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Self Additive Inverse Elements of Neutrosophic Rings and Fields

***T.Chalapathi*[1] and *R.V.M.S.S.Kiran Kumar*[2]**

[1] Department of Mathematics, Sree Vidyanikethan Eng.College Tirupati-517502
Andhra Pradesh, India. Email: chalapathi.tekuri@gmail.com
[2]Department of Mathematics, S.V.University, Tirupati-517502, Andhra Pradesh
India. Email: chalapathi.tekuri@gmail.com, [2]kksaisiva@gmail.com

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Abstract. In this paper, all Neutrosophic rings $N(R, I)$ are assumed to be finite commutative with identity element. An element $x \in N(R, I)$ is called a Neutrosophic self-additive inverse if $x+x=0$. A characterization is given for Neutrosophic self-additive inverse elements and their inverses in the classical finite commutative ring R with identity. The arithmetic functions $|S(R)|$ and $|S(N(R, I))|$ exists, which counts the total number of self-additive and Neutrosophic self-additive inverse elements in R and $N(R, I)$, respectively. The relations between $S(R)$ and $S(N(R, I))$ are explored, and $S(N(R, I)) \cong S(N(R', I))$ is proved, when $R \cong R'$. Furthermore, we obtain a formula for enumerating total number of self-additive and Neutrosophic self-additive inverse elements in finite fields F_{p^n} and Neutrosophic fields $(N(F_{p^n}), I)$, respectively.

Keywords: Neutrosophic rings; Neutrosophic fields; Neutrosophic self-additive inverse elements; Neutrosophic isomorphism; equivalent Neutrosophic rings.

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1. Introduction

Algebraic Neutrosophic theory is an abstract branch of modern mathematics that originated from classical algebra through the composition of Neutrosophic theory. Its development started few years ago, and now a days Neutrosophic analytic methods and results are important in various fields of engineering science and applied mathematics with its applications. The impetus came from mathematical logic and philosophical problems, whose theory had the greatest effect on the development and promotion of the modern and philosophical ideas in the real world problems. Neutrosophic mathematicians observed that the real world problems from different fields often enjoy related features and Neutrosophic properties. This fact was used for an effective unifying approach towards such Neutrosophic problems, the unification being obtained by the omission of unessential details. Hence, the advantage of such a Neutrosophic abstract approach is that it concentrates on the essential facts, so that these facts become clearly visible. In this respect the Neutrosophic abstract method is the simplest and most economical method for treating Neutrosophic mathematical systems.

In the Neutrosophic abstract approach, one usually starts from a set of Neutrosophic elements satisfying certain Neutrosophic axioms. The nature of the Neutrosophic element is left unspecified. This is done on purpose of Neutrosophic theory. The theory then consists of logical consequences which result from the Neutrosophic axioms and are derived as theorems once and for all.

The concept of finite Neutrosophic numbers, sets, structures and systems was introduced by Florentin Smarandache [9]. Kandasamy and Florentin Smarandache were shown in [10] how algebraic operations addition and multiplication could be performed in the Neutrosophic sets and Neutrosophic structures. These authors introduced in [11] the concepts of philosophical theory, in particular, the notion of indertminancy of the real world problem in algebra, and initiated the new way for the emergence of a new class of rings and fields, namely, Neutrosophic rings and Neutrosophic fields. In [2, 3], Agboola and others studied further properties of Neutrosophic rings with different illustrations and examples.

The problem of classifying the self additive inverse elements of an arbitrary finite commutative semi ring with identity is also another open problem in Neutrosophic theory. However, the problem will be solved for certain classes of semi rings and ordered semi rings, see [13-15].

Let R be a finite commutative ring with identity 1 and let $N(R, I)$ be its Neutrosophic ring with same identity 1and determinacy I , where $I^2 = I$. The order of R and the order of $N(R, I)$ will be denoted by $|R|$ and $|N(R, I)|$, respectively. in this paper, $S(R)$ and $S(N(R, I))$ denotes the set of self additive inverse elements of R and $N(R, I)$, respectively.

