Solving Incompletely Predictable problems: Riemann hypothesis, Polignac’s and Twin prime conjectures

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Abstract L-functions form an integral part of the 'L-functions and Modular Forms Database' with far-reaching implications. In perspective, Riemann zeta function is the simplest example of an L-function. Riemann hypothesis refers to the 1859 proposal by Bernhard Riemann whereby all nontrivial zeros of this function are conjectured to lie on the critical line. This proposal is equivalently stated in this research paper as all nontrivial zeros are conjectured to exactly match the 'Origin' intercepts of this function. Deeply entrenched in number theory, prime number theorem involves analysis of prime counting function for prime numbers. Solving Riemann hypothesis would enable complete delineation of this important theorem. Involving proposals on the magnitude of prime gaps and their associated sets of prime numbers, Twin prime conjecture deals with prime gap = 2 (representing twin primes) and is thus a subset of Polignac’s conjecture which deals with all even number prime gaps = 2, 4, 6,... (representing prime numbers in totality except for the first prime number ‘2’). Both nontrivial zeros and prime numbers are Incompletely Predictable entities which allow us to employ our novel Virtual Container Research Method to solve the associated hypothesis and conjectures.

Keywords Information-Complexity conservation; Gram points; Polignac’s conjecture; Riemann hypothesis; Sigma-Power Laws; Twin prime conjecture

Mathematics Subject Classification (2010) 11A41, 11M26

1 Introduction

Previous unsuccessful attempts to solve celebrated open problems such as Riemann hypothesis, Polignac’s and Twin prime conjectures have frustratingly been littered with false claims and counter claims. The historical Riemann hypothesis, proposed over 150 years ago by famous German mathematician Bernhard Riemann...
J. Y. C. Ting (September 17, 1826 – July 20, 1866) in 1859, stood out well in this regard. Thus readers should unsurprisingly expect that our engineered proofs on these three intractable open problems in this lengthy 2017-dated research paper will have to be inspirational, innovative and different when compared to any previous attempts.

With values traditionally given by parameter $t$, nontrivial zeros in Riemann zeta function consist of the countable infinite set (CIS) of irrational numbers [rounded off to six decimal places] 14.134725, 21.022040, 25.010858, 30.424876, 32.935062, 37.586178,.... Prime numbers consist of the CIS of rational numbers 2, 3, 5, 7, 11, 13,.... Both nontrivial zeros and prime numbers are Incompletely Predictable entities obeying Complex Elementary Fundamental Laws.

This paper deals with the task of solving and conveniently grouping together the above three mentioned open problems in number theory. Riemann hypothesis involves analysis on nontrivial zeros in Riemann zeta function – the study of which will, in addition, achieve the secondary objective of providing complete explanations for the existence of its closely related Gram points, or more specifically Gram[$x = 0$] and Gram[$y = 0$] points, in this same function. Both Polignac’s and [its subset] Twin prime conjectures involve analysis on even number prime gaps and their associated sets of odd prime numbers. A broad overview on how we execute our plans to derive solutions for the rigorous proofs on these three open problems is communicated next. Stemming from this 'Executive Summary’, we notice the different "flavors" involved for Riemann hypothesis versus that for Polignac’s and Twin prime conjectures. The former demands exact solutions for the conjectured single line location of all nontrivial zeros (which are known to be of infinite magnitude) while the later demands exact solutions for each set of odd prime numbers conjectured to be of infinite magnitude which is generated by each of the even number prime gaps conjectured again to be of infinite magnitude.

**Executive Summary**

**I. Riemann hypothesis** Riemann hypothesis refers to the proposal whereby all nontrivial zeros of Riemann zeta function are conjectured to be located on the critical line. Our required proof is overall dependent on fulfilling two mutually inclusive conditions: Condition 1. All nontrivial zeros in Riemann zeta function will be shown to be located on the critical line when this function rigidly complies with Dimensional analysis homogeneity which is associated with $\sigma = \frac{1}{2}$. Condition 2. None of the nontrivial zeros in Riemann zeta function will be shown to be located on the critical line when this function rigidly comply with Dimensional analysis non-homogeneity which is associated with $\sigma \neq \frac{1}{2}$.

**II. Polignac’s and Twin prime conjectures** Twin prime conjecture involves analysis of prime gap = 2 which is the very first even number prime gap representing all twin primes. This is a subset of Polignac’s conjecture involving analysis of prime gaps = 2, 4, 6, 8, 10,... which are the countable infinite set of all even number prime gaps representing prime numbers in totality except for the very first (and only) even prime number '2' representing the very first (and only) odd number prime gap = 1. Together these two conjectures are unambiguously represented by two closely related mini-proposals: Mini-proposal 1. Even number prime gaps are infinite (and arbitrarily large) in magnitude. Mini-proposal 2. Each individual even number prime gap will generate odd prime numbers which are again infinite
in magnitude. Our required proof is overall dependent on fulfilling two mutually inclusive aspects: Aspect 1. The "quantitive" aspect to the existence of the set of even number prime gaps and their associated sets of odd prime numbers, all as sets of infinite magnitude, will be shown to be correct by simultaneously utilizing concepts derived from Set theory (such as incorporating the cardinality of a set with the 'well-ordering principle' application and arguments based on the 'pigeonhole principle'). Aspect 2. The "qualitative" aspect to the existence of the set of even number prime gaps and their associated sets of odd prime numbers, all as sets of infinite magnitude, will be shown to be correct by the 'Plus-Minus Composite Gap 2 Number Alternating Law' which is applicable to all even number prime gaps apart from the special case of the first even number prime gap = 2 for twin primes. The prime gap = 2 situation will obey the 'Plus Composite Gap 2 Number Continuous Law'. These Laws are essentially "descriptive" laws inferring underlying intrinsic driving mechanisms that enable the infinity magnitude association for both the set of even number prime gaps and their associated sets of odd prime numbers to co-exist. By the same token, these Laws must have the all-important implication that they will be perpetually applicable to relevant even number prime gaps and each of their associated sets of odd prime numbers.

Footnote #1: We hereby specify that the phrase "countable infinite set of all even number prime gaps representing prime numbers [2, 3, 5, 7, 11,...] in totality" (or similarly worded phrases when alluded to in relevant parts of this paper) is not completely correct because this only encompass all known odd prime numbers [3, 5, 7, 11, 13,...] associated with all even number prime gaps = 2, 4, 6, 8, 10,... but without including the very first and only known even prime number '2' (which is associated with the very first and only known odd number prime gap = 1). In other words, prime numbers in totality will consist of all (multiple) odd prime numbers of infinite magnitude and the (single) even prime number of finite magnitude.

Footnote #2: Simply representing the expression of Mini-proposal 1 in full, the dissimilar looking phrases 'Even number prime gaps are infinite in magnitude' and 'Even number prime gaps are arbitrarily large in magnitude' with their different interpreted meanings will, of course, be equally valid when this mini-proposal in relation to the magnitude of prime gaps is proven to be true. Seemingly defying logic, this action is justifiable with the proviso that the term 'arbitrarily large' here must be clarified to denote the following caveats: The cumulative sum total of prime gaps is relatively much slower to attain the 'infinite in magnitude' status when compared to the cumulative sum total of prime numbers which will rapidly attain this status. The simple reason for this situation to exist is that every one of the even number prime gaps should generate (at the very least) more than one odd prime number, if not an unique set of odd prime numbers of infinite magnitude [which will happen when Mini-proposal 2 in relation to 'Each individual even number prime gap generating odd prime numbers which are infinite in magnitude' is proven to be true]. Failure to comply with the above situation will obviously occur just once at prime gap = 1 which represent the solitary even prime number '2'.

Contrary to the likely shared perception by many that the common denominator 'Incompletely Predictable entities being present in all three of our open problems' is a major liability; we counterintuitively regard the presence of this common denominator a decisive asset in permitting the convenient grouping of,
and in allowing employment of our novel mathematical tool coined 'Virtual Container Research Method' (VCRM) on, these three challenging problems. The recognition of this crucial idea is paramount in permissively allowing VCRM to literally act as foundation for the elegant mathematical framework that enables successful completion of the assigned monumental task to solve those three problems.

The essence of VCRM is best depicted below by two scenarios: Scenario 1. Completely Predictable mathematical problems involving Completely Predictable entities that are associated with Completely Predictable properties are amendable to treatment by conventional mathematical tools such as Calculus to solve these problems. Scenario 2. Incompletely Predictable mathematical problems involving Incompletely Predictable entities that are associated with Incompletely Predictable properties are not amendable to treatment by conventional mathematical tools such as Calculus to solve these problems. In Scenario 2, we need to initially derive certain Completely Predictable "meta-properties" out of the Incompletely Predictable properties. Only then can these problems in the guise of Completely Predictable "meta-properties" be amendable to treatment by conventional mathematical tools such as Calculus so that they can finally be solved. Thus it is by indirectly deriving certain Completely Predictable "meta-properties" from the Incompletely Predictable properties associated with the common denominator 'Incompletely Predictable entities being present in all three of our open problems' that will eventually allow us to successfully solve them. We will shortly explore some preliminary preambles after rendering all our claims and statements in this paper meaningful by defining the terms 'Completely Predictable entities' and 'Incompletely Predictable entities'; and explaining the Hybrid method of Integer Sequence classification with its associated Ratio Study. Further clarification for the meaningful use of the terms 'Simple Elementary Fundamental Laws' and 'Complex Elementary Fundamental Laws' is given in Section 10 below.

**Completely Predictable & Incompletely Predictable entities** At the outset here, we allude readers to the point that usage of the term 'Pseudorandom number' in scientific literature is largely synonymous with usage of our devised term 'Incompletely Predictable number' in this paper. Performing numerical analysis on the position of a selected Completely Predictable number (or entity) and a selected Incompletely Predictable number (or entity) will allow their succinct definitions to take place. Each can respectively be defined as a number (or entity) whose position is able to be fully specified without, and with, needing to know the associated details of the positions of all related preceding numbers (or entities) in its neighborhood. This property is demonstrated below using the example of odd number '99' (calculated by using a 'simple' formula and is a Completely Predictable number obeying Simple Elementary Fundamental Laws) and prime number '97' (computated by using a 'complicated' algorithm and is a Incompletely Predictable number obeying Complex Elementary Fundamental Laws).

Randomly picked odd number '99': Can we completely predict its associated details (i) specifying whether it could be classified as an odd number and (ii) whether its precise position could be specified without needing to know the positions of all preceding odd numbers? '99' satisfy the Odd number Label "Always end with a digit of 1, 3, 5, 7, or 9" and hence is truly an odd number. Its precise $i^{th}$ position can be calculated as follows: $i = (99+1)/2 = 50$. This implies that 99 is the $50^{th}$ odd number. Note that '99' is odd and composite but not prime as it consists of factors derived as $99 = 3 \times 33 = 3 \times 3 \times 11$. Therefore the an-
Answer is affirmative to both questions, and '99' as an odd number is a Completely Predictable number.

Randomly picked prime number '97': Can we completely predict its associated details (i) specifying whether it could be classified as a prime number and (ii) whether its precise position could be specified without needing to know the positions of all preceding prime numbers? '97' satisfy the Prime number Label "Always evenly divisible only by 1 or itself & must be whole numbers greater than 1" as confirmed by the trial division method resulting in 97 = 1 X 97. Hence '97' is truly prime. However, its precise position can only be determined by computing the positions of all preceding 24 prime numbers using the Sieve of Eratosthenes to eventually conclude that 97 is the 25th prime number. Therefore the answer is affirmative to the first question but negative to the second question, and '97' as a prime number is a Incompletely Predictable number. {We would have already realize that '97' is also the \[i = (97+1)/2\] 49th odd number as it satisfy the Odd number Label "Always end with a digit of 1, 3, 5, 7, or 9". Note that '97' when treated as an odd number is Completely Predictable and will conform to similar numerical analysis conducted in the previous paragraph for odd number '99'.}

We emphasize that this Incompletely Predictable number property is equally applicable to, for instance, any Incompletely Predictable algebraic or transcendental irrational number. We importantly discern that all Incompletely Predictable nontrivial zeros of Riemann zeta function as transcendental numbers will manifest the Incompletely Predictable number property twice, firstly in the countable infinite set of all nontrivial zeros, and secondly in the countable infinite set of all numerical digits after the decimal point in each nontrivial zero. In particular, the exact position of a nominated nontrivial zero and the exact position of a nominated numerical digit after the decimal point in each nontrivial zero, will not be known without calculating the positions of all relevant preceding nontrivial zeros and the positions of all relevant preceding numerical digits after the decimal point of that nontrivial zero. Immediately one would intuitively sense that any mathematical algorithm or equation required to deal with, for instance, "discrete-type" Incompletely Predictable prime numbers as opposed to, for instance, "discrete-type" (and "continuous-like") Incompletely Predictable nontrivial zeros would manifest features, respectively, of a "discrete" nature as opposed to features of a "continuous" nature. We point out here that this double manifestation of Incompletely Predictable number property will similarly occur for the Gram\[x=0\] and Gram\[y=0\] points of Riemann zeta function since they are transcendental numbers as well.

