullet would.

Reduce the size of the entire system:

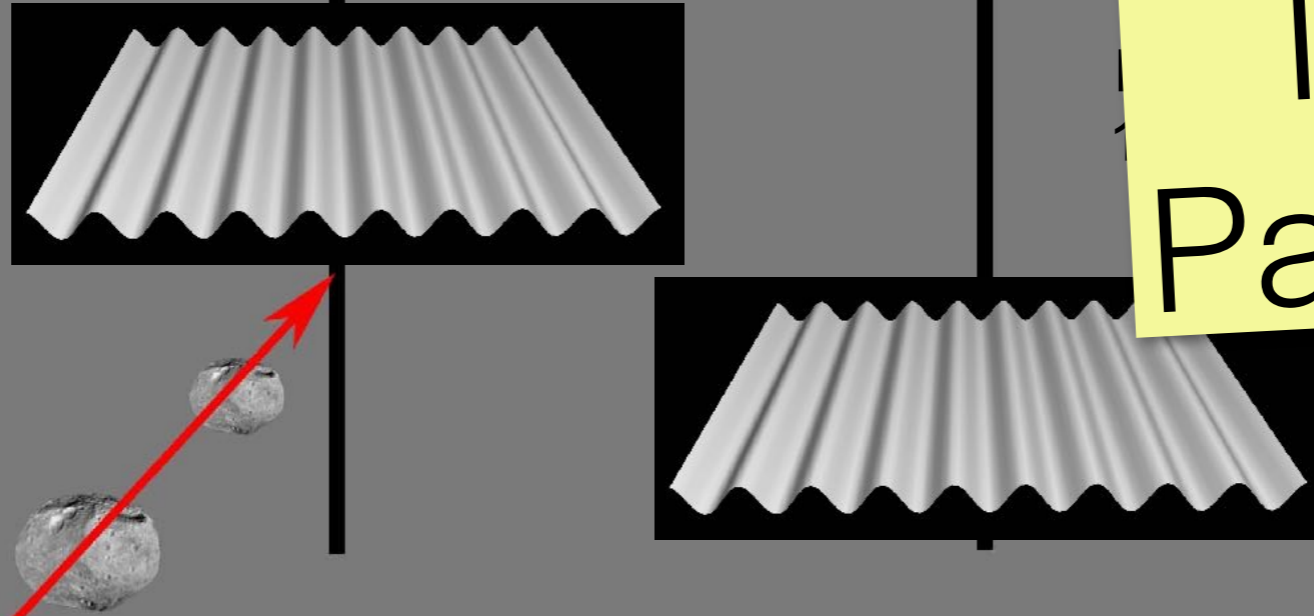
A tiny electron or photon, would "feel" any tiny ripple in the electromagnetic force **Field** within each slit.

A bullet would not.

Wave Interference?

or
Particle Scatter

Slapped-on
Interpretation:
It's a
Particle!



Is the scattering **Field** smooth or rippled?

A 10 km asteroid would not "feel" a 1 meter ripple on a steel plate within each slit, as it scatters off a plate.

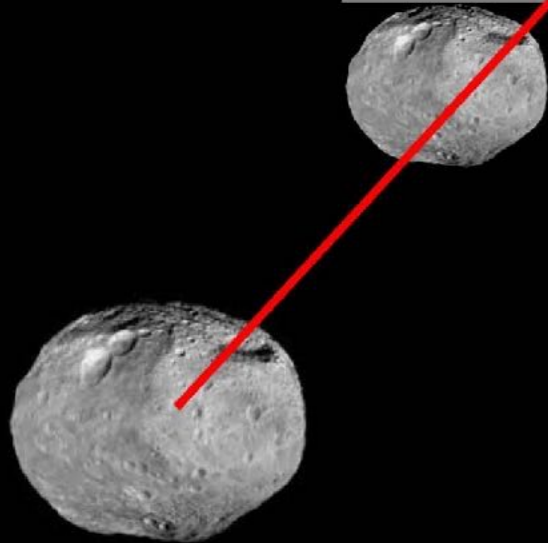
A 1 cm bullet would.

Reduce the size of the entire system:
A tiny electron or photon, would "feel" any tiny ripple in the electromagnetic force **Field** within each slit.

A bullet would not.

The trajectory of an Asteroid passing through a giant slit is effected by the missing mass of a second slit: the **Field** at the first slit is altered by the mere existence of the second slit:

This *interference* within the gravitational **Field** causes *interference* in the scattering pattern



Motion Tracking: Enabled by attaching easy to follow lights, to the points being tracked

Source: en.wikipedia.org/wiki/Motion-capture_acting

Problem:

How can the trajectory of a quantum “particle” be tracked mathematically, as it “flows” through a differential equation - an equation of motion?

Solution:

de Broglie did the mathematical equivalent, of attaching easy to follow, sinusoidal “waves”, to each particle and then used Fourier “Superposition” to localize each particle’s position

But what happens if you turn off all the other lights, and ignore, as de Broglie did, everything except the “light” from the waves, attached to the trackers?