The main purpose of this paper is to investigate the set of self and Neutrosophic self additive inverses elements of finite rings and fields. Further, we determine $|N(R, I)|$ and $|S(N(R, I))|$. In particular, we compute $|S(N(Z_n, I))|$, $\left|S\left(N(F_{p^n}, I)\right)\right|$. Furthermore, we prove that the result, if two rings R and R' are isomorphic, then $S(N(R, I)) \cong S(N(R', I))$.

2. Finite Neutrosophic rings and its basic properties

This section reviews some basic and important notions about finite Neutrosophic rings and their properties. These results arise in important ways in this text to follow other sections. We assume that the reader of this paper is familiar with the fundamentals of finite commutative rings [4]. Therefore, this section solely intended to provide a brief over view of the basic concepts of ring theory [1] and to consider terminology and notation employed in our discussion of Neutrosophic theory [12].

We begin with definition of Neutrosophic ring with few properties and results.

Definition 2.1. Let $(R, +, \cdot)$ be a finite ring. The set $N(R, I) = \langle R \cup I \rangle = \{a + bI : a, b \in R\}$ is called a Neutrosophic finite ring generated by R and I , where I is the Neutrosophic element with the properties $I^2 = I$, $0I = 0$, $I + I = 2I$ and I^{-1} does not exist.

The Neutrosophic ring contains the following properties

1. $R \subset N(R, I)$.
2. $N(R, I) \not\subset R$.
3. R is a ring with unity if and only if $N(R, I)$ is a Neutrosophic ring with unity.
4. $N(R, I)$ is commutative if and only if $rs = sr$ for all $r, s \in \langle R \cup I \rangle$.
5. Every Neutrosophic ring is a ring.

Theorem [12] 2.2. The Neutrosophic ring is a classical ring under the operations

1. $(a + bI) + (c + dI) = (a + c) + (b + d)I$
2. $(a + bI)(c + dI) = (ac) + (bc + ad + bd)I$ for all $a + bI$, $c + dI \in N(R, I)$.

Definition 2.3. Let S be a subring of a ring R . Then $N(S, I)$ is a Neutrosophic subring of $N(R, I)$ if $N(S, I)$ is itself a Neutrosophic ring .

Definition 2.4. Let $N(R, I)$ and $N(R', I)$ be any two Neutrosophic rings. The mapping $f : N(R, I) \to N(R', I)$ is called a Neutrosophic ring homomorphism if f satisfies the following axioms.

1. f is a ring homomorphism
2. $f(I) = I$.

3. Equivalent Neutrosophic rings

In this section, we obtain a formula for enumerating total number of Neutrosophic elements in the finite Neutrosophic ring $N(R, I)$. In particular, we compute $|N(Z_n[i], I)|$. Also this section covers equivalent and non-isomorphic classical rings of $N(R, I)$ with R is isomorphic to Z_n , $Z_n \times Z_n$, $Z_n[i]$ and $\dfrac{Z_n[x]}{\langle \pi(x) \rangle}$.

Definition 3.1. Let R and R' be any two finite rings. Then R is equivalent to R' if and only if there exists a one-one correspondence between R and R', and we write $R \sim R'$.

If R is isomorphic to R' , then there exist a bijective ring homomorphism between R and R', it can be written as $R \cong R'$.

The following theorem about Neutrosophic finite rings is a basic result. It play an important role in finding the orders of various Neutrosophic rings of finite order.

Theorem 3.2. Let R be a finite commutative ring with unity of order n. Then $|N(R, I)| = n^2$.

Proof: We have $R^* = R - \{0\}$, $R^*I = \{aI : a \in R^*\}$ and $R^* + R^*I = \{a + bI : a, b \in R^*\}$. Therefore, $N(R, I) = R \cup R^*I \cup (R^* + R^*I)$, where $R \cap R^*I = \varphi$, $R^*I \cap (R^* + R^*I) = \phi$ and $(R^* + R^*I) \cap R = \phi$. Hence,

$$|N(R, I)| = |R| + |R^*I| + |R^* + R^*I| = n + (n-1) + (n-1)^2 = n^2.$$

Theorem 3.3. Let $N(Z_n[i], I)$ be the Neutrosophic ring of Gaussian integers $Z_n[i]$ over modulo n. Then, $|N(Z_n[i], I)| = n^4$.