**Hybrid method of Integer Sequence classification** We tentatively advocate for the Hybrid method of Integer Sequence classification to act as a simple mathematical tool enabling meaningful division of all integer sequences into either Hybrid integer sequences and non-Hybrid integer sequences. In regards to usage of the terms 'Hybrid integer sequence' and 'non-Hybrid integer sequence', we now mention here the curious A228186 integer sequence [1] – the first ever novel [infinite length] Hybrid integer sequence artificially synthesized from Combinatorics Ratio [constituting an inequality criteria according to 'Ratio Study']. In the 'Position i' notation, let \(i = 0, 1, 2, 3,...\) be the set of natural numbers of infinite magnitude. A228186 "Greatest \(k > n\) such that ratio \(R < 2\) is a maximum rational number with \(R = \text{Combinations with repetition} / \text{Combinations without repetition}\) is equal to [infinite length] non-Hybrid (usual "garden-variety") integer sequence A100967 [2] except for the finite number of 21 'exceptional' terms at Positions 0, 11, 13, 19,
21, 28, 30, 37, 39, 45, 50, 51, 52, 55, 57, 62, 66, 70, 73, 77, and 81 with their values given by the relevant A100967 term plus 1. It was previously published by us on The On-line Encyclopedia of Integer Sequences website in 2013. The first 49 terms of A100967 "Least k such that binomial(2k+1, k-n) ≥ binomial(2k, k)" are listed below: 3, 9, 18, 29, 44, 61, 81, 104, 130, 159, 191, 225, 263, 303, 347, 393, 442, 494, 549, 606, 667, 730, 797, 866, 938, 1013, 1091, 1172, 1255, 1342, 1431, 1524, 1619, 1717, 1818, 1922, 2029, 2138, 2251, 2366, 2485, 2606, 2730, 2857, 2987, 3119, 3255, 3394, and 3535. Then at Position 0: A228186 (= 4) is given by A100967 (=3) + 1; at Position 11: A228186 (= 226) is given by A100967 (=225) + 1, etc. Commencing from Position 0 onwards, we can usefully envision the following attractive idea: "In the limit" that i approaches 82, A228186 becomes (and is identical to) A100967 for all i ≥ 82 values used to denote Position i. In the relevant parts of this paper, we refer to the unique concepts stemming from our Integer Sequence classification and Ratio Study predominantly during the derivation of our rigorous proof for Riemann hypothesis.

1.1 On the Virtual Container Research Method

We can arbitrarily and usefully create three groups of entities: Group I Completely Predictable entities, Group II Incompletely Predictable entities, and Group III Completely Unpredictable entities. Group III could also be appropriately coined Completely Chaotic or Completely Indeterministic entities. Only certain correctly selected and naturally occurring physical processes can ever give rise to true [measured] random numbers in Group III since these physical processes are totally indeterministic or chaotic, and thus they can consequently be regarded entirely as Completely Unpredictable entities. In this sense, the [computational] pseudorandom number generators based solely on deterministic logic can never be regarded as sources for true random number (or true Completely Unpredictable entities). Any given set of computed numbers derived from Group I and Group II will always be perfectly reproducible as the main distinguishing feature [at one end of a spectrum] when compared to any given set of measured numbers derived from Group III which will never be perfectly reproducible [at the opposite end of a spectrum].

Remark 1.1. Virtual Container Research Method could be applied to both Completely Predictable and Incompletely Predictable entities.

We now provide further clarification on the Virtual Container concept. Relevant Virtual Containers must be correctly recognized and understood as constituting the basic foundation underlying any research methodology to ensure that derived mathematical proofs using this Virtual Container technique will always be valid. Carefully explained with various examples below, we must justify the responsible use of this core Virtual Container Research Method (VCRM) which is essential for mathematically enabling theoretical derivation of various convincing proofs for Riemann hypothesis, Polignac’s and Twin prime conjectures to mature. This is now demonstrated when applied to Group I which we will classify as belonging to the 'General-Class-of-Mathematical-Problems with Multiple-Proof-Solutions' [or simply 'Completely Predictable problems'] using "discrete" even & odd numbers as two nominated examples and "continuous" $y = 2x$ & $y = 2x - 1$ equations as another two nominated examples; and Group II which we will classify as belonging to the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-
Solution’ [or simply ‘Incompletely Predictable problems’] using "discrete" prime & composite numbers as two nominated examples and "continuous" Riemann zeta function as another solo nominated example.

Consider x for all real number values \( \geq 1 \). Let y be the set of real numbers such that \( y = 2x \). Then this \( y = 2x \) "continuous" linear equation is literally the virtual container mathematically "containing" the [complete set] straight line of infinite length commencing from the Cartesian point \( (x=1, y=2) \). This straight line will fully represent the \( y = 2x \) output real number values for all the specified \( x \geq 1 \) input real number values. Computing \( y = 2x \) values an infinite number of times will not per se result in obtaining the gradient or slope of 2 for this equation. This gradient can be obtained by utilizing more than one method – either via trigonometrically calculating the tangent of the \( y = 2x \) straight line [which equals to 2] or via mathematically analyzing the intrinsic property of the \( y = 2x \) equation using Differential Calculus [viz. \( dy/dx = d(2x)/dx = 2 \)]. As a side note, we observe that by applying Integral Calculus together with Fundamental Theorem of Calculus to the continuous \( y = 2x \) equation for the interval \( [1, +\infty) \), viz. \( \int_{1}^{\infty}(2x)dx = [x^{2} + C]_{1}^{\infty} = (\infty^{2} + C) - (1^{2} + C) = \infty \), will result in the "area of infinite size enclosed by the [straight line] curve and the x-axis".

We can carry out an identical treatment to the \( y = 2x - 1 \) "continuous" equation for the same \( x \geq 1 \) real number values to obtain the infinite length straight line but commencing this time from the different Cartesian point \( (x=1, y=1) \). Its gradient of 2 can similarly be obtained either using the tangent method or the \( dy/dx = d(2x-1)/dx \) Differential Calculus method. As a side note, we observe in a similar fashion that by applying Integral Calculus together with Fundamental Theorem of Calculus to the continuous \( y = 2x - 1 \) equation for the interval \( [1, +\infty) \), viz. \( \int_{1}^{\infty}(2x - 1)dx = [x^{2} - x + C]_{1}^{\infty} = (\infty^{2} - \infty + C) - (1^{2} - 1 + C) = \infty \), will result in the "area of infinite size enclosed by the [straight line] curve and the x-axis".

By carrying out this identical treatment using the same \( y = 2x \) and \( y = 2x - 1 \) as "discrete" equations by considering \( x \) for all integer number values \( \geq 1 \) [instead of \( x \) for all real number values \( \geq 1 \)], we easily obtain (respectively) the complete set of even and odd numbers [with both sets of infinite magnitude in size]. These "discrete" equations are the virtual containers "containing" all known even and odd numbers. Computing even and odd numbers infinitely often will not per se enable us to ever conclude that the gap between any two consecutive even numbers (even gap) and any two consecutive odd numbers (odd gap) will both always equal to 2. This "gradient-equivalent" even gaps and odd gaps can simply be obtained by transforming those equations from their "discrete" formats into the equivalent "continuous" formats [viz. "discrete" \( \Delta x = 1 \) \( \rightarrow \) "continuous" \( \Delta x = 0 \)] to obtain their gradients either using the tangent method or Differential Calculus method (as outlined in the preceding two paragraphs). Then the even and odd gaps, both equal to 2, is numerically identical and mathematically equivalent to the relevant obtained gradients, both also equal to 2. Similar in nature to our mentioned process of transforming equations from their "discrete" formats into the equivalent "continuous" formats; we importantly point out here that one of the many crucial steps, as depicted in the relevant parts of this paper below, required to successfully solve Riemann hypothesis will involve applying Riemann integral to the "discrete-like" Riemann-Dirichlet Ratio (in its summation format) in order to obtain the "continuous-like" Riemann-Dirichlet Ratio (in its integral format).
Two crucial points to note here are (i) the two equations \( y = 2x \) and \( y = 2x - 1 \) in both their "discrete" and "continuous" formats are totally independent of each other as we can successfully obtain their respective gradient or gap values by individually analyzing each relevant equation by itself, and (ii) there are more than one way to obtain those gradient or gap values as clearly illustrated above using the tangent method or Differential Calculus method. Specifically by this two points, we imply that Completely Predictable entities will always belong to the 'General-Class-of-Mathematical-Problems with Multiple-Proof-Solutions' [or simply the 'Completely Predictable problems'].

We now logically apply in an analogous manner similar treatment to (a) non-trivial zeros & its closely related two types of Gram points – all computationally obtained directly from the "continuous" Riemann zeta function; and (b) prime & composite numbers – all computationally obtained directly & indirectly from the "discrete" Sieve of Eratosthenes algorithm. They are all of infinite magnitude in size and are typical representations of Incompletely Predictable entities. For the "continuous" Riemann zeta function, the axes intercepts at the 'Origin' (viz. nontrivial zeros or Gram\([x=0, y=0]\) points), the x-axis (viz. 'usual'/‘traditional’ Gram points or Gram\([y=0]\) points) and the y-axis (viz. Gram\([x=0]\) points) – all consisting of transcendental numbers – are the Incompletely Predictable entities of interest that we wish to study. As we shall subsequently observe in all of our obtained proofs for our three open problems based on Riemann zeta function and Sieve of Eratosthenes, both (a) the three axes intercepts sets of infinite nontrivial zeros, infinite Gram\([y=0]\) points and infinite Gram\([x=0]\) points, and (b) the two numerical sets of infinite prime numbers & infinite composite numbers, are totally dependent on each other in the following sense. There is just one solitary way to solve those open problems as we can only succeed in rigorously obtaining the relevant proofs when (a) nontrivial zeros, Gram\([y=0]\) points and Gram\([x=0]\) points are all simultaneously analyzed together and "contained" using the relevant Virtual Container [largely represented by Sigma-Power Laws derived via certain non-negotiable mathematical steps being correctly undertaken], and (b) prime numbers & composite numbers are simultaneously analyzed together using the relevant Virtual Container [largely represented by Information-Complexity conservation derived via certain non-negotiable mathematical steps being correctly undertaken] to "contain" them. In other words, we are only able to solve those open problems when (i) the axes intercepts of Riemann zeta function are dependently analyzed together using the derived 'solitary-style' relevant Virtual Container that "contains" them, and prime numbers & composite numbers are dependently analyzed together to derive this 'solitary-style' relevant Virtual Container to "contain" them, and (ii) these representative Virtual Containers can only be derived via certain non-negotiable mathematical steps being correctly undertaken. In a nutshell, satisfying criteria (i) and (ii) is the *sine qua non* of the requirements to fulfill the condition that solving the three open problems of Riemann hypothesis, Polignac’s and Twin prime conjectures – all endowed with Incompletely Predictable entities – will always be classified as solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' [or simply the 'Incompletely Predictable problems'].

We intuitively sense that to ultimately solve open problems related to non-trivial zeros (\& the two types of Gram points) of Riemann zeta function and prime numbers (\& composite numbers) generated from the Sieve of Eratosthenes would subsequently/concurrently require the correct analysis of certain [finite
number of intrinsic properties and behaviors arising from those representative Virtual Containers. In particular, we are dealing with entities such as Incompletely Predictable 'varying gaps' [which is the equivalent of Incompletely Predictable 'varying gradients'] between consecutive prime numbers (prime gaps) & between consecutive composite numbers (composite gaps); and our hereby conjured-for-illustration-purpose Incompletely Predictable 'varying gaps' [which is the equivalent of Incompletely Predictable 'varying gradients'] between consecutive nontrivial zeros (dubbed nontrivial zero gaps), between consecutive Gram[y=0] points (dubbed Gram[y=0] points gaps) & between consecutive Gram[x=0] points (dubbed Gram[x=0] points gaps).

Of utmost mathematical significance, these Incompletely Predictable 'varying gaps' or 'varying gradients' of infinite magnitude are de novo natural phenomena arising out of Incompletely Predictable function or algorithm – which in these cases refer to Riemann zeta function and Sieve of Eratosthenes algorithm. They will inevitably never be amendable to direct or conventional treatments, for example, by using tangent method or Differential Calculus method which can only ever be validly applied to Completely Predictable functions or algorithms [such as those associated with the above mentioned y = 2x & y = 2x - 1 equations, which are both endowed with 'non-varying gaps' or 'non-varying gradients' of infinite magnitude].

We can now deduce that a critical step in using our VCRM technique to successfully obtain the proofs for Riemann hypothesis, Polignac’s and Twin prime conjectures [which are respectively connected with Riemann zeta function, and Sieve of Eratosthenes algorithm] will consist of analyzing the finite number of Completely Predictable "meta-properties" that are (intrinsically) already present in the relevant Incompletely Predictable function and algorithm [instead of analyzing the infinite number of Incompletely Predictable entities (extrinsically) generated by these same function and algorithm]. Then the tell-tale sign indicating Virtual Container use in this paper is epitomized by relevant statements or sentences incorporating expressions with wordings such as "...containing each and every conceivable nontrivial zeros [but not its actual identity]..." or "...containing each and every conceivable prime number [but not its actual identity]...".