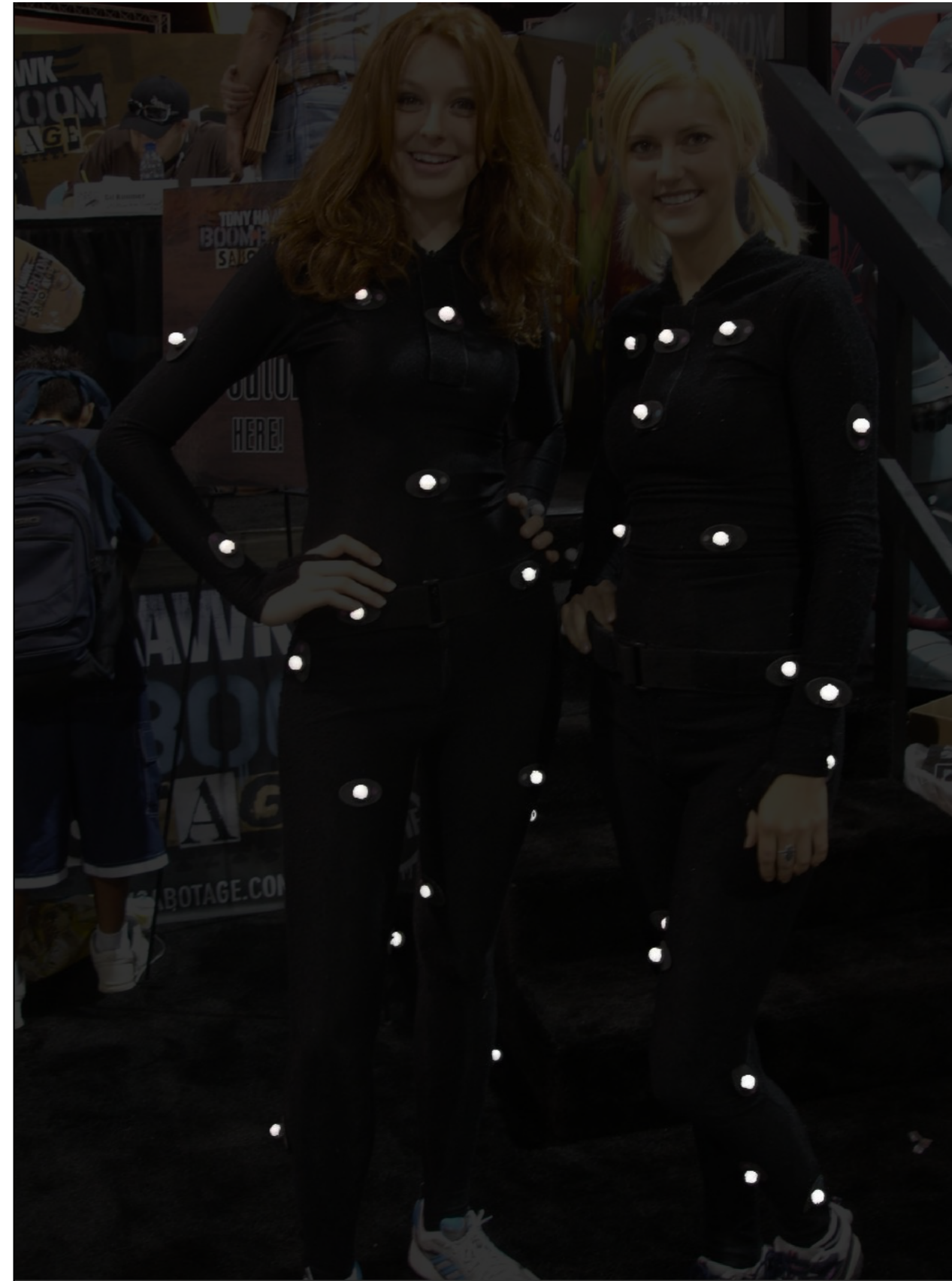


Motion Tracking: Enabled by attaching easy to follow lights, to the points being tracked

Source: en.wikipedia.org/wiki/Motion-capture_acting

What is this?
a Wave?
a Particle?
a Wave-Particle
Duality?

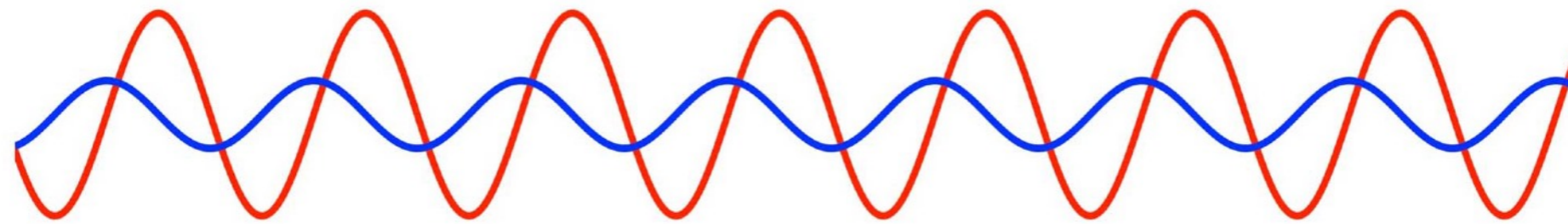
Origin of **Wave** vs. **Particle** Confusion:
This technique is great for tracking the positions and velocities of things!
But it is *useless* for determining the type of things the trackers are attached to!
Is it a Person? A Monkey? A Banana?
A Particle? A Wave?



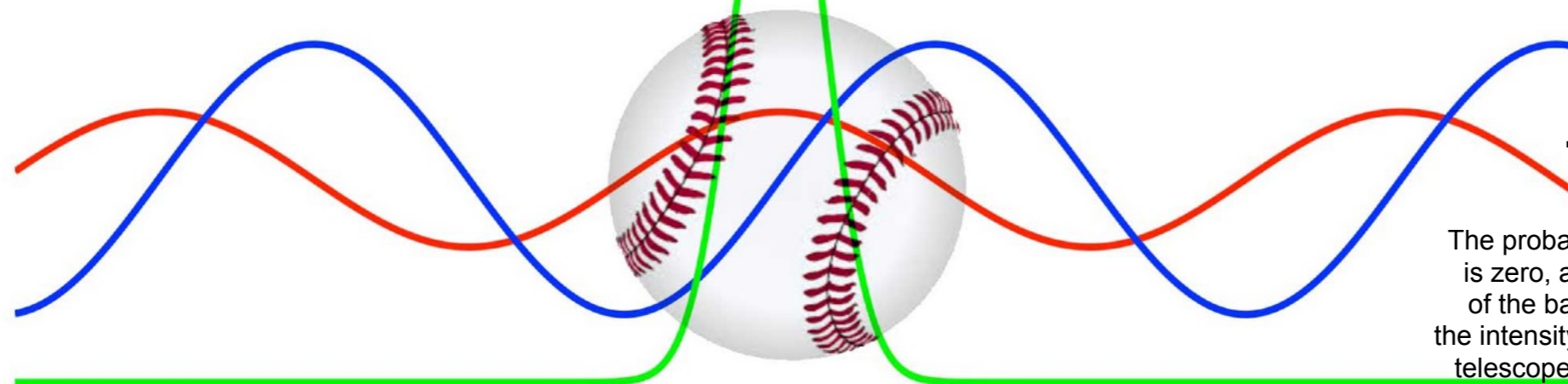
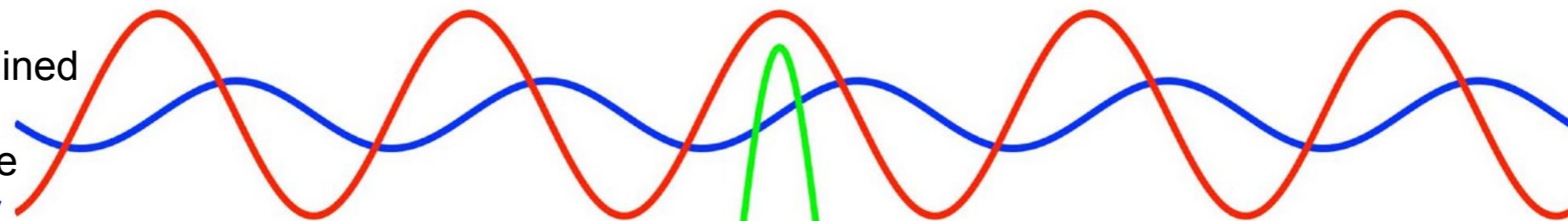
De Broglie **associated** a Wave with every Particle.

Each **Wave Packet**, which yields the probability of finding the particle at a given position, is mathematically constructed as a superposition of (**real & imaginary**) sinusoids: a Fourier Transform.

These sinusoids function like Motion Trackers, enabling the Packet's position and velocity to be tracked, as it passes through the equation of motion.



Phase is determined by the relative amplitudes of the **real & imaginary** components



The Born Rule:

The probability of finding the BaseBall is zero, at points beyond the radius of the ball. This is **exactly** equal to the intensity distribution, seen through a telescope with a narrow field of view, and pointed in the vicinity of the tracking light, associated with the ball. It is a histogram, that counts detected **quanta** of energy, within the field of view

The Trackers (waves) are *not* the same as the Thing (particle) being tracked

Mathematical Identities are *not* Physical Identities:

$$a(b+c) = ab+ac$$

But one side of the equation has twice as many multipliers as the other: Which side is the “correct” *physical* interpretation?

