The Mystery Behind the Fine Structure Constant
Contracted Radius Ratio Divided by the Mass Ratio?
A Possible Atomist Interpretation?

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Abstract
THIS IS AN UNEDITED VERSION with many spelling errors that will be fixed in the next version.
In this paper we look at various alternatives for what the fine structure constant can represent. In
particular we look at a speculative alternative where the fine structure constant represent the radius ratio
divided by the mass ratio of the electron versus the proton as newly suggested by Koshy[5], but here
derived and interpreted based on Haug atomism (see [7]). This ratio is remarkably very close to the
fine structure constant and it is a dimensionless number. We also look at other alternatives such as the
proton mass divided by the Higgs mass which also seems to be a possible candidate for what the fine
structure constant can represent.

Key words: Fine structure constant, atomism, electron, proton, radius ratio, mass ratio, Higgs

1 The Fine Structure Constant

In 1916 Arnold Sommerfeld [1] introduced the fine structure constant in relation to spectral lines. The
fine structure constant \( \alpha \approx 0.0072973525664 \) (2014 CODATA recommended values) plays an important
role in modern physics. It has been suggested it is related to the ratio of the velocity of the electron in
the first circular orbit of the Bohr model of the atom to the speed of light in vacuum.

It has alternatively been suggested related to the Bohr radius by
\[
\alpha = \frac{a_0}{\bar{\lambda}_e} = \frac{\alpha}{\sqrt{\frac{\bar{\lambda}_e}{2}}} = \frac{\alpha}{\sqrt{\frac{\bar{\lambda}_e}{2}}} = \frac{\alpha}{\sqrt{\frac{\bar{\lambda}_e}{2}}}
\]

where \( \bar{\lambda}_e \) is the reduced Compton wave length of the electron. Further the classical electron radius is given by
\( r_e = \alpha^2 a_0 = \alpha \bar{\lambda}_e \).

The fine structure constant is also related to the charge of an electron to the planck charge

\[
\alpha = \frac{e^2}{q_P^2} = \left( \frac{\sqrt{\frac{e}{2}} \sqrt{\alpha 10^7}}{\sqrt{\frac{e}{2} \sqrt{10^7}}} \right)^2
\]

Also the Rydbergs constant is a function of the fine structure constant. We will here not comment
much on the importance or relevance of these suggested connections. Still we ask why do the fine structure
constant have exactly this “magic” value it has, or as stated by Richard Feynman

It has been a mystery ever since it was discovered more than fifty years ago, and all good
theoretical physicists put this number up on their wall and worry about it. Immediately you
would like to know where this number for a coupling comes from: is it related to or perhaps
to the base of natural logarithms? Nobody knows. It’s one of the greatest damn mysteries of
physics: a magic number that comes to us with no understanding by man.

Others have suggested that atomic structures somehow are linked to the Golden ratio and that the
fine structure ratio is related to this, see [2, 3, 4]. The Golden angle is given by \( \frac{\alpha}{\phi} \approx 137.50844 \) which
is not far from one divided by the fine structure constant: \( \frac{1}{\alpha} \approx 137.036 \).

In this paper we will suggest other possible connections to the fine structure constant.
2 The Contracted Radius Ratio Divided by the Mass Ratio

Koshy has in a recent working paper [5] interestingly suggested that the fine structure constant could be linked to a radius ratio dividend by the mass ratio. Here we builds on that idea, but in a quite different way than Koshy. Here we assume all matter and energy consist of indivisible particles always moving at the speed of light in the void as assumed by Haug [7, 8]. Haug’s newly introduced atomism theory gives all the same mathematical end results as in Einstein’s special relativity when using Einstein-Poincar’e synchronized clocks. In addition the theory gives upper boundary conditions on such as relativistic mass and how close the speed of mass can be relative to that of light.

Each indivisible particle in the electron moves back and forth over the reduced Compton wavelength of the electron with the speed of light. Only at collision is the electron truly a mass. Each collision represent the Planck mass that last for one Planck second. This leads to a mass gap of \( m_p \epsilon_p \approx 1.17337 \times 10^{-51} \text{kg} \). The Electron is the mass gap \( \frac{m_e}{\epsilon_e} \approx 7.763 \times 10^{20} \text{times per second} \), which gives the well known electron rest-mass, see also [9]. The indivisible particle has a radius equal to the Planck length, [10]. This means that the electron has a radius equal to its reduced Compton wavelength when extended \(^1\). Further it has only a radius equal to the Planck length when contracted.