We diagrammatically depict Group I Completely Predictable entities (giving rise to Completely Predictable problems), symbolized by the Completely Predictable "——CP Straight line INTERFACE" for "discrete" even-odd numbers (in Table 1) and "continuous" y = sin x function (in Table 3); and Group II Incompletely Predictable entities (giving rise to Incompletely Predictable problems), symbolized by the Incompletely Predictable "vvvvvvIP Jagged line INTERFACE" for "discrete" prime-composite numbers (in Table 2) and "continuous" Riemann zeta function (in Table 4). There is a finite number of intrinsic properties with special characteristics that will manifest themselves infinitely often at both 'numerical relationship interface' and 'axes intercepts relationship interface' for only Group II Incompletely Predictable entities. As elaborated below, it is these intrinsic properties with special characteristics – which are our so-called Completely Predictable "meta-properties" – observed at those relevant interfaces that can be perceived as automatic sequelae arising from various "interactions" between the two dependent sets of [Incompletely Predictable] prime and composite numbers, and between the three dependent sets of [Incompletely Predictable] axes intercepts viz. Gram[y=0], Gram[x=0] & Gram[x=0,y=0] points in Riemann zeta function. A simple overall Completely Predictable "meta-properties" could then be perceived
Table 1 The Completely Predictable problem of Even-odd number pairing

| Set of even number | CP Straight line INTERFACE | Set of odd number |

Table 2 The Incompletely Predictable problem of Prime-composite number pairing

| Set of composite number | Jagged line INTERFACE | Set of prime number |

Table 3 The Completely Predictable problem of \( y = \sin x \) function

| Set of x-axis intercepts in \( y = \sin x \) | CP Straight line INTERFACE | Set of 'Origin' intercept in \( y = \sin x \) |

Table 4 The Incompletely Predictable problem of Riemann zeta function

| Set of Gram\( [y=0] \) points ('usual' Gram points) | Jagged line INTERFACE | Set of Gram\( [x=0,y=0] \) points (nontrivial zeros) | Set of Gram\( [x=0] \) points |

as the consistent [solitary] critical line location of all nontrivial zeros of Riemann zeta function (indicating Riemann hypothesis to be true); and as the consistent [dual] infinity magnitude size of the set consisting all even number prime gaps and infinity magnitude sizes of the individual sets of prime numbers that are derived from each and every even number prime gaps (together indicating Polignac’s and Twin prime conjectures to be true).

**Numerical relationship interface** For the Completely Predictable problem of even-odd number pairing and the Incompletely Predictable problem of prime-composite number pairing, the following collective statements are valid:

*For the Completely Predictable problem of even-odd number pairing in Table 1:*  
Set of natural number = Set of even number + Set of odd number. Set of even number = Set of odd number equality relationship is an "exact non-varying" Completely Predictable relationship. The numerical relationship interface for this even-odd number pairing is symbolized by the Straight line INTERFACE.

*For the Incompletely Predictable problem of prime-composite number pairing (involving the Sieve of Eratosthenes) in Table 2:*  
Set of natural number = Set of prime number + Set of composite number. Set of composite number > Set of prime number inequality relationship is an "exact varying" Incompletely Predictable relationship. The numerical relationship interface for this prime-composite number pairing is symbolized by the Jagged line INTERFACE. Classical examples of intrinsic properties with special characteristics ("meta-properties") arising out of the "interactions" between the two dependent sets of prime and composite numbers are the 'Plus-Minus Composite Gap 2 Number Alternating Law' and the 'Plus
Composite Gap 2 Number Continuous Law’. Outlined in the relevant parts of this paper below, these are essentially Completely Predictable laws that are computationally applicable [albeit with Incompletely Predictable timing on an eternal basis] to certain designated prime numbers (which are respectively generated from all even number prime gaps apart from prime gap = 2 in the first law, and from even number prime gap = 2 in the second law).

Axes intercepts relationship interface For the Completely Predictable problem of $y = \sin x$ function and the Incompletely Predictable problem of Riemann zeta function, the following collective statements are valid:

For the Completely Predictable problem of $y = \sin x$ function in Table 3: Set of all axes intercepts = Set of $x$-axis intercepts + Set of 'Origin' intercept. Set of [infinite] $x$-axis intercepts $>>$ Set of [single/finite] 'Origin' intercept inequality relationship is an "exact non-varying" Completely Predictable relationship. The axes intercepts relationship interface for this even-odd number pairing is symbolized by the Straight line INTERFACE.

For the Incompletely Predictable problem of Riemann zeta function (involving the three axes intercepts) in Table 4: Set of all axes intercepts = Set of $x$-axis intercepts + Set of $y$-axis intercepts + Set of 'Origin' intercepts. Set of [infinite] $x$-axis intercepts = Set of [infinite] $y$-axis intercepts = Set of [infinite] 'Origin' intercepts equality relationship is an "exact varying" Incompletely Predictable relationship. The axes intercepts relationship interface for this Riemann zeta function is symbolized by the Jagged line INTERFACE. Classical examples of intrinsic properties with special characteristics ("meta-properties") arising out of the "interactions" between the two dependent sets of 'usual' Gram points ($x$-axis intercepts) and non-trivial zeros ('Origin' intercepts) are the Completely Predictable periodic, albeit Incompletely Predictable timing of, occurrences of Gram’s Law and its violation, Gram block, etc on an eternal basis. These phenomena are outlined in relevant parts of this paper below.

Unless stated otherwise in this paper, the symbol 'log' will refer to natural logarithm; and we will generally outline materials on the proof for Riemann hypothesis ahead of materials on the proofs for Polignac’s and Twin prime conjectures.

The term 'hypothesis' is often taken to connote a 'conjecture' once it has been rigorously proven to be true. For instance, the traditionally-dubbed 'Riemann hypothesis' should strictly be previously labeled as 'Riemann conjecture' because chronologically this reference to 'Riemann hypothesis' has [ideally] not been used correctly in the era prior to rigorous proof being obtained for this conjecture. Therefore utilizing more accurate terms and metaphorically speaking in the relevant parts of this paper below, we aim to produce easy-to-understand [initially] geometrical-format-version & [subsequently] mathematical-format-version of the wider proposed-in-2017 Dirichlet-Gram-Riemann conjecture which also encompasses the proposed-in-1859 Riemann conjecture; with both conjectures, once proven, being able to be denoted by (respectively) Dirichlet-Gram-Riemann hypothesis and Riemann hypothesis. On the other hand, this designated conjecture-to-hypothesis terminology usage with Polignac’s and Twin prime conjectures is
appropriate whereby the adjoined term ‘conjecture’ can now be replaced by the term ‘hypothesis’ to give rise to Polignac’s and Twin prime hypotheses once these conjectures are proven to be true.

1.2 Brief overview of Riemann hypothesis

Riemann hypothesis refers to the famous 1859 conjecture explicitly equivalent to the mathematical statement that the critical line [which is a vertical straight line of infinite length defined by $\sigma = \frac{1}{2}$] in the critical strip [which is a vertical rectangular region of infinite area defined by $0 < \sigma < 1$] of Riemann zeta function is the location for all nontrivial zeros. At the most rudimentary level, Riemann hypothesis simply refers to the generated curves of Riemann zeta function graphically intersecting both x- and y-axes [formally named the 'Origin'] an infinite number of times and these 'Origin' intercepts are Incompletely Predictable entities constituting a countable infinite set (CIS) of irrational [transcendental] numbers. It is precisely because these nontrivial zeros are Incompletely Predictable entities thus obeying Complex Elementary Fundamental Laws that we will need to use our devised Virtual Container Research Method (VCRM) to solve Riemann hypothesis.

Remark 1.2. Computationally checking for nontrivial zeros to be correctly located on the critical line philosophically implies [but does not rigorously prove] Riemann hypothesis to be true.

In regards to Riemann hypothesis, it was postulated that nontrivial zeros of Riemann zeta function must all lie on the critical line ($\sigma = \frac{1}{2}$) or [equivalently stated in this paper] must all exactly match the 'Origin' intercepts. This has previously been computationally checked for the first 10,000,000,000,000 identities (solutions). But the exercise of checking these nontrivial zeros identities only represent the ‘tip of the iceberg’ as they have already been shown to be infinitely many of them lying on this critical line by Hardy in 1914 [3] and Hardy & Littlewood in 1921 [4] by considering moments of certain functions related to Riemann zeta function. However this discovery by Hardy and Littlewood does not constitute the rigorous proof for Riemann hypothesis because they have not mathematically exclude the possible existence of nontrivial zeros which are located away from the critical line. As we shall see, the use of our Virtual Container [essentially embodied in the relevant Theorem {Riemann} I to IV below] in "containing" all nontrivial zeros but without needing to know their true identities is paramount in allowing us to convincingly prove Riemann hypothesis via subsequent / concurrent "correct analysis" of this Virtual Container. Resulting crucial primary and secondary beneficiary by-products arising out of this accomplished feat (predominantly achieved through using the mathematical tool Sigma-Power Laws and concepts from the Hybrid method of Integer Sequence classification) promise to be aplenty.

For the purpose of this research paper, we are interested in various generated curves [Output] of Riemann zeta function which are "continuous" uncountable infinite set (UIS) when given the relevant values [Input] which are again "continuous" UIS. As opposed to the resulting curves generated from the overall "continuous" UIS Output; the resulting points generated from the "discrete" CIS Output of trivial and nontrivial zeros are subsets of this overall "continuous" UIS Output. All the negative even (integer) number values constituting "discrete-type" triv-
Riemann-Polignac-Twin prime viXra

1.3 Brief overview of Polignac’s and Twin prime conjectures

The ancient Euclid’s proof on the infinitude of prime numbers predominantly by *reductio ad absurdum* (proof by contradiction), occurring well over 2000 years ago (c. 300 BC), is the earliest known but not the only possible proof for this simple
problem in number theory. Since then dozens of proofs have been devised to show that prime numbers in totality are indeed infinite in magnitude such as the three chronologically listed below with the strangest candidate likely to be Furstenberg’s Topological Proof.

1. Goldbach’s Proof using Fermat numbers (written in a letter to Swiss mathematician Leonhard Euler, July 1730)

Then solving Polignac’s & Twin prime conjectures demands the rigorous proof that each and every single one of the infinitely many odd prime numbers is derived from the infinite (and arbitrarily large) magnitude of even number prime gaps = 2, 4, 6,... with each even number prime gap generating its own (unique) infinite magnitude of odd prime numbers. The exception to this trend occur for the very first and only even prime number ‘2’ which is associated with the only odd number prime gap = 1. These two conjectures can equivalently be explicitly stated as they demanding the rigorous proof that each and every single one of the infinitely many even composite numbers is derived from the finite magnitude of odd number composite gap = 1 & even number composite gap = 2. The exception to this trend occur for the very first and only odd composite number ‘1’ which is associated with the solitary odd number composite gap = 3.

There is a massive unification of prime numbers and Riemann hypothesis in the sense that a crucial primary or direct by-product arising out of the rigorous proof for Riemann hypothesis is theorized to result in complete formalization of prime number theorem which relates to prime counting function for prime numbers.

The Incompletely Predictable prime numbers will consist of prime numbers with ’small gaps’ and prime numbers with ’large gaps’ alike. Roughly speaking akin to the ’proof by contradiction’ technique, our lemmas, propositions & theorems in relation to Polignac’s and Twin prime conjectures in this paper are comparatively simple and prominently based on compulsorily using Virtual Containers to ”contain” the infinite magnitude of all known prime numbers but without needing to know their true identities simply because otherwise we have to infinitely often prove (without complete certainty) that every single prime number will comply with certain mathematical properties (satisfying Condition X or Y or Z). This inevitable mathematical snag could be perceived as the classical equivalent of persistently encountering the (fatal) mathematical error ”undefined” when dividing a non-zero number N by zero; viz. N÷0 = ”infinitely large arbitrary number” is undefined, whereas the reciprocal 0÷N = 0 is clearly defined. [We note that the analogy of this (fatal) mathematical error ”undefined” will similarly be encountered by all other Incompletely Predictable entities such as nontrivial zeros of Riemann zeta function.] Phrased in another way: Once successfully obtained, our Virtual Containers can then be used to rigorously prove Polignac’s and Twin prime conjectures (with complete certainty) via the required finite steps of subsequent / concurrent correct analysis on various derived mathematical ”meta-properties” (once again satisfying Condition X or Y or Z) obtained from these Virtual Containers. Furthermore, if our Virtual Container Research Method is apparently the only (solitary) way to ultimately solve these two conjectures, then they are to be regarded as belonging to the ’Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution’, or simply the ’Incompletely Predictable problems’.
The Virtual Containers for prime numbers must be endowed with the following key properties and behaviors. They must mathematically (i) incorporate the ability to accurately and completely "contain" the relevant prime (& composite) numbers without being "contaminated" by non-prime (& non-composite) numbers entities, and (ii) not utilize the ability to either fully or partially "calculate" identities of relevant prime (& composite) numbers in an intrinsic manner.