The *fundamental* problem with the **Scientific Method**:

Mathematically identical equations, that all yield the exact, same, theoretical result, to be compared to observations, may thus have *wildly* different Physical **Interpretations**

The standard interpretations are like Huygen’s miraculously appearing waves. But an alternative, is that the math describes an array (filter bank) of “telescopic” detectors, counting the arrivals of equal-energy particles

The math *only* describes how things behave, the observed **effects** (theory); it does *not* describe the underlying **causes** (interpretation), responsible for producing those effects

Fourier Synthesis = a Superposition

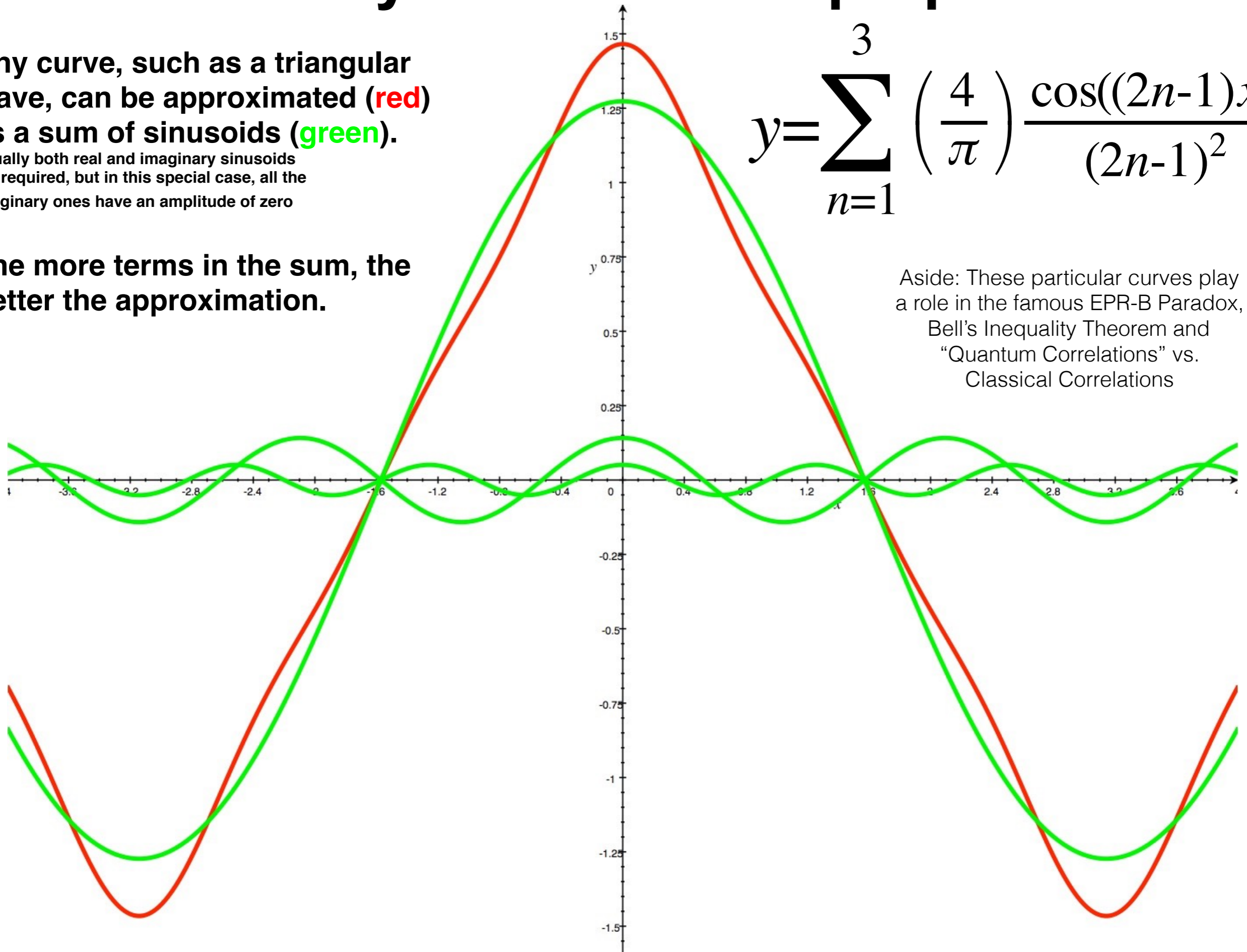
Any curve, such as a triangular wave, can be approximated (red) as a sum of sinusoids (green).

Usually both real and imaginary sinusoids are required, but in this special case, all the imaginary ones have an amplitude of zero

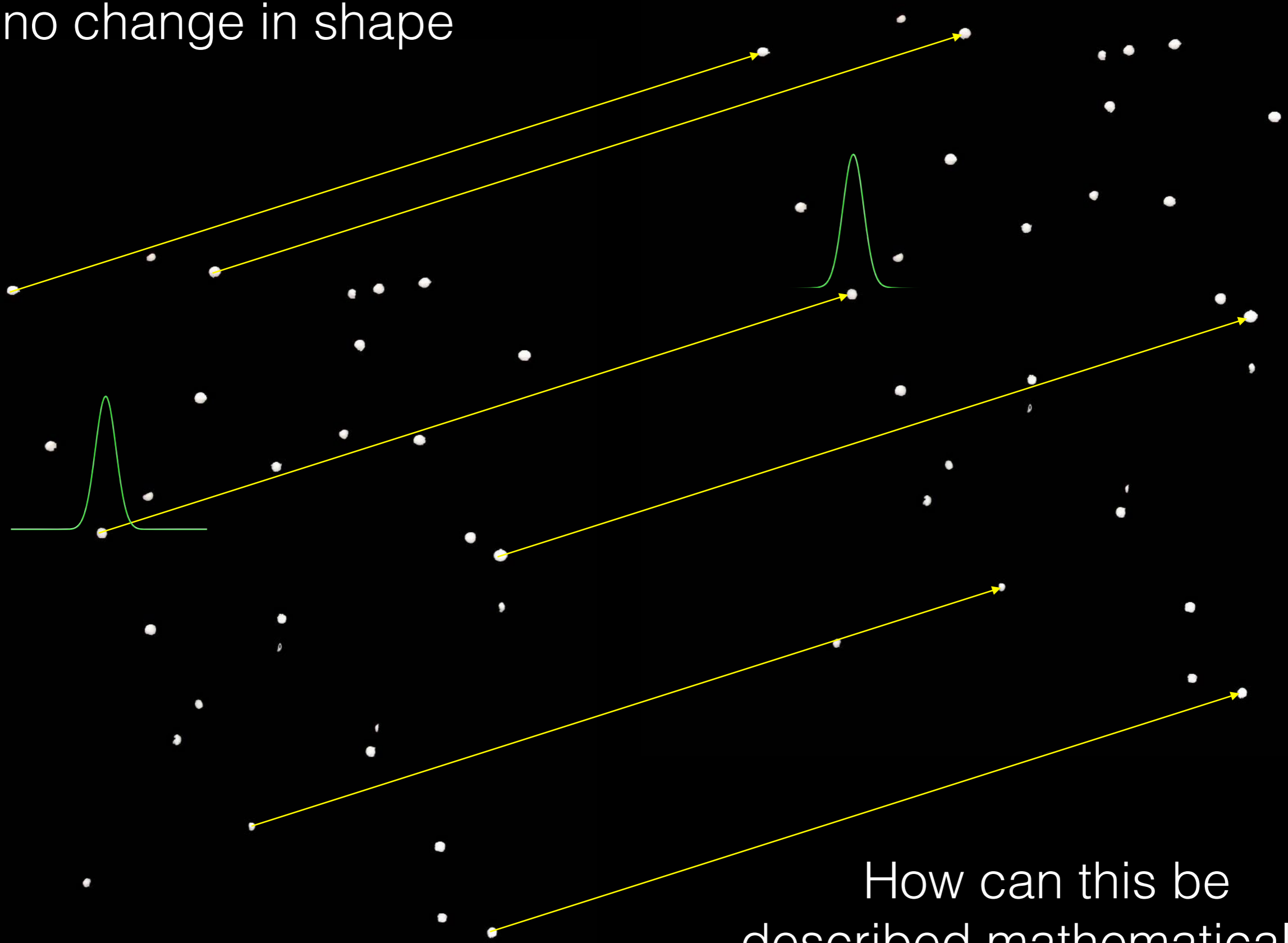
The more terms in the sum, the better the approximation.

$$y = \sum_{n=1}^3 \left(\frac{4}{\pi} \right) \frac{\cos((2n-1)x)}{(2n-1)^2}$$

Aside: These particular curves play a role in the famous EPR-B Paradox, Bell's Inequality Theorem and "Quantum Correlations" vs. Classical Correlations



Freely moving particle trackers (no force acting on them)
all moving together as a single object,
with no change in shape



How can this be
described mathematically?