Proof: We know that, $|Z_n[i]| = n^2$. In view of the Theorem [3.2],

$$|N(Z_n[i], I)| = |Z_n[i]| + |Z_n[i]^* I| + |Z_n[i]^* + Z_n[i]^* I|$$
$$= n^2 + (n^2 - 1) + (n^2 - 1)^2 = n^4.$$

Definition [7] 3.4. Let Z_n be the ring of integers modulo *n*. Then $Z_n \times Z_n$ is a commutative ring with unity $(1, 1)$ under addition and multiplication defined by

1. $(a, b) + (c, d) = (a + c, b + d)$ and
2. $(a, b)(c, d) = (ac, bd)$ for every (a, b), (c, d) are in $Z_n \times Z_n$.

The following theorem tells us that the finite commutative ring and Neutrosophic commutative ring are both equivalent but not isomorphic.

Theorem 3.5. Let $n \geq 1$ be a positive integer. Then the following conditions are hold.

1. $Z_n \times Z_n \sim N(Z_n, I)$.
2. $Z_n \times Z_n \ncong N(Z_n, I)$.

Proof: For each positive integer $n > 1$, define a map $f : Z_n \times Z_n \to N(Z_n, I)$ by the relation $f((a, b)) = a + bI$ for every $(a, b) \in Z_n \times Z_n$ with, $f((0, 0)) = 0$ $f((1, 0)) = 1$, $f((0, 1)) = I$. Clearly, f is a well-defined and one-one function because $(a, b) = (c, d)$ $\Leftrightarrow a = c, b = d$ $\Leftrightarrow a = c, bI = dI$ $\Leftrightarrow a + bI = c + dI$ $\Leftrightarrow f((a, b)) = f((c, d))$. Also, for any $a + bI \in N(Z_n, I)$, there exist $(a, b) \in Z_n \times Z_n$ such that $f((a, b)) = a + bI$, as f is surjective. Thus the classical ring $Z_n \times Z_n$ is equivalent to Neutrosophic ring $N(Z_n, I)$.

Further, f is not a ring homomorphism, since $f((1, 0)(0, 1)) = f((0, 0)) = 0$ and $f((1, 0)) f((0, 1)) = 1I = I$. Hence, $Z_n \times Z_n$ is not isomorphic to $N(Z_n, I)$.

Theorem [3.5] has a number of useful consequences.

Corollary 3.6. Let n be a positive integer. Then the following are true.

1. $Z_n[i] \sim N(Z_n, I)$
2. $Z_n[i] \ncong N(Z_n, I)$, where $Z_n[i]$ is the ring of Gaussian integers over modulo n.

Proof: It is obvious from the bijective map $a+bi \mapsto a+bI$ with $0 \mapsto 0$, $1 \mapsto 1$ and $i \mapsto I$, where $i^2 = -1$ and $I^2 = I$.

Corollary 3.7. Let n be a positive integer and $\pi(x)$ is a quadratic irreducible polynomial over Z_n. Then the following are true

1. $\dfrac{Z_n[x]}{(\pi(x))} \sim N(Z_n, I)$
2. $\dfrac{Z_n[x]}{(\pi(x))} \ncong N(Z_n, I)$.

Proof: Follows from the bijective map $a+b\alpha \mapsto a+bI$ with $0 \mapsto 0$, $1 \mapsto 1$ and $\alpha \mapsto I$ where $I^2 = I$ and $\pi(\alpha) = 0$.

4. Self additive inverse elements of finite Neutrosophic rings

In this section, we define Neutrosophic self-additive inverse elements of Neutrosophic rings and studied their basic properties. Also, we obtain formulae for enumerating total number of self-additive inverse elements of various finite Neutrosophic rings. Furthermore, we compute the relation between $S(R)$ and $S\left(N(R, I)\right)$.