We will provide a brief synopsis pertaining to important treatises on prime numbers. The main goal is to solve Polignac’s conjecture which can be succinctly stated as whether even number prime gaps are infinite (and arbitrarily large) in magnitude with each individual even number prime gap generating prime numbers which are again infinite in magnitude. Polignac’s conjecture involves analysis of all possible even number prime gaps = 2, 4, 6,... which slowly become infinitely large at prime number examination on larger ranges (in the opposite direction to that of the smallest possible prime gap = 2). Bearing in mind that Twin prime conjecture involves analysis of even number prime gap = 2 (for twin primes), we can regard this conjecture as a mathematical subset of Polignac’s conjecture. We use our Virtual Container Research Method, which neatly incorporates the novel mathematical tool coined Information-Complexity conservation, to solve those conjectures. Having obtained the relevant Virtual Container [which is essentially embodied in the relevant Theorem \{Polignac-Twin prime\} I to IV below], the subsequent / concurrent correct analysis of this Virtual Container will result in the rigorous proofs for these two conjectures to materialize. The proofs will only succeed with deployment of sound mathematical principles which are mostly derived from our Information-Complexity conservation and Set theory. Prime numbers and composite numbers are intimately related simply because the complementary set of composite numbers constitutes the set of natural numbers with the exact set of prime numbers excluded in its entirety. The Information-Complexity conservation has its core foundation based on this [complete] prime-composite number relationship. In addition, a key mathematical law dubbed ‘Plus-Minus Composite Gap 2 Number Alternating Law’ (and its companion law dubbed ‘Plus Composite Gap 2 Number Continuous Law’) arising naturally from applying this Information-Complexity conservation form vital mathematical bridges in achieving those rigorous proofs.

The phrase “If I have seen a little further it is by standing on the shoulders of giants” used by Isaac Newton in a 1676-dated letter to his rival Robert Hooke is eminently applicable to all modern researchers, scientists, and authors alike from the current 21st Century. Generally speaking, a relevant valid finding arising from the finalized recent/past research would usually represent a mathematical step closer to achieving that elusive proof for a well-defined conjecture.

In 2013, Yitang Zhang proved a spectacular landmark mathematical result showing that there is some unknown even number ‘N’ smaller than 70 million such that there are infinitely many pairs of primes that differ by ‘N’ \[7\]. Without going into specific details concerning optimizing Zhang’s bound, subsequent Polymath Project collaborative efforts employing a new refinement of the GPY sieve in 2013 lowered ‘N’ to 246; and assuming the Elliott-Halberstam conjecture and its generalized form have managed to further lower ‘N’ to 12 and 6, respectively. Thus ‘N’ has (intuitively) more than one valid values such that there are infinitely many pairs of primes that differ by each of those ‘N’ values. No matter what, we can only theoretically lower ‘N’ to 2 (in regards to prime numbers with ‘small gaps’), and unfortunately there are still an infinite number of even number prime gaps
(in regards to prime numbers with 'large gaps') that will require "the proof that each will generate a set of infinite prime numbers". Colloquially speaking, we are just observing the 'tip of the iceberg' in regards to the infinitely (and arbitrarily) large number of even number prime gaps. In other words, the belief here is that we can justifiably use our Virtual Container (predominantly achieved through Information-Complexity conservation) to mathematically contain the complete set of infinite prime numbers [apart from the first prime number '2' represented by the solitary odd prime gap = 1] generated by all infinite even number prime gaps. Then subsequent/concurrent correct analysis of this Virtual Container will ultimately yield the reward for successfully proving those two conjectures on prime numbers.

On the [dual] infinity of the set of even number prime gaps and of each associated set of prime numbers derived from individual even number prime gap, we can see that the potential mathematical source(s) of prime number infiniteness could feasibly arise in two ways via (i) one or more than one or all of the even number prime gap(s) with those nominated even number prime gap(s) each generating an infinite magnitude of distinct prime numbers &/or (ii) the infinite (and arbitrarily) large magnitude of even number prime gaps collectively able to generate an infinite magnitude of prime numbers such that the criterion in (ii) will hold true even if none of the individual even number prime gap were to ever generate an infinite magnitude of prime numbers. Stated differently for the "none of the individual even number prime gaps were to ever generate an infinite magnitude of prime numbers’ scenario, we are alleging that groups of these [imaginary] finite prime numbers derived from each even number prime gap are [wrongly] classified as countable finite sets (CFS) but this arrangement will still culminate in producing a [incomplete] countable infinite set (CIS) of prime numbers as long as there are infinitely many even number prime gaps. We now arrive at the realization that rigorously proving Polignac’s and Twin prime conjectures in essence would entail the need to show that (i) the solitary set of all even number prime gaps and (ii) the infinite sets of prime numbers (comprising of prime numbers in totality apart from prime number '2') arising from all even number prime gaps, must all be CIS.

We next outline some interesting properties of prime numbers.

English mathematician John Horton Conway coined the term 'jumping champion' in 1993. An integer n is a jumping champion if n is the most frequently occurring difference (prime gap) between consecutive prime numbers < x for some x. Example: for any x with 7 < x < 131, n = 2 (indicating twin prime numbers) is the jumping champion. It has been conjectured that (i) the only jumping champions are 1, 4 and the primorials 2, 6, 30, 210, 2310, 30030,... and (ii) jumping champions tend to infinity. Their required proofs will likely need the proof of the k-tuple conjecture. [For i = 1, 2, 3, 4, 5, 6,...; primordial P_i# is the analog of the usual factorial for prime numbers (2, 3, 5, 7, 11, 13,...). For instance P_1# = 2, P_2# = 2 X 3 = 6, P_3# = 2 X 3 X 5 = 30, P_4# = 2 X 3 X 5 X 7 = 210, P_5# = 2 X 3 X 5 X 7 X 11 = 2310, P_6# = 2 X 3 X 5 X 7 X 11 X 13 = 30030, etc.]

We now look at the data of all prime numbers obtained when extrapolated out over a wide range of x values. Generally speaking, as the sequence of prime numbers carries on, prime numbers with ever larger prime gaps will tend to appear. For the given range of x values, we say that prime gap = n_2 is a 'maximal prime gap' if prime gap = n_1 < prime gap = n_2 for all n_1 < n_2. In other words, the largest such prime gaps in the sequence are called maximal prime gaps. The ratio n_2 : log (prime number associated with prime gap = n_2) is called the 'merit' of prime gap
Table 5 First 17 prime gaps depicted in the format utilizing maximal prime gaps [depicted with the asterisk symbol (*)] and non-maximal prime gaps [depicted without this symbol].

<table>
<thead>
<tr>
<th>Prime gap</th>
<th>Following the prime number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>2</td>
</tr>
<tr>
<td>2*</td>
<td>3</td>
</tr>
<tr>
<td>4*</td>
<td>7</td>
</tr>
<tr>
<td>6*</td>
<td>23</td>
</tr>
<tr>
<td>8*</td>
<td>89</td>
</tr>
<tr>
<td>10</td>
<td>139</td>
</tr>
<tr>
<td>12</td>
<td>199</td>
</tr>
<tr>
<td>14*</td>
<td>113</td>
</tr>
<tr>
<td>16</td>
<td>1831</td>
</tr>
<tr>
<td>18*</td>
<td>523</td>
</tr>
<tr>
<td>20*</td>
<td>887</td>
</tr>
<tr>
<td>22*</td>
<td>1129</td>
</tr>
<tr>
<td>24</td>
<td>1669</td>
</tr>
<tr>
<td>26</td>
<td>2477</td>
</tr>
<tr>
<td>28</td>
<td>2971</td>
</tr>
<tr>
<td>30</td>
<td>4297</td>
</tr>
<tr>
<td>32</td>
<td>5591</td>
</tr>
</tbody>
</table>

= n₂ (maximal prime gap). Thus the merit of a prime gap is a normalized number representing how “soon” in the sequence a maximal prime gap appears relative to the natural logarithm of the associated larger prime number of interest. To the best of our knowledge, there is no clear-cut correlation between the largest known merit value and either the relative size of the relevant prime number with prime gap = n₂ or the relative size of that prime gap = n₂.

The term ‘first occurrence prime gaps’ commonly refers to first occurrences of maximal prime gaps whereby maximal prime gaps can also be perceived here as prime gaps of ”at least of this length”. The CIS of ’maximal prime gaps’ and the (complementary) CIS of ’non-maximal prime gaps’ can be fully derived and depicted as below. We endorse non-maximal prime gaps with the interesting nickname ‘slow jumpers’ in this paper. We coin the term ’slow jumpers’ here because non-maximal prime gaps always lag behind their maximal prime gaps counterparts for their onset appearances in the prime number sequence. This is tabulated for the first 17 prime gaps in Table 5 consisting of maximal prime gaps and non-maximal [slow jumper] prime gaps.

Note that the progressive resultant prime numbers generated here in Table 5 solidly represent only a single prime number for each prime gap and this will always be less than the complete set of all prime numbers generated from, for instance, the Sieve of Eratosthenes. The initial seven of the [majority] ”missing” prime numbers are 5, 11, 13, 17, 19, 29, 31,...; and they all belong to the subset of prime numbers with ‘residual’ prime gaps which must be the potential source of prime numbers in relation to the proposal that each of the even number prime gaps of 2, 4, 6, 8, 10,... will generate its specific CIS of prime numbers.

Remark 1.3. Maximal and non-maximal prime gaps supply crucial indirect evidence to intuitively and philosophically support, but does not prove, the mathematical statement ”Each even number prime gap will generate an infinite magnitude of prime numbers on its own accord”. 

From the above brief analysis on prime number distribution, we easily deduce that [predominantly the groups of] prime numbers with jumping champion prime gaps, [the individual/groups of] prime numbers with maximal prime gaps, and [the individual/groups of] prime numbers with non-maximal prime gaps would seem to make perpetual repeating appearances amongst the complete CIS of prime numbers. A vitally crucial observation is that all prime numbers generated by (i) non-maximal (slow jumper) prime gaps and (ii) maximal prime gaps, will still not generate the complete CIS of prime numbers. This is simply because, apart from the one-off prime gap = 1 associated with the very first prime number '2', all other [infinite magnitude] prime gaps 2, 4, 6, 8, 10,... must each generate more than one, if not a CIS of, prime numbers in order to account for all prime numbers. This clear-cut observation constitutes indirect evidence to intuitively and philosophically support [but does not prove] the proposition that each even number prime gap will likely generate an infinite magnitude of prime numbers on its own accord.

Although not crucial for the purpose of this paper, we could potentially study exciting behaviors from the subset of prime numbers with 'residual' prime gaps as obtained when the subset of prime numbers with maximal prime gaps and the subset of prime numbers with non-maximal prime gaps are progressively removed from the complete set of prime numbers derived from all known prime gaps. This can mathematically be visualized as: Complete set of prime numbers with all prime gaps = Subset of prime numbers with maximal prime gaps + Subset of prime numbers with non-maximal prime gaps + Subset of prime numbers with 'residual' prime gaps. In addition, prime numbers with 'residual' prime gaps must include all the correctly selected (odd) prime numbers representing all even number prime gaps 2, 4, 6, 8, 10,... [except the one-off very first odd number prime gap = 1 representing the very first (and only even) prime number '2'].

2 The eight categories of intercepts for 2-Variable Equations

In the Class of n-Variable Equations with n = 2 [which translate to 2-Variable Equations], when computationally depicted by 2-dimensional graphs with their x- and y-axes relevantly defined; they often have one or more points of intersection on (i) x-axis, and/or (ii) y-axis, and/or (iii) both x- and y-axes [formally known as the 'Origin']. The Origin, often labeled with capital letter 'O', is defined as the point where the vertical y-axis and the horizontal x-axis intersect each other. Not all functions, though, will have intercepts; which are where the graph crosses either the x-axis (viz. the x-axis intercept, often referred to as "zeros or roots of the equation"), or the y-axis (viz. the y-axis intercept), or both the x- and y-axes (viz. the Origin intercept). There are eight possible Categories of Intercepts for 2-Variable Equations, as detailed below:

- Category I Intercept: comprising of nil intercept
- Category II Intercept: comprising of single x-axis intercept(s) only
- Category III Intercept: comprising of single y-axis intercept(s) only
- Category IV Intercept: comprising of single Origin intercept(s) only
- Category V Intercept: comprising of double x- and y-axes intercept(s)
- Category VI Intercept: comprising of double x-axis and Origin intercept(s)
- Category VII Intercept: comprising of double y-axis and Origin intercept(s)
- Category VIII Intercept: comprising of triple x-, y-axes and Origin intercept(s)
Fig. 1 Sample graphs for Category I Intercept using \( y = \frac{1}{x} \) with nil intercept; Category V Intercept using \( y = (x + 3)(x - 3) = x^2 - 9 \) with two x-axis intercepts and one y-axis intercept; Category VI Intercept using \( y = \sin(x) \) with one Origin intercept and infinite number of x-axis intercepts; and Category VIII Intercept using \( y = x^3 \) with one Origin intercept.

Combinatorics language wise, the eight categories are from the sum total of (a) choosing zero item from three as in Category I; (b) choosing one item from three as in Category II, III, and IV; (c) choosing two items from three as in Category V, VI, and VII; and (d) choosing three items from three as in Category VIII. Theoretically, the various intercepts could numerically consist of whole numbers from 0, 1, 2, ..., \( \infty \). Permutational wise, this observation would apparently result in (not unexpectedly) limitless, or almost limitless, 'exotic-flavored' equations; typified (for instance) by the simple case of 2-Variable Equation \( y = \sin(x) \) (shown as part of Figure 1 and discussed under Remark 1.1 in Section 1.1 above as an example of 'Axes intercepts relationship interface'), which is easily seen as belonging to Category VI Intercept containing a Completely Predictable solitary Origin intercept combined with a Completely Predictable infinite number of x-axis intercepts.