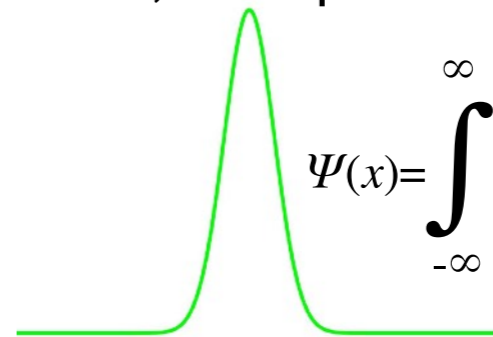
Whence the Wave Equation?

The Fourier Transform merely describes freely-moving (no forces) tracking-wave-packets, attached to each particle, in one dimension:

it's just Math, not Physics - no "Laws of Nature" are involved

It is necessary that the *Group Velocity* of a particle's wave packet, be equal to a classical particle's velocity: this implies that the angular frequency $\omega = \frac{\hbar^2 k^2}{2m}$

(1) The defining equation for any Fourier Transform:



$$\Psi(x) = \int_{-\infty}^{\infty} \phi(k) \exp[i(kx)] dk$$

(2) Or, as a function of time:
the attached wave-packet,
moving in a straight line,
as a function of time

$$\Psi(x, t) = \int_{-\infty}^{\infty} \phi(k) \exp[i(kx - \omega t)] dk$$

$$\Psi(x, t) = \int_{-\infty}^{\infty} \phi(k) \exp\left[i\left(kx - \frac{\hbar k^2}{2m} t\right)\right] dk$$

(3) first derivative of (2) with respect to time:

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \int_{-\infty}^{\infty} \phi(k) \frac{\hbar^2 k^2}{2m} \exp\left[i\left(kx - \frac{\hbar k^2}{2m} t\right)\right] dk$$

(4) second derivative of (2) with respect to x:

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) = \int_{-\infty}^{\infty} \phi(k) \frac{\hbar^2 k^2}{2m} \exp\left[i\left(kx - \frac{\hbar k^2}{2m} t\right)\right] dk$$

The last two equations are equal, so we have the **Schrödinger Wave Equation** for describing the motion of the tracking-wave-packets, attached to each particle

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t)$$

problem if $m=0$: photon

How can a force be introduced into this description of motion?

By demanding that the derivatives of the *average* momentum and the *average* position of the wave-packet agree with Newton's laws of motion (Bohm 9.26):

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x, t)$$

or

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[\frac{p^2}{2m} + V(x) \right] \Psi(x, t)$$

kinetic + potential energy

This *should* have been called the Schrödinger Heat Equation!
Not the Schrödinger Wave Equation!

$$\frac{\partial}{\partial t} T(x, t) = \left[k \frac{\partial^2}{\partial x^2} + S(x, t) \right] T(x, t)$$

Classical Heat Equation for the temperature, T, in a rod, with a heat source, S, proportional to the local temperature. (Berg and McGregor 3.2.5)

Waves, Heat or Heat-Wave dualities?

Light spreads out when a narrow beam flows through a small slit

But so does heat

So why assume only waves behave like this?

Classically, heat is thought of as being caused by particle motion

CHAPTER 1

LIGHT RAYS

Optics, the study of light, is conveniently divided into three fields, each of which requires a markedly different method of theoretical treatment. These are (a) *geometrical optics*, which is treated by the method of light rays, (b) *physical optics*, which is concerned with the nature of light and involves primarily the theory of waves, and (c) *quantum optics*, which deals with the interaction of light with the atomic entities of matter and which for an exact treatment requires the methods of quantum mechanics. This book deals almost entirely with (a) and (b), although some of the salient features of (c) will be outlined in the last chapter. These fields might preferably be called macroscopic, microscopic, and atomic optics as giving a more specific indication of their domains of applicability. When it is a question of the behavior of light on a large scale, the representation by means of rays is almost always sufficient.

1.1. Concept of a Ray of Light. The distinction between geometrical and physical optics appears at once when we attempt by means of diaphragms to isolate a single ray of light. In Fig. 1A let S represent

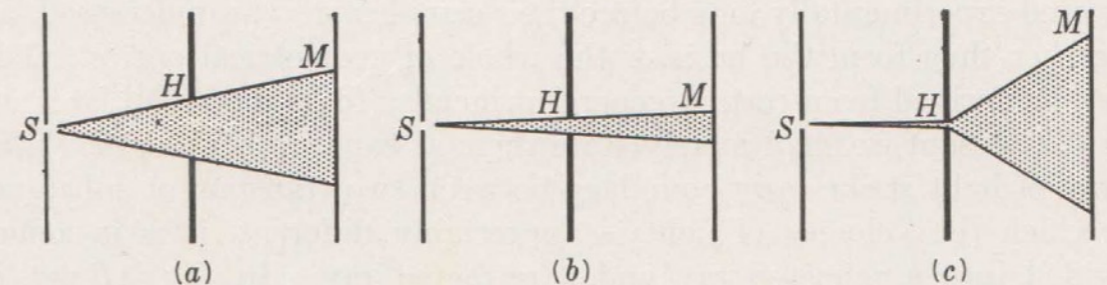


FIG. 1A. Attempt to isolate a single ray of light.

a source of light of the smallest possible size, a so-called *point source*. Such a source is commonly realized by focusing the light from the white-hot positive pole of a carbon arc on a metal screen pierced with a small hole.* If another opaque screen H provided with a much larger hole is now interposed between S and a white observing screen M [Fig. 1A(a)], only the portion of the latter lying between the straight lines drawn from

* The concentrated-arc lamp to be described in Sec. 21.2 also furnishes a very convenient way of approximating a point source.

Why are Observations Quantized?

Claude Shannon, the father of Information Theory, provided an answer, that has been ignored by the physics community, for 70 years:

Quantized Observations result from observations having:

a small Information content

not

a small physical size

of the object being observed!

What are these theories actually describing?

An unobserved emitter?
(think of a black hole)

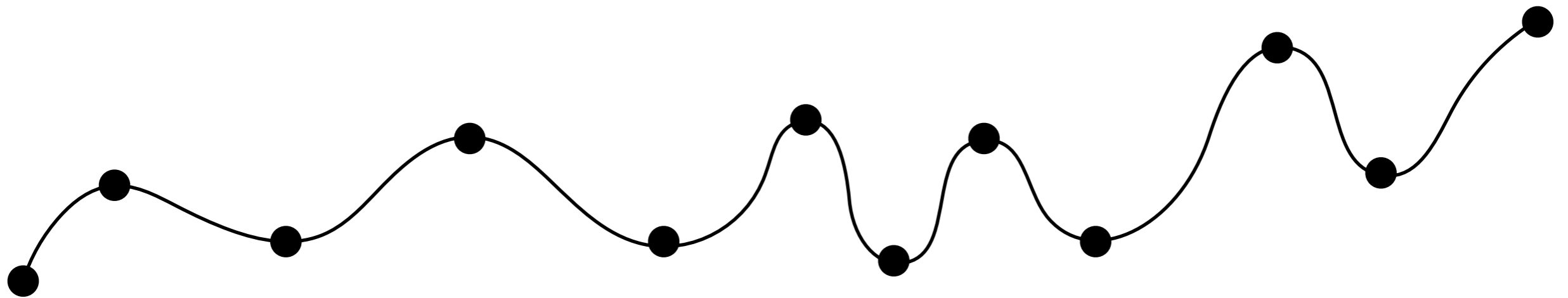
An observed emission?