Definition 4.1. Let $N(R, I)$ be a finite commutative Neutrosophic ring with unity. An element x in $N(R, I)$ is called self-additive inverse element if $x+x=0$. Otherwise x is called mutual additive inverse element in $N(R, I)$.

The set of all self additive inverse elements in $N(R, I)$ denoted by $S\left(N(R, I)\right)$ that is, $S\left(N(R, I)\right) = \{x \in N(R, I) : 2x = 0\}$ and the of set all mutual additive inverse elements in $N(R, I)$ denoted by $M\left(N(R, I)\right)$, that is,

$M\left(N(R, I)\right) = \{x \in N(R, I) : 2x \neq 0\}$.

Always, the indeterminacy I is never self-additive inverse element for any Neutrosophic ring $N(R, I)$ if and only if $S(R) \neq R$, because $I + I \neq 0$.

Theorem 4.2. 1. $S(R)$ is a subring of a finite commutative ring R.

2. $S\left(N(R, I)\right)$ is not a Neutrosophic subring of a finite Neutrosophic commutative ring $N(R, I)$ with unity.

Proof: 1. We have $S(R) = \{a \in R : 2a = 0\}$. For any $a, b \in S(R)$, we have $2a = 0$ and $2b = 0$. Therefore, $2(a+b) = 2a + 2b = 0$, $2(a-b) = 2a - 2b = 0$ and

$2(ab)=(2a)b=0b=0$. This implies that $a+b,\ a-b,\ ab\in S(R)$. Thus $S(R)$ is a subring of R.

2.The set $S\left(N(R,\ I)\right)=\{x\in N(R,\ I):2x=0\}$ is not a Neutrosophic subring of $N(R,\ I)$. For instance, $S\left(N(Z_8,\ I)\right)=\{0,\ 4,\ 4I,\ 4+4I\}$ is the set of self additive inverse elements of the Neutrosophic ring $N(Z_8,\ I)$, but $S\left(N(Z_8,\ I)\right)$ is not Neutrosophic subring because $S\left(N(Z_8,\ I)\right)$ does not contain the indeterminacy I.

Remark 4.3. The Theorem [4.2] shows that $S\left(N(R,\ I)\right)$ is not a Neutrosophic subring of $N(R,\ I)$, but it is a semi Neutrosophic commutative subring of $N(R,\ I)$ with unity.

Theorem 4.4. Let R be a finite ring with unity. Then, $S(R)=R$ if and only if $S\left(N(R,\ I)\right)=N(R,\ I)$.

Proof: Let $a,\ b$ be any two elements in a finite ring R with unity. Then, $S(R)=R$
$\Leftrightarrow 2a=0,\ 2b=0$
$\Leftrightarrow 2a=0,\ 2bI=0$
$\Leftrightarrow 2\left(a+bI\right)=0$, $\forall a+bI\in N(R,\ I)$
$\Leftrightarrow S\left(N(R,\ I)\right)=N(R,\ I)$.

Lagrange's Theorem [1] 4.5. Let A be a subring of a finite ring R. Then $|A|\,\big|\,|R|$.

Theorem 4.6. Let Z_n be a ring of integers modulo n. Then

$$\left|S\left(N(Z_n,\ I)\right)\right|=\begin{cases}1 & \text{if } n \text{ is odd}\\ 4 & \text{if } n \text{ is even}\end{cases}.$$

Proof: First, by way of contradiction, suppose that $\left|S\left(N(Z_n,\ I)\right)\right|>1$ if n is odd. So without loss of generality we may assume that $\left|S\left(N(Z_n,\ I)\right)\right|=2$. Then there exist a subring $A=\{0,\ a:2a=0,\ a\in R\}$ in Z_n such that, by the Lagrange's Theorem [4.5] for finite rings, $|A|\,\big|\,|Z_n|$ that is, $2\,\big|\,|Z_n|$ which is not true because 2 is even and $|Z_n|$ is odd. So our assumption is not true, and hence $\left|S\left(N(Z_n,\ I)\right)\right|=1$ when n is odd.