3 Riemann zeta and Dirichlet eta functions

The iconic Riemann zeta function, \( \zeta(s) \), is a function of the complex variable \( s = \sigma \pm it \) that analytically continues the sum of an infinite series

\[
\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots.
\]

The arguments of this function are traditionally denoted by two letters: sigma (\( \sigma \)) for the real part, and \( t \) for the imaginary part where \( i = \sqrt{-1} \) is the imaginary number. Alternatively stated, \( \zeta(s) \) is the famous complex number infinite series constituting a real and an imaginary part determined by its complex variable \( s \); whereby \( s \) itself is further constituted by a real part \( \sigma \), and an imaginary part \( t \). In practice, the positive (0 < \( t < +\infty \)) and numerically equal to the negative (\( -\infty < t < 0 \)), counterpart of the conjugate pairs for the x-axis, y-axis, and Origin intercepts is usually quoted or employed for calculation purposes.

The Dirichlet eta function, \( \eta(s) \), also known as the alternating zeta function, must act as the proxy/surrogate for \( \zeta(s) \) in the critical strip (0 < \( \sigma < 1 \)) whereby the critical line (\( \sigma = \frac{1}{2} \)) lies. This is because \( \zeta(s) \) only converges when \( \sigma > 1 \), implying that it is essentially undefined to the left of this region [viz. 0 < \( \sigma < 1 \)] which then requires its proxy \( \eta(s) \) representation instead. Their mathematical relationship is defined by \( \zeta(s) = \gamma \cdot \eta(s) \), whereby the proportionality factor \( \gamma \) is defined as \( \gamma = \frac{1}{(1 - 2^{1-s})} \) and

\[
\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots.
\]
The above paragraphs are further discussed below in terms of Simplicity and Complexity. The concept of Simplexity [as opposed to Simplicity] could be amplified as the process by which nature strives towards simple ends by complex or complicated means. In other words, Simplexity may be defined as the combination of Simplicity and Complexity within the context of a dynamic relationship between means and ends. We observe the extra presence of alternating + and - signs in $\eta(s)$ conferring an extra layer of Complexity as opposed to just the + sign present in $\zeta(s)$ with relative Simplicity – bear in mind that there are in general "Subjective to Semi-objective to Objective views on the Simplicity to Complexity continuum-spectrum range". Stated in a slightly different manner, the words 'Simplicity' and 'Complexity' can be seen to roughly progress along the following equivalent but opposing continuum-spectrum range.

\[ \text{Left} \overset{\text{Decreasing Order from Left to Right for Maximal Simplicity to Minimal Simplicity range}}{\longrightarrow} \text{Right} \]

\[ \text{Left} \overset{\text{Increasing Order from Left to Right for Minimal Complexity to Maximal Complexity range}}{\longrightarrow} \text{Right} \]

\[ \text{Fig. 2} \quad \text{INPUT for } \sigma = \frac{1}{2}, \frac{2}{5}, \text{ and } \frac{3}{5}. \] For Riemann zeta function, the set of zeros, or roots, in $\zeta(s)$ consist of the easily identifiable (Completely Predictable) trivial zeros located at $\sigma = -2, -4, -6, -8, -10,...$ and the not-so-easily identifiable (Incompletely Predictable) nontrivial zeros located at $\sigma = \frac{1}{2}$ for various computed $t$ values. Both trivial and nontrivial zeros are of infinite magnitude.

\[
\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}
\]

\[
= \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots
\]

\[
= \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}
\]

\[
= \frac{1}{(1 - 2^{-s}) \cdot (1 - 3^{-s}) \cdot (1 - 5^{-s}) \cdot (1 - 7^{-s}) \cdot (1 - 11^{-s}) \cdots}
\]
Eq. (1) can only be defined for the $1 < \sigma < \infty$ region whereby $\zeta(s)$ is absolutely convergent. There are no zeros located in this region. Note the equivalent Euler product formula with product over prime numbers [instead of summation over natural numbers] can also be used to represent Riemann zeta function.

$$\zeta(s) = 2^s \pi^{s-1} \sin \left( \frac{\pi s}{2} \right) \Gamma(1-s) \zeta(1-s)$$

With $\sigma = \frac{1}{2}$ as a symmetry line of reflection, Eq. (2) is the Riemann’s functional equation fully satisfying $-\infty < \sigma < \infty$ and can be used to find all trivial zeros on the horizontal line at $it = 0$ and $\sigma = -2, -4, -6, -8, -10, \ldots, \infty$ [all negative even numbers] whereby $\zeta(s) = 0$ because the factor $\sin(\frac{\pi s}{2})$ vanishes. $\Gamma$ is the gamma function, an extension of the factorial function [a product function denoted by the $!$ notation; $n! = n(n-1)(n-2)\ldots (n-(n-1))$] with its argument shifted down by 1, to real and complex numbers. That is, if $n$ is a positive integer, $\Gamma(n) = (n - 1)!$

$$\zeta(s) = \left( \frac{1}{1 - 2^{1-s}} \right) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$$

$$= \left( \frac{1}{1 - 2^{1-s}} \right) \left( \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \ldots \right)$$

Eq. (3) is defined for all $\sigma > 0$ except for a simple pole at $\sigma = 1$. As just alluded to above, $\zeta(s)$ without the $\left( \frac{1}{1 - 2^{1-s}} \right)$ proportionality factor, viz. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$ is also known as Dirichlet eta ($\eta$) or alternating zeta function. This $\eta(s)$ function is
a holomorphic function of $s$ as defined by analytic continuation and can mathematically be seen to be defined at $\sigma = 1$ whereby an analogous trivial zeros [with presence only] for $\eta(s)$ [and not for $\zeta(s)$] on the vertical straight line $\sigma = 1$ are obtained at $s = 1 \pm i \frac{2\pi k}{\log(2)}$ where $k = 1, 2, 3, \ldots, \infty$. All nontrivial zeros are conjectured to be located on the critical line ($\sigma = \frac{1}{2}$) in the critical strip ($0 < \sigma < 1$) of this region.

Any given function or equation including our $\zeta(s)$ can be supplied with an INPUT and resulting in an OUTPUT. Figure 2 pictorially depict complex variable $s = (\sigma \pm it)$ as INPUT with x-axis denoting the real part $\text{Re}\{s\}$, equating to $\sigma$; and y-axis denoting the imaginary part $\text{Im}\{s\}$, equating to $t$. Figures 3, 4, and 5 schematically depict (respectively) $\zeta(s)$ as OUTPUT for real values of $t$ running from 0 to 34 at $\sigma = \frac{1}{2}$ (critical line), $\sigma = \frac{5}{6}$, and $\sigma = \frac{3}{5}$ with x-axis denoting the real part $\text{Re}\{\zeta(s)\}$ and y-axis denoting the imaginary part $\text{Re}\{\zeta(s)\}$. Riemann hypothesis can be computationally visualized as the appearance of infinite number of Origin intercepts by its generated spirals/curves occurring only when $\sigma = \frac{1}{2}$; with this previously computed for the first 10,000,000,000,000 nontrivial zeros.

There will be an infinite types-of-spirals possibilities associated with each and every $\sigma$ value arising from all possible infinite $\sigma$ values in the critical strip. From this, we derive the following two "interim" propositions for $\zeta(s)$ with their proofs naturally arising out of the other proofs obtained for subsequent lemmas, propositions and theorems below.

**Proposition ["interim"] 3.1.** Only at $\sigma = \frac{1}{2}$ value will the generated single [finite] type-of-spiral have Category VIII Intercept comprising of triple x-axis, y-axis and Origin intercepts and with all these intercepts consisting of transcendental numbers having Incompletely Predictable properties and be of infinite magnitude.

**Proposition ["interim"] 3.2.** For all other $\sigma \neq \frac{1}{2}$ (viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$) values, the generated multiple [infinite] types-of-spirals have Category V Intercept comprising of double x-axis and y-axis intercepts and with all these intercepts consisting of transcendental numbers having Incompletely Predictable properties and be of infinite magnitude.

The conceptually important mathematical ideas depicted under Remark 1.2 in subsection 1.2 above are that the 'varying gap' between two consecutive nontrivial zeros (dubbed nontrivial zeros gap), two consecutive 'usual' Gram points (dubbed Gram points gap) and two consecutive Gram[x=0] points (dubbed Gram[x=0] points gap) can be seen as the equivalent 'varying gradient' dubbed, respectively, nontrivial zeros gradient, 'usual' Gram points gradient and Gram[x=0] points gradient. We can now geometrically visualize these 'varying gaps' or 'varying gradients' as being related to the size and shape of those spirals/loops present in Figure 3. The foundation mathematics targeted to concisely prove the infinite magnitude existence of all intercepts associated with both propositions above would naturally belong to the realm of rigorously proving Dirichlet-Gram-Riemann conjecture which literally consists of Gram[x=0] conjecture, Gram[y=0] or 'usual' Gram conjecture, and Gram[x=0,y=0] conjecture (Riemann hypothesis). The expanded explanations on each respective conjecture are elaborated in the relevant sections below. But, firstly, preliminary $\zeta(s)$ and $\eta(s)$ nomenclature materials with interpretational meanings are provided by the following six clear-cut correlation points.

(A) At the one specific $\sigma = \frac{1}{2}$ value whereby the term Gram points is understood to denote the "Critical line-Gram points" official notation;
Point 1. The Origin intercepts are synonymous with Gram[x=0,y=0] points or the traditionally denoted 'nontrivial zeros'. The associated Riemann hypothesis is synonymous with Gram[x=0,y=0] conjecture.

Point 2. The x-axis intercepts are synonymous with Gram[y=0] points or the traditionally denoted 'Gram points'. This is associated with Gram[y=0] conjecture.

Point 3. The y-axis intercepts are synonymous with Gram[x=0] points. This is associated with Gram[x=0] conjecture.

(B) For all other infinite $\sigma \neq \frac{1}{2}$ (viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$) values whereby the term 'near-identical' (virtual) Gram points is understood to denote the "Non-critical lines-Gram points" official notation;

Point 4. The Origin intercepts are non-existent.

Point 5. The x-axis intercepts are synonymous with 'near-identical' (virtual) Gram[y=0] points. These points have totally different numerical values to the Gram[y=0] points mentioned in Point 2.

Point 6. The y-axis intercepts are synonymous with 'near-identical' (virtual) Gram[x=0] points. These points have totally different numerical values to the Gram[x=0] points mentioned in Point 3.

4 {Geometrical-format-version} conjectures

Our Gram[y=0] {geometrical-format-version} conjecture is explicitly equivalent to the statement that the infinite number of x-axis intercepts / Gram[y=0] points derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$.

Our Gram[x=0] {geometrical-format-version} conjecture is explicitly equivalent to the statement that the infinite number of y-axis intercepts / Gram[x=0] points derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$.

From previous reasoning above, we can now justifiably coin the 1859 Riemann {geometrical-format-version} hypothesis as Gram[x=0,y=0] {geometrical-format-version} conjecture; which is explicitly equivalent to the statement that the infinite number of Origin intercepts / Gram[x=0,y=0] points / nontrivial zeros derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$. In descriptive terms, this can be seen in Figure 3 as Riemann zeta function in the critical strip generating an infinite number of spirals graphically intersecting the Origin an infinite number of times only on the critical line which is denoted by $\sigma = \frac{1}{2}$.

Then, as honor and tribute to the three famous namesake mathematicians, it is pure common sense to create the 'glorified' Dirichlet-Gram-Riemann {geometrical-format-version} conjecture which is compatibly equivalent to the statement that the infinite number of Gram[x=0] points, Gram[y=0] points, and Gram[x=0,y=0] points derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$. 
5 Prerequisite lemma, corollary and propositions for Gram[x=0,y=0] conjecture (Riemann hypothesis)

We treat and closely analyze Riemann zeta and Dirichlet eta functions as unique mathematical objects looking for key intrinsic properties and behaviors. As original true equations containing all possible x-axis, y-axis and Origin intercepts, Riemann zeta and Dirichlet eta functions by themselves viz. without computationally supplying "input information" as depicted in Figure 2 [with horizontal axis: $\sigma$ and vertical axis: $t$] so as not to generate the necessary "output complexity" as depicted in Figure 3 [with horizontal axis: $\text{Re}\{\zeta(\frac{1}{2} + it)\}$ and vertical axis: $\text{Im}\{\zeta(\frac{1}{2} + it)\}$]; these two functions will both intrinsically incorporate the actual presence [but not the actual locations] of the complete set of Gram[x=0] points, Gram[y=0] points, and Gram[x=0,y=0] points. We use the typical case of nontrivial zeros or Gram[x=0,y=0] points to obtain the required lemma, corollary, propositions and their proofs prior to ultimately proving the relevant Theorem {Riemann} I to IV below. The lemma, corollary, propositions and proofs associated with the other two cases of Gram[x=0] and Gram[y=0] points will simply reflect "slight mathematical variations to the same theme for nontrivial zeros". These are outlined in Appendix 1.