Both?

Physics often confuses the two
(entropy vs. information)
Information Theory
resolves the confusion

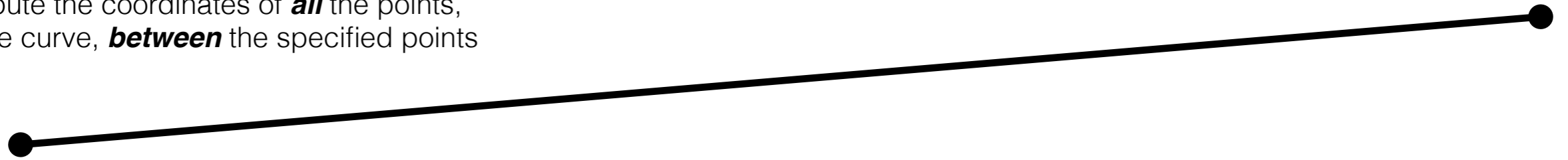
The Map vs. the Territory

What is the connection between a continuous function, such as those described by the differential equations of theoretical physics, and the discrete (quantized) measurements and observations produced by experimental physics?

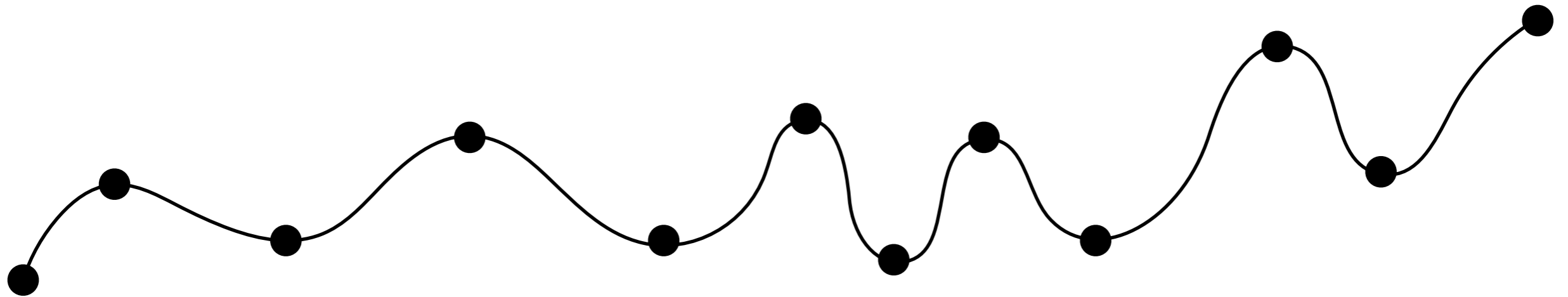


Two Points *Determine* a Line Segment

Determine = being able to accurately compute the coordinates of ***all*** the points, on the curve, ***between*** the specified points

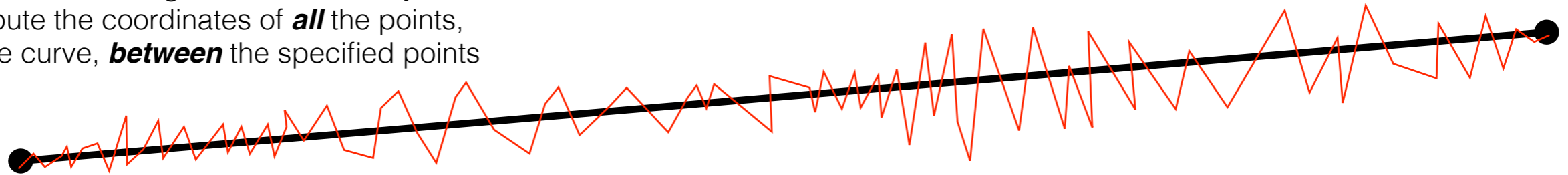


How many points are required to determine an arbitrary curve?



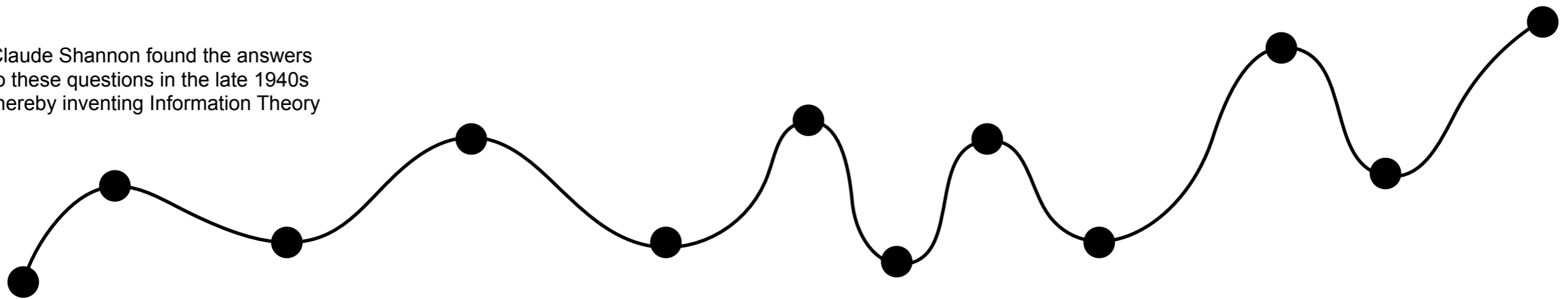
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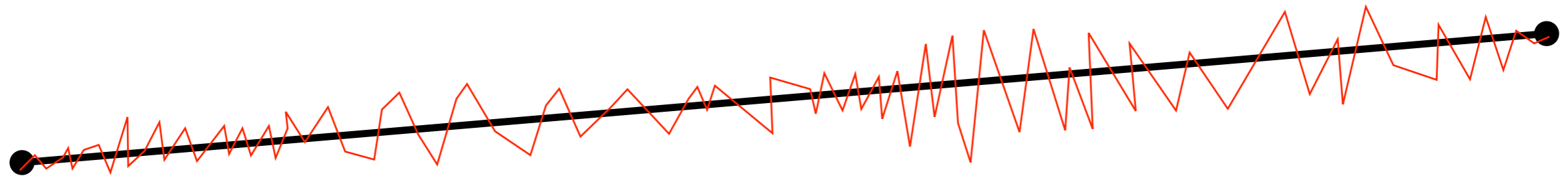
How many points are required to determine an arbitrary curve?