Next, suppose n is even, then, by the Theorem [3.2], $|N(Z_n,\ I)|$ is also even. Now let $x=a+bI\in N(Z_n,\ I)$ for any $a,\ b\in Z_n$. Therefore,

$$\begin{aligned}2x=0 &\Leftrightarrow 2(a+bI)=0\\ &\Leftrightarrow 2a+2bI=0+0I\\ &\Leftrightarrow 2a=0,\ 2b=0\end{aligned}$$

$$\Leftrightarrow a,\, b \in \left\{0,\, \frac{n}{2}\right\},\ \text{since}\ \frac{n}{2}+\frac{n}{2} \equiv 0\ (\text{mod}\ n)$$

$$\Leftrightarrow x \in \left\{0,\, \frac{n}{2},\, \frac{n}{2}I,\, \frac{n}{2}+\frac{n}{2}I\right\}.$$

Hence $\left|S\left(N(Z_n,\, I)\right)\right| = 4$ when n is even.

Theorem 4.7. Let R and R' be two finite commutative rings with unity. If $R \cong R'$, then $S\left(N(R,\, I)\right) \cong S\left(N(R',\, I)\right)$. But converse is not true.

Proof: Suppose $R \cong R'$. Then there exist an isomorphism f from a ring R onto a ring R' such that $f(x) = x'$, where $f(0_R) = 0_{R'}$ and $f(1_R) = 1_{R'}$. We now show that $S\left(N(R,\, I)\right) \cong S\left(N(R',\, I)\right)$. For this we define a map $\varphi : S\left(N(R,\, I)\right) \to S\left(N(R',\, I)\right)$ by the relation

$$\varphi(x) = \begin{cases} x' & \text{if}\ x \in R \\ x'I & \text{if}\ x \in R^*I \\ x' + x'I & \text{if}\ x \in R^* + R^*I \end{cases}$$

For every $x \in N(R,\, I) = R \cup R^*I \cup (R^* + R^*I)$ and $R^* = R - \{0\}$ .It is straight forward to see that φ is a bijective Neutrosophic semi ring homomorphism because $R \mapsto R',\ R^*I \mapsto R'^*I$ and $R^* + R^*I \mapsto R'^* + R'^*I$ are bijective maps. Next, let x be any self-additive inverse in the Neutrosophic ring $N(R,\, I)$, then $x \in S\left(N(R,\, I)\right)$ $\Leftrightarrow 2x = 0_R$

$$\Leftrightarrow 2\varphi(x) = \varphi(x) + \varphi(x) = \varphi(x + x) = \varphi(0_R) = 0_{R'}$$

$$\Leftrightarrow \varphi(x) \in S\left(N(R',\, I)\right).$$

This shows that φ preserves self-additive inverse elements between Neutrosophic semi rings $S(N(R,\, I))$ and $S(N(R',\, I))$. Hence, $S\left(N(R,\, I)\right) \cong S\left(N(R',\, I)\right)$.

The converse of the Theorem [4.7] is not true, in general. Let $R = Z_4$ and $R' = Z_6$ we see that R and R' are both commutative rings with unity. By the Definition [4.1], we have $S\left(N(Z_4,\, I)\right) = \{0,\, 2,\, 2I,\, 2+2I\}$ and $S\left(N(Z_6,\, I)\right) = \{0,\, 3,\, 3I,\, 3+3I\}$. Clearly, $S\left(N(Z_4,\, I)\right) \cong S\left(N(Z_6,\, I)\right)$ but $Z_4 \ncong Z_6$.

5. Self additive inverse elements of Neutrosophic finite fields

The concepts of finite fields and their relations play a central role in number theory, algebraic number theory, and in applications of abstract algebra to communication theory, design theory, algebraic coding theory, algebraic cryptography, control theory and several other computer related areas, see [5,6, 8].

Basically, the finite cyclic groups $Z_n = \frac{Z}{nZ} = \{a + nZ : a \in Z\}$ may be given the structure of a finite commutative ring with unity. But, just as the addition on Z induced addition on Z_n, and similarly the multiplication on Z induced a multiplication on Z_n. So, the algebraic structure $(Z_n, \oplus, \odot)$ is a finite commutative ring with unity 1 of integers $0, 1, 2, ..., n-1$ with respect to addition $\oplus$ and multiplication $\odot$ modulo n. Further, we know that Z_n is a finite field of order n if and only if n is a prime. Notationally, if p is a positive prime integer, then F_p for the field with p elements. In particular, $F_p = Z_p$. Also, we shall notate F_{p^n} as a field with p^n elements over modulo p.