The Proton electron mass ratio is \( \frac{m_p}{m_e} \approx 1836.1525 \). We could assume the mass of a proton consisted of 1836.1525 electrons (or alternatively 1836). Each of these electrons we for a moment assume is a sphere with radius equal to the Planck length. What if we sphere packed these 1836.1525 electrons how much volume would they take up? In 1831, Gauss [11] proved that the most densely one could pack spheres amongst all possible lattice packings was given by

\[
\frac{\pi}{3\sqrt{2}} \approx 0.74048 \tag{2}
\]

In 1611, Johannes Kepler suggested that this was the maximum possible density for both regular and irregular arrangements; this is known as the Kepler conjecture. The Kepler conjecture was supposedly finally proven in 2014 by Hale [12]. Based on this the radius of the large sphere consisting of large numbers of densely packed sphere with radius \( r \) is approximately given by (see the appendix)

\[
R \approx \lambda \sqrt[3]{\frac{N}{\pi}} \sqrt[3]{\frac{18}{18}} \tag{3}
\]

This means we get a contracted radius of the Proton is

\[
R \approx r \sqrt[3]{\frac{1836.1525}{\pi}} \sqrt[3]{\frac{18}{18}} \approx 13.535r \tag{4}
\]

Next we will define the contracted radius ratio as

\[
R_R = \frac{R}{r} = \frac{r \sqrt[3]{\frac{1836.1525}{\pi}} \sqrt[3]{\frac{18}{18}}}{r} = \sqrt[3]{\frac{1836.1525}{\pi}} \sqrt[3]{\frac{18}{18}} \tag{5}
\]

That is the contracted radius of the Proton divided by the contracted radius of the electron.

If we next divide this contracted radius ratio with the mass of the proton divided by the mass of the electron we get a number very close to the fine structure constant:

\[
\alpha \approx \frac{R}{R_R} \approx 0.0073715 \tag{6}
\]

Since \( \frac{\lambda_e}{\lambda_p} = \frac{m_e}{m_p} \) we could alternatively have written this on the following form

\[
\alpha = \frac{R \lambda_p}{r \lambda_e} \approx 13.535 \times \frac{2.10309 \times 10^{-16}}{3.86159 \times 10^{-13}} \approx 0.0073715 \tag{7}
\]

Still this is somewhat off from the fine structure constant (0.0072973525664 CODATA 2014), the number is too large. However the approximation used to calculate the radius of the sphere packed electrons making up the proton mass will actually slightly over estimate the radius of the sphere packed radius. This because the outer surface of the sphere packed sphere not is smooth, but must be “jagged”. We could measure the average radius of the sphere packed spheres by measuring the radius from the inside radius and the outside radius and divide by two, see figure 1.

Figure 1 illustrates how we take into account the jagged surface of the sphere packed sphere. The diameter the average of the blue line and the green line. To find the green line we can use Pythagoras theorem to find the distance as shown in figure 2. If one properly adjust for the jagged surface of the

\(^1\)And it is extended \( \frac{\lambda_e}{\lambda_p} \approx 7.763 \times 10^{20} \text{times per second} \).
Figure 1: The figure illustrates the contracted radius of a sphere (here we only see a section cut of the sphere). As the surface of a sphere packed sphere must be “jagged” a good approximation for the radius is found by taking half of the the average of the black lined diameter and the green lined diameter. To find the green lined diameter we need to use Pythagoras theorem as illustrated in the subfigure below. The contracted proton radius can in the same way be seen as 1836 sphere packed spheres. The green lined diameter is equal to the Black lined diameter minus $2r - 2(\sqrt{3} - 1)r \approx 0.54r$, where $r$ is the radius of the small spheres, which based on recent development in mathematical atomism must be $r = l_p$, that is the Planck length.

$$R \approx \frac{\sqrt{\pi} \sqrt[3]{78} + \left(\frac{\sqrt{\pi} \sqrt[3]{78} - 1 + (\sqrt{3} - 1)}{2}\right)}{2} = \frac{2 \sqrt{\pi} \sqrt[3]{78} + \sqrt{3} - 2}{2} = \sqrt{\frac{N}{\pi} \sqrt[3]{78} + \frac{\sqrt{3}}{4} - 1 = 13.4012}$$

And from this we can calculate the fine structure constant by dividing the contracted radius ratio by the mass ratio

$$\alpha = \frac{R_R}{\frac{\overline{m_e}}{m_e}} \approx \frac{13.4012}{1836.152} \approx 0.00729854$$

(8)

(9)

This also means the fine structure constant can be represented by the contracted radius ratio multiplied by the ratio of the reduced Compton wavelengths. The calculated value is extremely close compared to $\alpha_e = 0.0072973525664$ which is the fine structure constant given by CODATA 2014. The difference between the two numbers are just about $\frac{\alpha - \alpha_e}{\alpha_e} = 0.0161\%$. We do not claim that this is what the fine structure constant must represent, but again it is interesting that this is a dimensionless number. It is even more interesting possibly in the view of recent development in mathematical atomism.