Claude Shannon found the answers to these questions in the late 1940s thereby inventing Information Theory



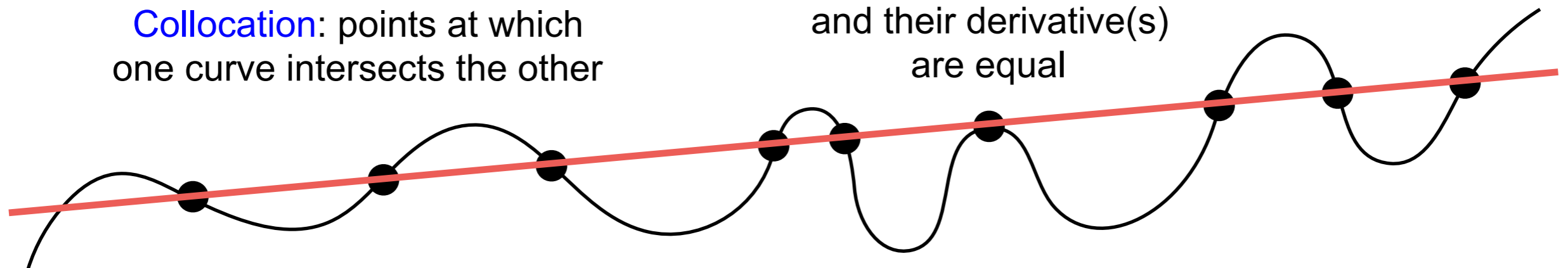
How many significant bits are required to specify the coordinates of each of those points, if the curve is *noisy*?

In what sense is one curve approximately equal to another? - *curve fitting*

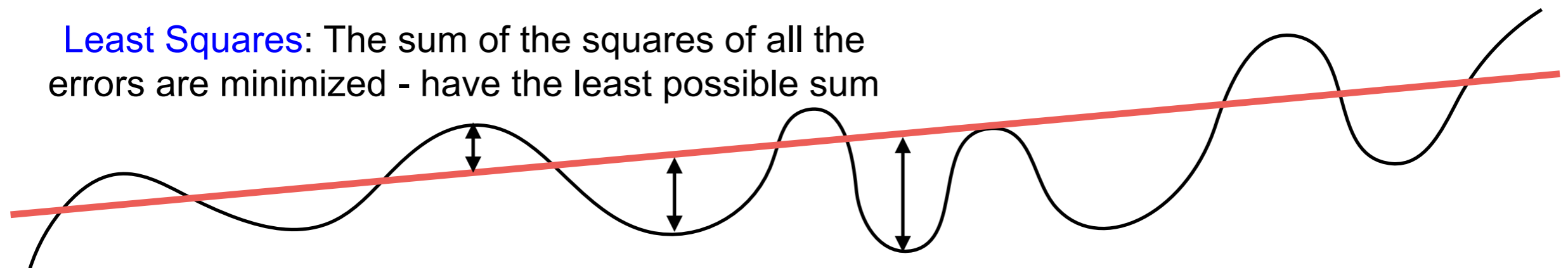


Collocation: points at which one curve intersects the other

Osculation: points at which both the curves and their derivative(s) are equal



Least Squares: The sum of the squares of all the errors are minimized - have the least possible sum



Fourier Analysis and Information Theory

Fourier Analysis will fit a curve in both the least squares and the collocation sense. But the curve it fits will be the signal *plus* the noise! Hence, a wave-function *only* fits (models) the noisy observations. It is *not* a clean, noise-free, physical model of the underlying *being*.

Information Theory uses an *entirely* different conception, of what it means, for discrete measurements to approximate a continuous function:
Perfect Reconstruction of *Symbols*

Perfect Reconstruction of *only* those aspects of the curve that you, the observer, actually care about: the Signal (symbols), not the Noise

In a sent Message, an emitter does not *ever* need to send (emit) anything the receiver (observer) already knows, in order to achieve perfect reconstruction: and that includes everything that the receiver can predict!

Everything that is predictable is *non* Information!!!

This is totally **contrary** to the Physics Perspective:
what is important and interesting about much of the
world, is precisely what *cannot* be predicted
rather than what *can* be predicted - like free will

Physics is entirely concerned with those few natural phenomenon,
that are almost *devoid* of any information
(low information density/capacity) and thus - predictable

And that includes all the equations being used to describe the
“laws” of physics!

Almost all of the *information*, in every physical situation, and in
every mathematical description of those situations, exists in the
form of the auxiliary conditions, such as initial values and boundary
values. It does not exist within the “laws” of physics or the
equations describing those laws!

The devil (Maxwell's Demon) is in the details.

In other words, Shannon discovered the maximum number of bits of *information*, required to perfectly reconstruct an arbitrary curve - like a wave-function!

$$\# \text{ bits} = (\# \text{ samples}) (\# \text{ bits/sample})$$
$$(\Delta T \Delta B) \quad \log_2(1+S/N)$$

or, to put it another way, Shannon discovered the maximum number of bits of *information* that can be recovered from *any* set of measurements of such a continuous function.

The famous Heisenberg Uncertainty Principle,
is simply equivalent to saying: $\# \text{ bits} \geq 1$

But $S/N = 1$, means that there is *intrinsic* noise in the very *definition* of a *bit* of information

This goes to the heart of the very concept of what it means for
Particles to behave as if they are *Identical*

Noise, intrinsic to the very definition of a bit, means that particles
must behave identically, without actually *being* identical

This, in turn, goes to the heart of what it means for something to
be an *elementary* particle. How can the most elemental particle,
possibly contain more information (have an infinite number of
significant bits encoded into its measurable attributes), than
anything else in the cosmos? An *elementary* particle ought to
have the *minimum* number of recoverable bits of information,
not the *maximum* - and it *does* in Shannon's Theory!

The Map versus the territory

Epistemology versus Ontology

These are all measurably different. Yet under many circumstances, you make a *decision* to treat them as if they are all identical: a quarter of a dollar.

This decision making, process, which designates some things as “signal” and others as “noise”, is what is being mistaken for a “collapse of the wave-function”

2
0
0
4

2
0
0
5

2
0
0
6

2
0
0
7

2
0
0
8



Michigan 2004P



Florida 2004P



Texas 2004P



Iowa 2004P



Wisconsin 2004P



California 2005P



Minnesota 2005P



Oregon 2005P



Kansas 2005P



West Virginia 2005P



Nevada 2006P



Nebraska 2006P



Colorado 2006P



North Dakota 2006P



South Dakota 2006P



Montana 2007P



Washington 2007P



Idaho 2007P



Wyoming 2007P



Utah 2007P



Oklahoma 2008P



New Mexico 2008P



Arizona 2008P



Alaska 2008P



Hawaii 2008P

Conservation Laws are the ultimate example of this lack of *information*:

It is trivial to predict the future value of every constant!

No one has to bother with *ever* sending you a message, to “update” the value of a constant.

This is why physics has a hard time, dealing with time. If your predictions are perfect, then the predictive model *never* needs to be updated with a message containing *any new information*, about the world. Thus, such a *static* model has become, in a peculiar sense, independent of both time and the world it purports to model - disconnected from a reality, in which **all** new (emergent) behaviors, are driven by the arrival of **new information**.

Conservation Laws are the ultimate example of this lack of *information*:

It is trivial to predict the future value of a constant!
No one has to bother with *ever* sending a message, to
“update” the value of a constant. “BlockTime!”

Slapped-on
Interpretation:

It's a constant, to

The map of the world's physics is
timeless, even though the world is not

This is why physics has a hard time, dealing with time. If your predictions are perfect, then the predictive model *never* needs to be updated with a message containing *any new information*, about the world. Thus, such a *static* model has become, in a peculiar sense, independent of both time and the world it purports to model - disconnected from a reality, in which **all** new (emergent) behaviors, are driven by the arrival of **new information**.

Signal (symbols) Vs. Noise

A A A a A A A A A A A A A A

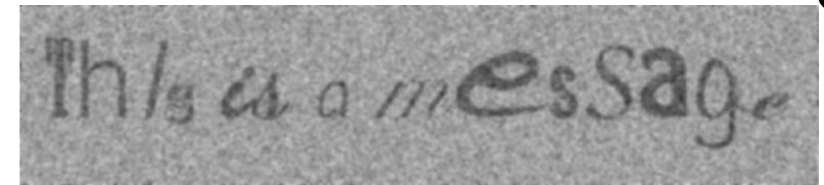
Are these all examples of the same symbol
in a known/familiar alphabet?