Lemma 5.1. The Riemann-Dirichlet Ratio can be derived from Riemann zeta or Dirichlet eta function with absolute ability to incorporate the actual presence [but not the actual locations] of the complete set of nontrivial zeros.

Proof. Euler formula is commonly stated as $e^{i\pi} = \cos \pi + i.\sin \pi$. The magnificent Euler identity (where $\pi = \pi$) is $e^{i\pi} = \cos \pi + i.\sin \pi = -1 + 0$, commonly stated as $e^{i\pi} + 1 = 0$. The $n^s$ of Riemann zeta function can be expanded to $n^s = n^{(\sigma + it)} = n^{\sigma}.e^{t.\log(n)}$ since $n^s = e^{t.\log(n)}$. Apply the Euler formula to $n^s$ will result in $n^s = n^{\sigma} \cdot (\cos(t.\log(n)) + \sin(t.\log(n))$ – designated here with the short-hand notation $n^s(Euler)$ – whereby $n^\sigma$ is the modulus and $t.\log(n)$ is the polar angle.

Apply $n^s(Euler)$ to Eq. (1), we have $\zeta(s) = \text{Re}\{\zeta(s)\} + i.\text{Im}\{\zeta(s)\}$ whereby $\text{Re}\{\zeta(s)\} = \sum_{n=1}^{\infty} n^{-\sigma} \cdot \cos(t.\log(n))$ and $\text{Im}\{\zeta(s)\} = i. \sum_{n=1}^{\infty} n^{-\sigma} \cdot \sin(t.\log(n))$. As Eq. (1) is defined only for $\sigma > 1$ where zeros never occur, we will not carry out further treatment related to this subject area.

Apply $n^s(Euler)$ to Eq. (3), we have $\zeta(s) = \gamma.\eta(s) = \gamma.[\text{Re}\{\eta(s)\} + i.\text{Im}\{\eta(s)\}]$ whereby

\[
\text{Re}\{\eta(s)\} = \sum_{n=1}^{\infty} ((2n - 1)^{-\sigma} \cdot \cos(t.\log(2n - 1)) - (2n)^{-\sigma} \cdot \cos(t.\log(2n)))
\]

\[
\text{Im}\{\eta(s)\} = i. \sum_{n=1}^{\infty} ((2n)^{-\sigma} \cdot \sin(t.\log(2n)) - (2n - 1)^{-\sigma} \cdot \sin(t.\log(2n)))
\]

Here $\gamma$ is the proportionality factor $\frac{1}{(1 - 2^{1-\sigma})}$. 
Apply the trigonometry identity $\cos(x) - \sin(x) = \sqrt{2}\sin\left(x + \frac{3}{4}\pi\right)$ to $\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\}$. Then,

$$\sum ReIm\{\eta(s)\} = \sum_{n=1}^{\infty} \left[(2n-1)\cos(t.\log(2n-1))\right] \tag{4}$$

Note our self-explanatory TAG legend used to illustrate where each term in the equations above originated from. It can easily be seen that both terms in the final equation consist of a mixture of real and imaginary portions. As Riemann hypothesis on nontrivial zeros based on $\zeta(s)$ is identical to that based on its proxy $\eta(s)$, then it is satisfied when

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\} = 0 \tag{5}$$

Ignoring the $\sqrt{2}$ term temporarily and with the application of Eq. (5), Eq. (4) becomes

$$\sum_{n=1}^{\infty} (2n-1)^{-\sigma}.\sin(t.\log(2n-1)+\frac{3}{4}\pi) = \sum_{n=1}^{\infty} (2n)^{-\sigma}.\sin(t.\log(2n)+\frac{3}{4}\pi) \tag{6}$$

We note from the above sequential mathematical derivation of Eq. (6) that this equation will completely and intrinsically fulfills the 'presence of the complete set of nontrivial zeros without knowing their actual location' criteria.

$$\frac{\sum_{n=1}^{\infty} \sin(t.\log(2n)+\frac{3}{4}\pi)}{\sum_{n=1}^{\infty} \sin(t.\log(2n-1)+\frac{3}{4}\pi)} = \frac{\sum_{n=1}^{\infty} (2n)^{\sigma}}{\sum_{n=1}^{\infty} (2n-1)^{\sigma}} \tag{7}$$

Eq. (7) above will also abide to this specified criteria as it is simply the result of rearranging the terms in Eq. (6) thus giving rise to our desired Riemann-Dirichlet Ratio. The proof is now complete for Lemma 5.1. $\square$

Denote the left hand side ratio as Ratio R1 (of a 'cyclical' nature due to the presence of sine function) and the right hand side ratio as Ratio R2 (of a 'non-cyclical' nature). Then Riemann-Dirichlet Ratio can be deemed to be represented by a more complicated ‘dynamic’ version of [infinite length] Hybrid integer sequence in that besides consisting of a particular ‘Class function’ expressed in Ratio R1’s numerator and denominator, this first Ratio R1 is again given as an equality to another seemingly different looking Ratio R2 whose numerator and denominator are expressed by another different ‘Class function’. One may intuitively think of a non-Hybrid integer sequence to metaphorically arise from a Hybrid integer.
sequence in the limit these two different 'Class functions’ in Hybrid integer sequence becomes one same 'Class function’ in the new non-Hybrid integer sequence. Note the absence and presence of σ variable in Ratio R1 and R2 respectively.

The Riemann-Dirichlet Ratio calculations, valid for all continuous real number values of t, would theoretically result in infinitely many non-Hybrid integer sequences [here arbitrarily] for the $0 < \sigma < 1$ critical strip region of interest with $n = 1, 2, 3, \ldots, \infty$ being discrete integer number values, or $n$ being continuous real numbers from 1 to $\infty$ with Riemann integral applied in the interval from 1 to $\infty$. This infinitely many integer sequences can geometrically be interpreted to representatively cover the entire plane of the critical strip bounded by $\sigma$ values of 0 and 1, thus (at least) allowing our proposed proof to be of a 'complete’ nature.

**Proposition 5.2.** The Sigma-Power Laws can be rigorously derived from Riemann-Dirichlet Ratio.

**Proof.** In Calculus, integration is defined as the reverse process of differentiation. Integration is geometrically viewed as the area enclosed by the curve of the function and the axis. Using the definite integral $I$ between the points $a$ and $b$ (i.e. in the interval $[a, b]$ where $a < b$) and computing the value when $\Delta x \rightarrow 0$, we get $I = \lim_{\Delta x \rightarrow 0} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_{a}^{b} f(x)dx$ - this is the Riemann integral of the function $f(x)$ in the interval $[a, b]$. We apply Riemann integral to the four continuous functions of Ratio R1 and Ratio R2 in Eq. (7) thus depicting the Riemann-Dirichlet Ratio in the integral forms - see the subsequent Eq. (12) below.

Thereafter, step-by-step we derive the closely related Dirichlet σ-Power Law [expressed in real numbers] and the Riemann σ-Power Law [expressed in real and complex numbers]. Due to the resemblance to various power-law functions in that the $\sigma$ variable from $s (= \sigma + it)$ being the exponent of a power function $n^\sigma$, the log scale use, and the harmonic $\zeta(s)$ series connection in Zipf’s law; we explain here why we have elected to endow our newly derived formula with the name Sigma-Power Law. Its Dirichlet and Riemann versions are directly related to each other via Dirichlet $\eta(s)$ being the equivalence of Riemann $\zeta(s)$ but without the $\left(\frac{1}{1-2\cdot\sigma}\right)$ proportionality factor. We stress that it is the main underlying mathematically-consistent properties of symmetry and constraints arising from this power law that also allowed our most direct, basic and elementary proof for the Riemann hypothesis to mature. An important characteristic to note of $\sigma$-Power Law is that its exact formula expression in the usual mathematical language [$y = f(x_1, x_2)$ format description for a 2-variable function] consists of $y = \{2n\}$ or $\{2n - 1\} = f(t, \sigma)$ with $n = 1, 2, 3, \ldots, \infty$ or $n = 1$ to $\infty$ with Riemann integral application; $-\infty < t < +\infty; \text{ and } \sigma$ being of real number values $0 < \sigma < 1$ corresponding to the [arbitrarily defined] critical strip of interest in this particular case scenario.

For the, initially, $\{2n\}$ parameter integration of R1, $\int_{1}^{\infty} \sin(t \cdot \log(2n) + \frac{3}{4}\pi) \cdot dn$

Use integration by $u$-substitution technique to obtain $u = t \cdot \log(2n) + \frac{3}{4}\pi$,

$n = \frac{1}{2}e^{\frac{1}{2}(u - \frac{3}{2}\pi)}, \frac{du}{dt} = \frac{2t}{2n} = \frac{1}{n}, du = t \cdot \frac{du}{dt}, dn = 2n \cdot \frac{du}{dt} = n \cdot \frac{du}{t}$

$\int_{1}^{\infty} \sin(u) \cdot \frac{du}{n} = \int_{1}^{\infty} \sin(u) \cdot \frac{1}{2} e^{\frac{1}{2}(u - \frac{3}{2}\pi)} \cdot du = \frac{1}{2} \cdot \frac{4}{2\pi} \int_{1}^{\infty} \sin(u) e^{\frac{4}{\pi}u} \cdot du$

Use the Products of functions proportional to their second derivatives, namely the indefinite integral

$\int \sin(a \cdot u) e^{b \cdot u} \cdot du = \frac{e^{nu}}{a^2 + b^2} (b \cdot \sin(a \cdot u) - a \cdot \cos(a \cdot u)) + C.$

Then $a = 1, b = \frac{3}{4}$, and temporarily ignore the $\frac{1}{2\pi}e^{\frac{3}{4}\pi}$ term, we have
\[ \int_1^{\infty} \sin(u) e^{\frac{\pi}{u}} \, du \]
\[ = \left[ \left( e^{\frac{\pi}{u}} / (1 + \frac{1}{t^2}) \right) / (t^2 + 1) \right]^{\infty}_1 \sin(u) - \cos(u) + C \]
\[ = \left[ \left( e^{\frac{\pi}{u}} / (t^2 + 1) \right) / (t^2 + 1) \right]^{\infty}_1 \sin(u) - \cos(u) + C \]

Now apply the non-linear combination of sine and cosine functions identity, namely
\[ a \sin(u) + b \cos(u) = c \sin(u + \phi) \text{ where } c = \sqrt{a^2 + b^2} \text{ and } \phi = \arctan(b/a). \]

Here \( a = \frac{1}{t}, b = -1, c = \sqrt{\left( \frac{1}{t} \right)^2 + 1} = \sqrt{\frac{t^2 + 1}{t}}. \) Then we have
\[ \int_1^{\infty} \sin(u) e^{\frac{\pi}{u}} \, du \]
\[ = \left[ \left( e^{\frac{\pi}{u}} / (t^2 + 1) \right) / (t^2 + 1) \right]^{\infty}_1 \sin(u + \arctan(t)) + C \]

But there was a \( \frac{1}{2\pi} e^{\frac{\pi}{2}} \) term in front of this integral as can be seen above. Then after substituting this term and simplifying, the integral
\[ \int_1^{\infty} \sin(u) e^{\frac{\pi}{u}} \, du \]
\[ = \left[ \left( e^{\frac{\pi}{u}} / (t^2 + 1) \right) / (t^2 + 1) \right]^{\infty}_1 \sin(u - \arctan(t)) + C \]

But \( u = t \log(2n) + \frac{3}{4} \pi. \) Reverting back to the \( n \) variable, and incorporating \( \sqrt{2} \) originating from the beginning during Eq. (6) derivation, the equation for the \( \{2n\} \) parameter finally becomes
\[ \sqrt{2} \int_1^{\infty} \sin(t \log(2n) + \frac{3}{4} \pi) \, dn \]
\[ = \left[ \sqrt{2} \left( \{2n\} e^{\frac{\pi}{2}} \right) / (2\sqrt{(t^2 + 1)} e^{\frac{\pi}{2}} \sin(t \log(2n) + \frac{3}{4} \pi - \arctan(t)) + C) \right]^{\infty}_1 \]

(8)

In a similar manner integration for the \( \{2n-1\} \) parameter, this equation becomes
\[ \sqrt{2} \left( \{2n-1\} e^{\frac{\pi}{2}} \right) / (2\sqrt{(t^2 + 1)} e^{\frac{\pi}{2}} \sin(t \log(2n-1) + \frac{3}{4} \pi - \arctan(t)) + C) \right]^{\infty}_1 \]

(9)

In R2 using \( \{2n\} \) parameter,
\[ \int_1^{\infty} (2n)^\sigma \, dn \]
\[ = \left[ 1 / (2(\sigma + 1)) \right] \left( (2n)^{\sigma + 1} + C \right]^{\infty}_1 \]
\[ = \left[ \frac{1}{3} \{2n\} (2n)^2 + C) \right]^{\infty}_1 \text{ when } \sigma = \frac{1}{2} \]