Alternatively we could have used the classical electron radius $r_e = \frac{1}{4\pi\varepsilon_0 m_e c^2} = \alpha \lambda_e \approx 2.81794 \times 10^{-15}$.
The classical electron radius divided by the reduced Compton wavelength of the proton\(^2\) is.

\[
\text{Radius ratio} = \frac{\alpha \lambda_e}{\lambda_P} \approx 13.39905249
\]

and the fine structure constant is given by

\[
\alpha \approx \frac{\alpha \lambda_e}{\lambda_P} = \frac{13.39905249}{1836.152} = \alpha \times 1 \approx 0.007297353
\]

In this case it is the radius of the electron divided by the radius of the proton while in the above analysis it was the contracted radius of proton divided by the contracted radius of the electron (according to the atomism model.). We think the classical electron radius likely do not exist in physical sense, it is just an imaginary unit that has the fine structure constant embedded. On the other hand the contracted radius ratio is something that possibly exist if the depth of reality is atomism.

When it comes to the relationship between the classical electron radius and the radius of the proton or neutron and their mass ratio Koshy has in a recent \[5\] suggested a similar relationship as a possibly interpretation of the fine structure constant. However we think the Haug atomist model has more going in its favor as from that one get all the Einstein special relativity mathematical end result when using Einstein-Poincaré’s synchronized clocks. This theory also seems to make the most sense when the diameter of the indivisible particle is the Planck length and its mass is Planck mass, and in addition a series of infinity problems are removed in a simple way.

On its own this result could possibly be seen as nothing more than numerology. It is when we see this together with the recent development in mathematical atomism that it possibly truly get’s interesting. The mathematical atomism model of Haug is very simple and has so far been shown to give all the same mathematical end result as Einstein’s special relativity theory when using Einstein-Poincaré synchronized clocks.

3 Proton mass divided by Higgs mass

The 2014 CODATA recommended proton mass is \(1.672621898 \times 10^{-27}\) kg. That is equivalent to about \(938.271137\) MeV/\(c^2\). On 4 July 2012, CMS announced the discovery of a previously unknown boson with mass \(125.3 \pm 0.6\) GeV/\(c^2\), see \[13\]. There is still considerable uncertainty about the mass of the Higgs boson \[14\]. For a moment assume the Higgs mass was approximately \(128577.056\) MeV/\(c^2\). In this case the proton mass divided by the Higgs mass would be basically identical to the fine structure constant, and it would be a dimensionless constant.

\[
\alpha \approx \frac{m_P}{m_H} \approx \frac{938.2721137}{128577.056} \approx 0.007297353
\]

while a Higgs mass of \(125.3\) GeV would give a fine structure constant of

\[
\alpha \approx \frac{m_P}{m_H} \approx \frac{938.2721137}{125300} \approx 0.007488205
\]

The later calculation seems to bee to far of from the value of the fine structure constant, this indicates the Higgs mass has nothing directly to do with the fine structure constant.

Still this suggested value of the Higgs boson seems to be too far away from what it need to be to be related to the fine structure constant? Further it is also in relation to electrons the fine structure constant seems to be most important.

4 Summary

The fine structure constant plays an important role in modern physics, still it is a mystery exactly what it represent and why it has the mystical value it has. We have in this paper suggested two new possibilities for what the fine structure constant could represent. It could be related to what we would call the contracted radius ratio of the electron versus the proton divided by the mass ratio. The contracted radius ratio is given from sphere packing of Planck diameter spheres and adjust for the jagged surface of this sphere packed sphere. This new ratio seems to be extremely close to the fine structure constant given by CODATA. Alternatively we have suggested that the fine structure constant could be related to the Higgs

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\(^2\) Here using CODATA: \(\lambda \approx\) and \(\lambda_e \approx\)
mass over the proton mass, but this later suggestion seems to give a fine structure constant considerably off from the one given by CODATA.

We have in this paper not concluded what the fine structure constant truly represent, but think the speculative idea that spins off from atomism could deserve further investigation.

**Radius of Spheres constructed from a Large number of small spheres**

Assume small spheres with radius \( r \) the volume of such a sphere is

\[
V = \frac{4}{3} \pi r^3
\]

When we pack the Planck spheres as densely as possible they will take up a volume of

\[
V_t = \frac{4}{3} \pi l_p^3 = l_p^3 \sqrt{32}
\]

The total volume is then \( NV_t \). This means we need a larger sphere with radius

\[
NV_t = \frac{4}{3} \pi R^3
\]

\[
R = \sqrt[3]{\frac{3}{2} NV_t} = \sqrt[3]{\frac{3}{2} N r^3 \sqrt{32}}
\]

\[
R = r \sqrt[3]{\frac{N}{\pi}} \sqrt{18} \tag{14}
\]

It is important to be aware that this formula only will be a good approximation for a very large numbers of spheres. In the case of a proton we will assume it consist of 1836 spheres, which is a number of spheres where this formula should be quite accurate.

**References**