Or are they each a different letter
in an unknown (to you) alphabet?

If you know the alphabet being used, then
you know *exactly* how to *ignore* the noise - the
misshaping of symbols - like bad **handwriting**

Message as transmitted: This is a message

Message as received: *noisy and distorted*



Message as reconstructed: This is a message

Information Recovery and Perfect Reconstruction are based upon the exploitation of a priori knowledge

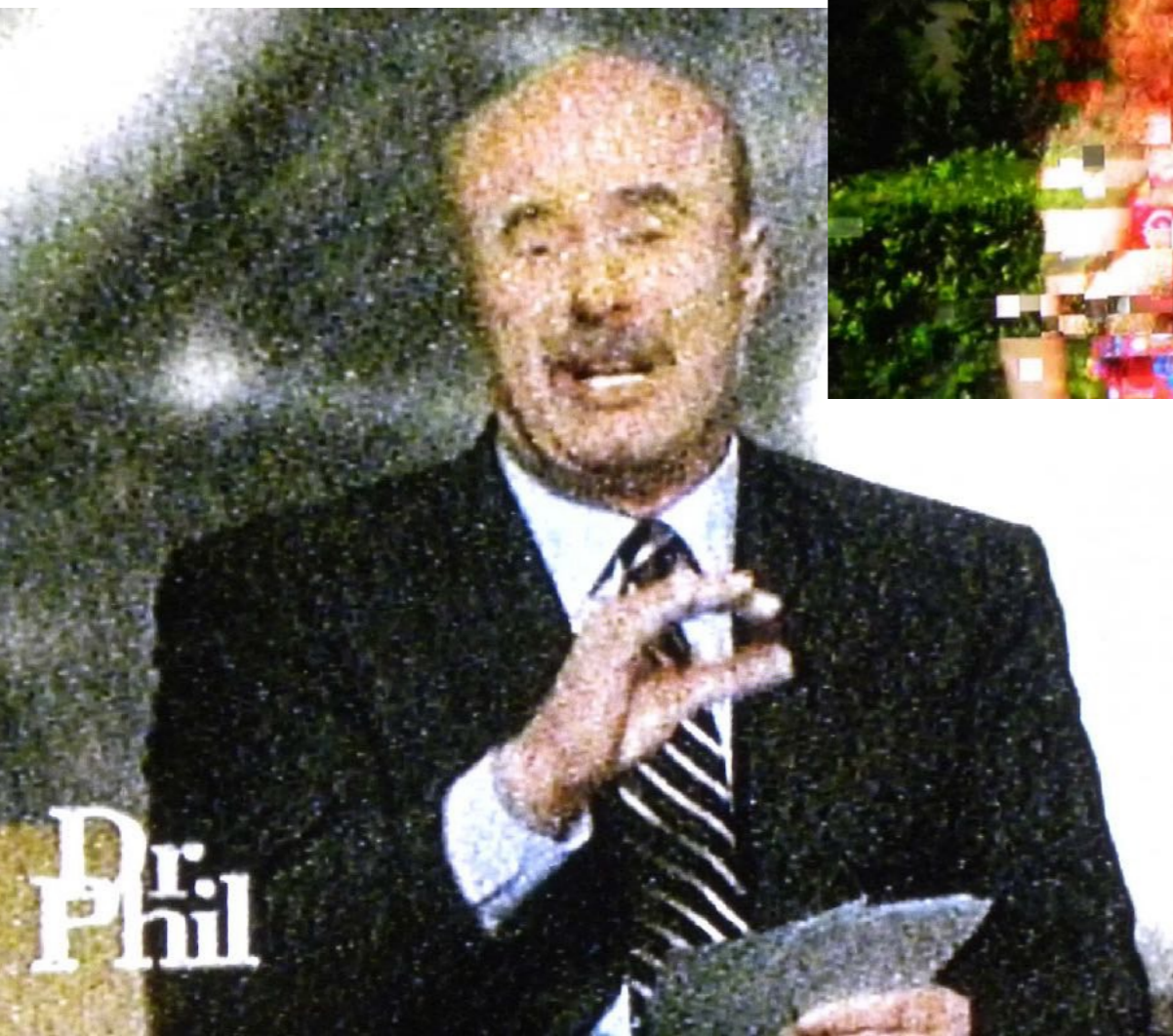
Like what is the alphabet being used

To *completely eliminate* every distorted symbol within every received message and/or sequence of measurements - by substituting the receiver's "best estimated *allowable* symbol" for each actual, received symbol

If the estimate is correct, the noise and distortion is GONE!
This is why an HDTV picture is so much cleaner than an old analog TV's picture. But if the guess is wrong - you get a catastrophic failure in the reconstruction