We next prove that the result for finding a formula for enumerating the total number of self additive inverse elements in F_{p^n}.

Theorem 5.1. Let $p \neq 2$ be a prime and $n \geq 1$ a positive integer. Then

$$\left|S(F_{p^n})\right| = \begin{cases} 1 & \text{if } S(F_{p^n}) \neq F_{p^n} \\ p^n & \text{if } S(F_{p^n}) = F_{p^n} \end{cases}.$$

Proof: Case 1. Suppose that $S(F_{p^n}) \neq F_{p^n}$. Then there exist at least one element $a \neq 0$ in F_{p^n} such that $a \neq -a$. Assume that $a + a = 0$ for some $a \neq 0$ in F_{p^n}, then a^{-1} exist in F_{p^n} such that $(2a)a^{-1} = 0a^{-1} = 0 \Rightarrow 2(aa^{-1}) = 0 \Rightarrow 2 = 0$, which is not true because $p \neq 2$. So our assumption is wrong, so that $a = 0$ is the only one self additive inverse element in F_{p^n}. So, in this case $\left|S(F_{p^n})\right| = 1$.

Case 2. If $S(F_{p^n}) = F_{p^n}$, then obviously, each and every element in F_{p^n} is self additive inverse element. Hence, $2a = 0$, for every $a \in F_{p^n}$.

Corollary 5.2. For each positive integer $n \geq 1$, we have $\left|S(F_{2^n})\right| = 2^n$.

Proof: It is obvious since $2a = 0$ for every $a \in F_{2^n}$.

Now starts Neutrosophic fields in the Neutrosophic theory. The study of Neutrosophic fields was introduced for the first time by Vasantha Kandaswamy and Florentin Samarandache in [12]. In this section we recall the definition of Neutrosophic finite field, and we are going to computing a formula for enumerating total number of self additive inverse elements in that field.

Definition 5.3. Let F_{p^n} be a finite field of order p^n. Then the Neutrosophic field generated by F_{p^n} and I under the operations on F_{p^n} denoted by $N(F_{p^n}, I) = \left\langle F_{p^n} \cup I \right\rangle$, where $I^2 = I,\ I \neq 1,\ I \neq 0$ and I^{-1} does not exist.

It is important to note that $N(F_{p^n}, I)$ is a Neutrosophic finite field but not a classical field, and it is only finite commutative ring with unity under the operations defined on the Theorem [2.2].

Theorem 5.4. Let $n \geq 1$ be a positive integer. Then,

$$\left| S\left(N\left((F_{p^n}, I) \right) \right) \right| = \begin{cases} 1 & \text{if } p \text{ is odd} \\ p^{2n} & \text{if } p \text{ is even} \end{cases}.$$

Proof: Case 1. If p is odd, then $S(F_{p^n}) \neq F_{p^n}$. Then, by the Theorem [ ], $\left| S\left(N\left((F_{p^n}, I) \right) \right) \right| = 1$.

Case 2. If p is even, then, in view of Theorem [3.2], we have $\left| N\left((F_{p^n}, I) \right) \right| = (p^n)^2 = p^{2n}$. This shows that $S(F_{p^n}) = F_{p^n}$. Hence, by the Theorem [5.1], $\left| S\left(N\left((F_{p^n}, I) \right) \right) \right| = p^{2n}$.

6. Conclusions

An enumerating procedure of the self additive inverses elements of a finite Neutrosophic commutative ring with identity was presented. The self additive inverse elements of finite fields were examined through the Neutrosophic finite fields. Complete characterizations of the finite rings and fields of determining all self additive inverse elements of R and R' such that $S\left(N(R, I)\right) \cong S\left(N(R', I)\right)$ when $R \cong R'$.

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