(10)
For the equivalent R2 based on \(\{2n\}\) parameter,

\[
\int_1^\infty (2n-1)^{\sigma} \, dn = \left[ \frac{1}{2} (2n-1)(2n-1)^{\frac{1}{2}} \right]_{1}^{\infty}\]

when \(\sigma = \frac{1}{2}\) \hspace{1cm} (11)

The Ratio R1 and Ratio R2 of Riemann-Dirichlet Ratio (for \(\sigma = \frac{1}{2}\)) is defined by the integral

\[
\frac{[(2n),(e^{\frac{3}{4} \pi} / 2 \sqrt{t^2 + 1}, e^{\frac{3}{4} \pi} \cdot \sin(t \log(2n) + \frac{3}{4} \pi} - \arctan(t))]_{1}^{\infty}}{[(2n-1),(e^{\frac{3}{4} \pi} / 2 \sqrt{(t^2 + 1)}, e^{\frac{3}{4} \pi} \cdot \sin(t \log(2n-1) + \frac{3}{4} \pi} - \arctan(t))]_{1}^{\infty}} = \left[ \frac{1}{3} (2n)^{\frac{1}{2}} \right]_{1}^{\infty}
\]

Canceling out the common parameter \(\{2n\}\) and \(\{2n-1\}\) terms,

\[
\frac{[(e^{\frac{3}{4} \pi} / 2 \sqrt{(t^2 + 1)}, e^{\frac{3}{4} \pi} \cdot \sin(t \log(2n) + \frac{3}{4} \pi} - \arctan(t))]_{1}^{\infty}}{[(e^{\frac{3}{4} \pi} / 2 \sqrt{(t^2 + 1)}, e^{\frac{3}{4} \pi} \cdot \sin(t \log(2n-1) + \frac{3}{4} \pi} - \arctan(t))]_{1}^{\infty}} \left\downarrow \text{this is R1} \right. \hspace{2cm} \left\uparrow \text{this is R2} \right.
\]

\[
= \left[ \frac{1}{3} (2n)^{\frac{1}{2}} \right]_{1}^{\infty}
\]

The \(\gamma\) proportionality factor term in Riemann \(\zeta\) function, viz. \(\frac{1}{\zeta(1-2s)}\), can also be expressed with the aid of Euler formula as follows (with the formula for \(\sigma = \frac{1}{2}\) substitution depicted last).

\[
\frac{1}{(1 - 2^{1-s})} = \frac{(2^{\sigma}, 2^{\sigma})}{(2^{\sigma}, 2^{\sigma} - 2)} = \frac{(2^{\sigma}, e^{t \log(2\sigma)})}{(2^{\sigma}, e^{t \log(2\sigma)} - 2)}
\]

\[
= (2^{\frac{1}{2}}, \cos(t \log(2) + \sqrt{t \log(2)}))
\]

\[
= \left[ \frac{1}{3} (2n)^{\frac{1}{2}} \right]_{1}^{\infty}
\]

The Dirichlet and Riemann \(\sigma\)-Power Laws are given by the exact formulae in Eqs. (14) to (17) below with \(\psi\) being the same proportionality constant valid for both power laws. We can now dispense with the constant of integration C. Using Dimensional analysis (DA) approach we can easily conclude that the 'fundamental dimension' [Variable / Parameter / Number X to the power of Number Y] has to be represented by the particular 'unit of measure' [Variable / Parameter / Number X to the power of Number Y whereby Number Y needs to be of the specific value \(\frac{1}{2}\)] for DA homogeneity to occur. This de novo DA homogeneity equates to the location of the complete set of nontrivial zeros and is crucially a fundamental
property present in all laws of Physics. The 'unknown' $\sigma$ variable, now endowed with the value of $\frac{1}{2}$, is treated as Number Y.

Dirichlet $\sigma$-Power Law using the $\{2n\}$ parameter:

$$\left[2^{\frac{1}{2}} \cdot \frac{e^{\frac{1}{2} \cdot \frac{\pi}{2}}}{2(t^2 + 1)^{\frac{3}{2}}}, e^{\frac{\pi}{2}} \cdot \sin(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{1}{3}(2n)^{\frac{1}{2}}\right]_1^\infty = \left[\psi \cdot \frac{1}{3}(2n)^{\frac{1}{2}}\right]_1^\infty
$$

With the common parameter $\{2n\}$ canceling out on both sides, the equation reduces to

$$\left[2^{\frac{1}{2}} \cdot \frac{e^{\frac{1}{2} \cdot \frac{\pi}{2}}}{2(t^2 + 1)^{\frac{3}{2}}}, e^{\frac{\pi}{2}} \cdot \sin(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{1}{3}(2n)^{\frac{1}{2}}\right]_1^\infty = 0 \quad (14)

Similarly for the $\{2n-1\}$ parameter, this equivalent equation is

$$\left[2^{\frac{1}{2}} \cdot \frac{e^{\frac{1}{2} \cdot \frac{\pi}{2}}}{2(t^2 + 1)^{\frac{3}{2}}}, e^{\frac{\pi}{2}} \cdot \sin(t \cdot \log(2n - 1) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{1}{3}(2n - 1)^{\frac{1}{2}}\right]_1^\infty = 0 \quad (15)

Finally, the Riemann $\sigma$-Power Law is given by the exact formulae using $\{2n\}$ and $\{2n-1\}$ parameters with the $\gamma = (2^{\frac{1}{2}} \cdot \cos(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{1}{3}(2n)^{\frac{1}{2}})$ substitution.

$$\left[2^{\frac{1}{2}} \cdot \frac{e^{\frac{1}{2} \cdot \frac{\pi}{2}}}{2(t^2 + 1)^{\frac{3}{2}}}, e^{\frac{\pi}{2}} \cdot \sin(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{2^{\frac{1}{2}} \cdot \cos(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{1}{3}(2n)^{\frac{1}{2}}}{2^{\frac{1}{2}} \cdot \cos(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \cdot \frac{1}{3}(2n)^{\frac{1}{2}}}\right]_1^\infty = 0 \quad (16)

$$

The proof is now complete for Proposition 5.3.

**Proposition 5.3.** Application of Dimensional analysis homogeneity to Sigma-Power Laws will always be associated with the one specific $\sigma = \frac{1}{2}$ value for Gram[x=0,y=0] points and this will enable the rigorous proof for Riemann hypothesis to mature.

**Proof.** We notice the $\gamma$ proportionality factor given by Eq. (13) above when depicted with the $2^{\frac{1}{2}}$ constant numerical value (derived using $\sigma = \frac{1}{2}$ as conjectured in the original Riemann hypothesis) further allowing, and enabling, de novo Dimensional analysis homogeneity compliance in Riemann $\sigma$-Power Law in Eqs. (16) and (17) above. There is only one type of $\frac{1}{2}$ exponent present in Riemann $\sigma$-Power Law indicating Dimensional analysis homogeneity. *This two mathematical statements*
essentially complete the proof for Proposition 5.3 with complimentary demonstration below for the Dimensional analysis non-homogeneity case scenario.

**Corollary 5.4.** Application of Dimensional analysis non-homogeneity to Sigma-Power Laws will never be associated with the one specific $\sigma = \frac{1}{2}$ value for $\text{Gram}[x=0,y=0]$ points and this will enable the rigorous proof for Riemann hypothesis to mature.

**Proof.** We illustrate the Dimensional analysis non-homogeneity property for a $\sigma = \frac{1}{4}$ arbitrarily chosen value [clear-cut case with \{2n\}-parameter] of Riemann $\sigma$-Power Law lying on a non-critical line (with total absence of nontrivial zeros) in the following formula derived using Eqs. (13) and (16). As Ratio R1 component of Riemann-Dirichlet Ratio is independent of $\sigma$ variable, unlike the Ratio R2 component of Riemann-Dirichlet Ratio and the $\gamma$ proportionality factor which are dependent on $\sigma$ variable, we now note the mixture of $\frac{1}{4}$ and $\frac{1}{2}$ exponents subtly, but nonetheless, present in this formula indicating Dimensional analysis non-homogeneity. Also the replacement of $\frac{1}{3}$ fraction with $\frac{2}{5}$ fraction [derived from substituting $\sigma = \frac{1}{4}$ into $\frac{1}{2(\sigma + 1)}$] has occurred. Mathematically, this Dimensional analysis non-homogeneity property for any real number value of $\sigma$, when $\sigma \neq \frac{1}{2}$ and $0 < \sigma < 1$, will always be present.

\[
\left[2^{\frac{1}{4}} \cdot \frac{e^{\frac{1}{4} \cdot \pi \cdot t \cdot \log(2n) + \frac{3}{4} \cdot \pi - \arctan(t)}}{2(t^2 + 1)^{\frac{1}{2}} \cdot e^{\frac{\pi}{2n}}} \cdot \sin(t \cdot \log(2n) + \frac{3}{4} - \arctan(t)) - \psi_{2^{\frac{4}{5}}}(\cos(t \cdot \log(2) + i \cdot \sin(t \cdot \log(2))) \cdot \frac{2^{\frac{2}{5}}(2n)^{\frac{1}{5}}} {5(2n)^{\frac{1}{5}}} \right]_{1}^{\infty} = 0
\]

The proof is now complete for Corollary 5.4.

6 {Mathematical-format-version} conjectures

We now explore the corresponding $\text{Gram}[x=0,y=0]$ (Riemann hypothesis), $\text{Gram}[y=0]$, $\text{Gram}[x=0]$, & Dirichlet-Gram-Riemann {mathematical-format-version} conjectures. The beautiful conjectures given in their geometrical-format-versions provide convincing but still insufficient evidence for their rigorous proofs. In this regard, mathematicians still demand that in reference to the 'grand' Dirichlet-Gram-Riemann {mathematical-format-version} conjecture fully constituted by the three subsets (i) Riemann hypothesis or $\text{Gram}[x=0,y=0]$, (ii) $\text{Gram}[y=0]$, and (iii) $\text{Gram}[x=0]$ {mathematical-format-version} conjectures; only when all are perfectly correct can they fulfill the absolute requirements for these rigorous proofs to be completely valid with [figuratively-speaking] 100% certainty. Fortunately, these mathematical-format-versions have conveniently been solved and proven not just beyond reasonable doubts, but beyond all doubts. The succinct summary on this point is appropriately expressed by the "Common Master Proof" outlined below for these three subsets using "Generic Gram conjecture" with their individualized "Generic Gram points". This Common Master Proof is centered on the four theorems (Virtual Container) below [with their proofs automatically obtained from the preceding Section 5 above and from Appendix 1].

**Theorem {Riemann} I.** The exact same {Generic Gram points}-Riemann-Dirichlet Ratio, directly derived from either the Riemann zeta or Dirichlet eta function, is an irrefutably accurate mathematical expression on the de novo criteria for the actual presence [but not the actual location] of the complete set of (identical) infinite Generic Gram points in both functions.
Proof. This 'overall' proof for Theorem (Riemann) I is now complete as it literally contain the successful incorporation of the rigorous proofs for Lemmas 5.1 and [Appendix] 1.1 which are associated with the complete set of the three types of Gram points [thus constituting the Generic Gram points].

Theorem (Riemann) II. Both the near-identical (by proportionality factor-related) {Generic Gram points}-Riemann Sigma-Power Law and {Generic Gram points}-Dirichlet Sigma-Power Law with their derivations based on either the numerator or denominator of {Generic Gram points}-Riemann-Dirichlet Ratio have Dimensional analysis homogeneity only when their common and unknown $\sigma$ variable has a value of $\frac{1}{2}$ as its solution.

Proof. This 'overall' proof for Theorem (Riemann) II is now complete as it literally contain the successful incorporation of the rigorous proofs from Propositions 5.2 & [Appendix] 1.2 on Sigma-Power Laws and Propositions 5.3 & [Appendix] 1.3 on Dimensional analysis homogeneity. These are applicable to the complete set of the three types of Gram points [which constitute Generic Gram points].

Theorem (Riemann) III. The $\sigma$ variable with value of $\frac{1}{2}$ derived using the {Generic Gram points}-Sigma-Power Laws [from Theorem (Riemann) II above] is the exact same $\sigma$ variable in Generic Gram conjecture which proposed $\sigma$ to also have the value of $\frac{1}{2}$ (representing the critical line with $\sigma = \frac{1}{2}$ in the critical strip with $0 < \sigma < 1$) for the location of all Generic Gram points of Riemann zeta function [and Dirichlet eta function by default], thus providing irrefutable evidence for this Generic Gram conjecture to be correct with further clarification from Theorem (Riemann) IV below.

Proof. This 'overall' proof for Theorem (Riemann) III is now complete as Theorem (Riemann) III simply reflect Theorem (Riemann) II with its {Generic Gram points}-Sigma-Power Laws having the exact same $\sigma$ variable as that referred to by the Generic Gram conjecture with each case independently referring to (i) the Generic Gram points being endowed with the same value of $\frac{1}{2}$ for their $\sigma$ variable and (ii) the critical line also simultaneously being endowed with the same value of $\frac{1}{2}$ for its $\sigma$ variable. In relation to Dimensional analysis homogeneity in Sigma-Power Laws, the condition "If $\sigma$ parameter is endowed with the $\frac{1}{2}$ value, then Dimensional analysis homogeneity will be satisfied" acts as the one (and only one) possible mathematical solution.