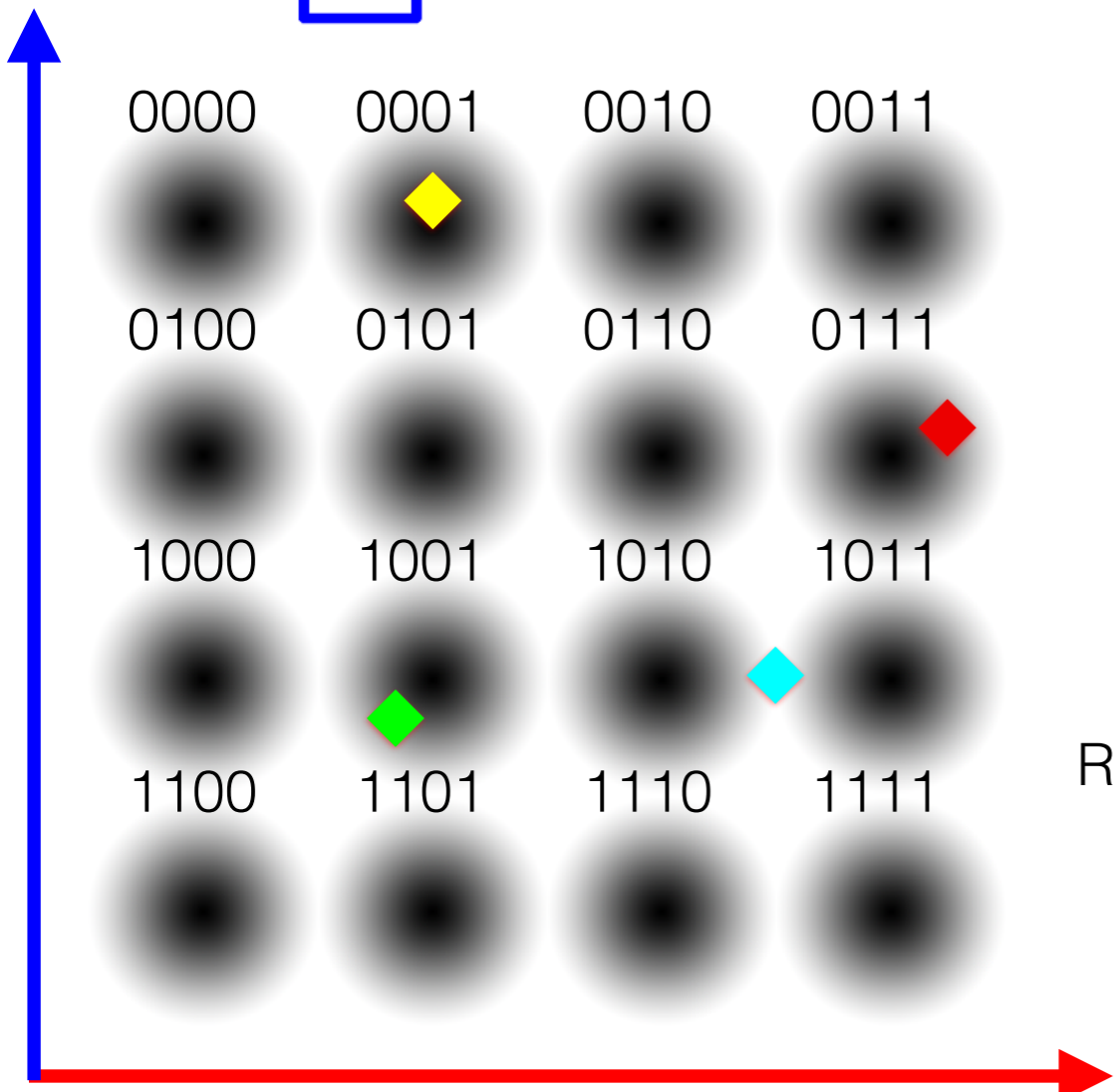
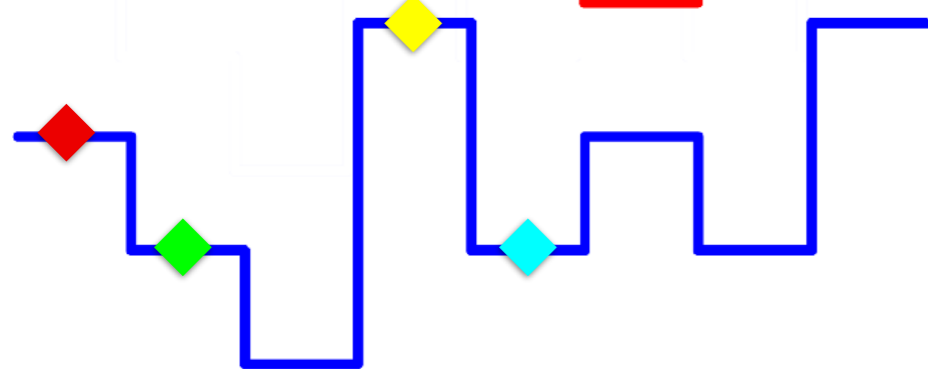
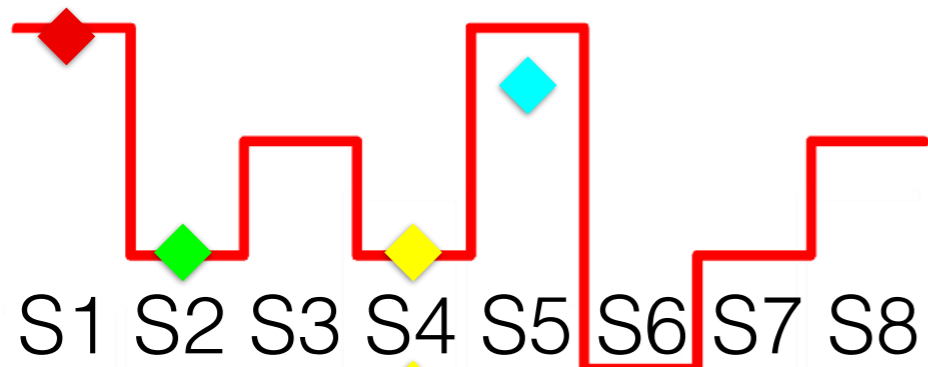
Badly Reconstructed HDTV Image

Actually, this is not all that bad. If it was *really* bad, most HDTVs would blank the screen, to prevent you from ever even seeing it!



Noisy Analog TV Image

Quadrature Amplitude Modulation



Information is all about making *decisions* (manifested in an action, like absorbing a quanta of energy, or not - Maxwell's Demon):
 Decisions about what to ignore and what not to ignore - Based on a prior knowledge about what is significant to the recipient and what is not

This is what *causes* non-continuous behaviors and why the equations of continuous, classical models, cannot be extended into the quantum realm

The continuous equations only work, for physical systems in which the information content *actually* being mathematically modeled, is negligible in comparison to the information content available, and that *could be* modeled: it is not necessary to model every atom in a planet in order to predict the planet's orbital motions.

Reconstructed Bit Sequence

◆ ◆ ◆ ◆
 0111 1001 0001 1010
 ?
 1011

This decision making, is ultimately (in the high information content case) equivalent to treating sequences of observations, not as *measurements*, with most and least significant bits, but as one long *serial-number*. Thus, such a system behaves, towards its input, symbolically rather than physically - a car stopping, because its driver detected (recovered information from) a red traffic-control-light, rather than because it hit a concrete wall.

A particle's *response* to a detected serial-number, is part of its built-in, a priori, behavioral repertoire. Such responses cannot be derived from observations of the encounter; they lack sufficient information content. Analogous to biological "receptors" detecting the specific type of molecule that they have been empowered to detect.

The devil (Maxwell's Demon) is in the details.

The coins can be thought of as spatially localized fields

asteroids passing through slits off-center, like NASA gravitational slingshot trajectory

Elementary particles ought to have minimal, rather than maximal, information content

Inverse square law (Gravity, EM, Holographic principle) is a consequence to how information carrying signals disperse - passing through a sphere enclosing the emitter - information density with constant noise level

Fraunhofer versus Fresnel diffraction and Gibbs phenomenon - the ripple in the scattering field

observables like position, momentum and spin, are inherently "relative" to the detector

David Bohm, ***Quantum Theory***, 1951, Prentice-Hall or 1989 Dover

Comments on particle scattering:

21.19: “Born Approximation “...approximation is equivalent to the assumption that the incident wave is not seriously distorted by the scattering potential.”

21.20: “the cross section is determined by the Fourier components of the potential.”

“This is because the particle is described by a wave packet, rather than by a trajectory.”

21.22: “A very important property of the deflections is that a given momentum change, $\Delta p = p - p_0$ can be produced only if the potential has such a shape that this Fourier component is present. A very large deflection can be produced in this way by a very small force, provided that the force varies rapidly enough in space.”

“This is in contrast to classical theory...”

“... in the Born approximation the deflection process is described as a single indivisible transition from one momentum state to another.”

“If there are enough successive deflections, the scattering process will begin to seem continuous, and it will approach a classical behavior. Thus, we see in another way why a strong force tends to produce a classical behavior; also we see how the apparently continuous classical deflection arises, despite the indivisible nature of the elementary processes of deflection.”

21.23: “The oscillatory nature of σ arises from the sharp “edge” in the square potential.”

21.27: “It can be treated rigorously only by a complete solution of the wave equation, which takes into account the change of wave amplitude in the slit resulting from electric currents that are induced in the slit by the total waves, *including* the part produced by the currents themselves. For a slit that is wide in comparison to a wavelength, the wave amplitude inside the slit is not very different from the incident amplitude, so that the Born approximation can be used in computing the diffraction pattern. For a narrow slit, however, the modification of the wave by the slit is so great that one needs a much better solution of the wave equation.”

21.34: “...the breakdown of the Born approximation means... the system may make many successive transitions... This, however, is exactly what will lead to the possibility of describing the scattering process classically.”

Additional References:

(1) A Classical System for Producing “Quantum Correlations”: <http://vixra.org/abs/1609.0129>

(2) Misinterpreting Reality - Confusing Mathematics for Physics: <http://fqxi.org/community/forum/topic/1372>

(3) Demystifying the Connection Between Physics and Mathematics: <http://fqxi.org/community/forum/topic/2315>