Theorem (Riemann) IV. Condition 1. Any other values of $\sigma$ apart from the $\frac{1}{2}$ value arising from $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$ in the critical strip does not contain any Generic Gram points ["the DA-wise mathematical impossibility argument" with resulting de novo DA non-homogeneity], together with Condition 2. The one and only one value of $\frac{1}{2}$ for $\sigma$ in the critical strip contains all the Generic Gram points ["the DA-wise one and only one mathematical possibility argument" with resulting de novo DA homogeneity] from Theorem (Riemann) III, fully support the rather mute, but nevertheless the whole, point of study in this paper that Generic Gram conjecture is proven to be true when these two (mutually inclusive) conditions are met.

Proof. This 'overall' proof for Theorem (Riemann) IV is now complete as Theorem (Riemann) IV simply reflect the rigorous proofs from Theorem (Riemann) III on Generic Gram points with the additional proofs from Corollaries 5.4 & [Appendix] 1.4 on non-Generic Gram points being tightly incorporated into this mathematical framework.
Dimensions are properties which can be measured. With global consensus, Systeme International d’Unites (SI Units) is the standard elements we use to scientifically quantify dimensions. In Dimensional analysis (DA), we are only concerned with the nature of the dimension i.e. its quality (and not its quantity). The following abbreviations are commonly used for examples of various dimensions (expressed in their SI base or SI derived units):

- angle = \( \theta \) in radian
- length = \( L \) in meter
- mass = \( M \) in kilogram
- time = \( T \) in second
- force = \( F \) in newton
- temperature = \( Q \) in kelvin

For instance, the common and traditional unit of measurement of angles is degree. Radian is considered the SI derived unit of measurement of angles equivalent to the angle subtended at the centre of a circle by an arc equal in length to the radius. One radian is equal to about 57.3 deg and \( \pi \) radian is exactly 360 deg. Thus the term ‘dimension’ is traditionally used to refer to the units of measurement associated with various terms of an equation. However, we arbitrarily utilized ‘dimension’ to also refer to other mathematical properties such as the power or exponent associated with various terms of an equation. In other words, it is nothing more than a convenient language tool when using the term ‘dimension’ to directly refer to ‘power’ or ‘exponent’ in the sense that we could legitimately coin parallel terms such as Power analysis (PA) or Exponent analysis (EA) homogeneity [and non-homogeneity]. So what is this mysterious DA homogeneity (and non-homogeneity)? Any equation describing a physical situation will only be true (false) if both sides of the equation have the same (different) dimensions; that is, it must possess DA homogeneity (non-homogeneity). Examples: 2 kg + 3 kg = 5 kg is a valid equation because it possess DA homogeneity. 2 kg + 3 meter = 5 ‘something undefined nonsense unit’ is meaningless and definitely not a valid equation because it possess DA non-homogeneity. It is a straight-forward exercise to arrive at this two verdicts.

The original equation \( y^\frac{1}{2} = x^\frac{1}{2} + 3 \) which is equivalent to \( y = x + 6x^\frac{1}{2} + 9 \) possess DA or PA or EA homogeneity, having the same power or exponent \( \frac{1}{2} \) [at least] in the original equation. But the original equation \( y^\frac{2}{3} = x^\frac{2}{3} + 3 \) which is equivalent to \( y = x^\frac{2}{3} + 6x^\frac{1}{3} + 9 \) possess DA or PA or EA non-homogeneity, having different powers or exponents \( \frac{1}{2}, \frac{1}{3} \) or \( \frac{2}{3} \) in both the original and equivalent equation. We make a brief comment here that determing the validity of the last two equations endowed with DA or PA or EA non-homogeneity is intuitively not such a straight-forward exercise in this setting. Alternatively stated, those last two equations looks like being physically invalid but may still be mathematically valid.

In this paper, it is to be explicitly elaborated here that a totally invalid comment such as "Dimensional analysis homogeneity can prove Riemann hypothesis" will contextually never be used by us to indicate a connotation that "Laws of Physics along with Scientific Principles even when all of them put together fully satisfying DA homogeneity per se can purportedly prove mathematical theorems". Rather, of utmost significance is this DA homogeneity (and non-homogeneity) being the secondary consequence [in a mathematical consistent albeit seemingly indirect manner] arising naturally out of our Virtual Container Research Method used...
to fully prove Theorems \{\text{Riemann}\} I - IV above. As shown in the above section and in Appendix 1, the process to ultimately prove Theorems \{\text{Riemann}\} I - IV involves important mathematical tools such as Euler formula application, Ratio Study, Riemann integral, Calculus (Integration and Differentiation), Dimensional analysis, and concepts from the Hybrid method of Integer Sequence Classification.

Overall, the 6 steps (‘mathematical foot-prints’) in specific sequence required to prove Theorems \{\text{Riemann}\} I - IV are: Step 1: Riemann zeta or Dirichlet eta function [for the critical strip $0 < \sigma < 1$] → Step 2: Riemann zeta or Dirichlet eta function [with Euler formula application] → Step 3: Riemann zeta or Dirichlet eta function [simplified and identical version specifically indicating the criteria for the presence of the complete set of Generic Gram points without knowing their location] → Step 4: Riemann-Dirichlet Ratio [in discrete summation format] → Step 5: Riemann-Dirichlet Ratio [in continuous integral format] → Step 6: Riemann Sigma-power law and Dirichlet Sigma-power law [both with Dimensional analysis homogeneity].

In the process of deriving our rigorous proof, the seemingly small but utterly essential mathematical step in recognizing and representing a 2-variable function with parameters \{2n\} or \{2n-1\} allows crucial moments where cancellation of the relevant “common” parameters in Riemann-Dirichlet Ratio and various Sigma-Power Laws can occur, further allowing the proper DA process to happen in the absolute correct way. These “common” parameters must be mathematically viewed as $(2n)^\frac{1}{2}$ or $(2n - 1)^\frac{1}{2}$, viz. raised to a power (exponent) of $1$ which will hamper proper DA if not serendipitously deleted. Once deleted, we now have the presence of parameters $(2n)^\frac{1}{4}$ or $(2n - 1)^\frac{1}{4}$, viz. raised to the [same and uniform] power (exponent) of $\frac{1}{2}$, which will then enable proper DA (homogeneity) to proceed.

The key end-product equations arising out of, and fully conforming with, Theorems \{\text{Riemann}\} I - IV above then act as ultimate and final evidences for the complete proof of Riemann hypothesis, Gram[y=0] conjecture, and Gram[x=0] conjecture. These closely related equations, for the \{2n\}-parameter case with $\psi$ being the proportionality constant, will subtly manifest the necessary DA homogeneity for the single $\sigma = \frac{1}{2}$ value involving the $\frac{1}{2}$ exponents [and DA non-homogeneity for all other infinite $\sigma$ values with the given example of $\sigma = \frac{1}{4}$ involving the mixture of $\frac{1}{4}$ and $\frac{1}{2}$ exponents]. They are, respectively, regurgitated below from Section 5 and Appendix 1 for convenience.

\begin{equation}
[2^{\frac{1}{4}} \frac{e^{\frac{1}{2} \frac{3}{4} \pi}}{2(t^2 + 1)^\frac{1}{4}} \cdot \sin(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \frac{2^\frac{1}{4} \cdot (cos(t \cdot \log(2) + t \cdot \sin(t \cdot \log(2))))}{2^\frac{1}{4} \cdot (cos(t \cdot \log(2) + t \cdot \sin(t \cdot \log(2) - 2)))} \cdot \frac{1}{3} (2n)^\frac{1}{4}]_i^\infty = 0
\end{equation}

(19)

\begin{equation}
[2^{\frac{1}{4}} \frac{e^{\frac{1}{2} \frac{3}{4} \pi}}{2(t^2 + 1)^\frac{1}{4}} \cdot \sin(t \cdot \log(2n) + \frac{3}{4} \pi - \arctan(t)) - \psi \frac{2^\frac{1}{4} \cdot (cos(t \cdot \log(2) + t \cdot \sin(t \cdot \log(2))))}{2^\frac{1}{4} \cdot (cos(t \cdot \log(2) + t \cdot \sin(t \cdot \log(2) - 2)))} \cdot \frac{2}{5} (2n)^\frac{1}{2}]_i^\infty = 0
\end{equation}

(20)
The near-identical "physical manifestations" depicted in Section 5, Figure 2a &

\[ \frac{e^{\frac{t}{2}}}{2(t^2 + 1)^{\frac{1}{2}}} \cdot \sin(t \log(2n) - \arctan(t)) - \psi \cdot \frac{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)} \cdot \frac{1}{3} (2n)^{\frac{1}{2}} \bigg|_{n=1}^{\infty} = 0 \]

(21)

\[ \frac{e^{\frac{t}{2}}}{2(t^2 + 1)^{\frac{1}{2}}} \cdot \sin(t \log(2n) - \arctan(t)) - \psi \cdot \frac{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)} \cdot \frac{2}{3} (2n)^{\frac{1}{2}} \bigg|_{n=1}^{\infty} = 0 \]

(22)

The near-identical "physical manifestations" depicted in Section 5, Figure 2a &

\[ \frac{e^{\frac{t}{2}}}{2(t^2 + 1)^{\frac{1}{2}}} \cdot \sin(t \log(2n) + \arctan(t)) - \psi \cdot \frac{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)} \cdot \frac{1}{3} (2n)^{\frac{1}{2}} \bigg|_{n=1}^{\infty} = 0 \]

(23)

\[ \frac{e^{\frac{t}{2}}}{2(t^2 + 1)^{\frac{1}{2}}} \cdot \sin(t \log(2n) + \arctan(t)) - \psi \cdot \frac{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^\frac{1}{2} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)} \cdot \frac{2}{3} (2n)^{\frac{1}{2}} \bigg|_{n=1}^{\infty} = 0 \]

(24)

**Fig. 6** Combined graphs giving us a snapshot on the "physical manifestations" of various Ratio and Law formulae derived out of Riemann hypothesis, Gram[y=0] and Gram[x=0] conjectures.
Figure 4a & 4b, p. 12, Figure 5a & 5b, p. 13 from [9] on various Ratio and Law formulae which are obtained from our three conjectures demonstrating properties such as Mathematical Symmetry, Chaos (Chaos theory with Chaotic non-linear deterministic dynamics), and Fractals (Fractal geometry with Self-similarity) are portrayed in Figure 6. These miniaturized version graphs are grouped together from Left to Right as corresponding to Gram[x=0,y=0] (Riemann hypothesis), Gram[y=0] and Gram[x=0] conjectures; and depicted using $\sigma = \frac{1}{5}, \frac{1}{2}, \& \frac{5}{4}$ values.

7 Information-Complexity conservation

An Equation or Algorithm is simply a Black Box generating the necessary Output (with qualitative structural 'Complexity') when supplied with the given Input (with quantitative data 'Information'). In Set theory, the infinite sets are sets that are not finite sets, and they are further subdivided into two groups: "discrete" countable infinite sets and "continuous" uncountable infinite sets; with the later being conceptually larger than the former in magnitude [despite both being treated as objects endowed with the infinity property]. A set is countable if we can count its elements. If the set is finite, we can easily count its elements. If the set is infinite, being countable means that we are able to put the elements of the set in order (just like natural numbers are in order). The infinite sets of rational number and irrational number are, respectively, countable and uncountable. These two sets together give rise to the infinite set of real number which are uncountable.

The two-dimensional complex plane is typically specified by a one-dimensional real number line for horizontal or x-axis, and a one-dimensional imaginary number (i = $\sqrt{-1}$) line for vertical or y-axis. Complex numbers, each defined with a (pure) real number component and a (pure) imaginary number component, lie on this plane. Real numbers could alternatively be perceived as complex numbers with their imaginary number component being zero. In regards to the uncountable infinite set of real number line (with the understanding that every positive real number has its mirror image negative real number counterpart), this line is further seen to consist of both countable and uncountable infinite sets. The set and subsets of real numbers with some of their properties are comparatively illustrated below using the following legends & abbreviations: =, $<$, $>$, $>>$, $\subset$, $\in$, CFS, CIS & UIS denoting (respectively) 'equal to', 'less than', 'greater than', 'much greater than', 'subset of', 'belongs to', 'countable finite set', 'countable infinite set' & 'uncountable infinite set'.

We can use the cardinality relation to describe the size of a set by comparing it with standard sets. Any set X with cardinality less than that for the set of natural numbers (set N), or $|X| < |N|$, is said to be a CFS. Any set X that has the same cardinality as set N, or $|X| = |N|$, is said to be a CIS endowed with "cardinality of the natural numbers". Any set X with cardinality greater than that for set N, or $|X| > |N|$ (for example, when set X = real numbers), is said to be a UIS endowed with "cardinality of the continuum". From smallest to biggest sets, natural numbers (CIS) $\subset$ integer numbers (CIS) $\subset$ rational numbers (CIS) $\subset$ algebraic numbers (CIS) $\subset$ real numbers (UIS). By definition, for relatively "smaller" set X (= even numbers or odd numbers or prime numbers or composite numbers) $\in$ set of natural numbers, we [counterintuitively] note that $|X|$ still = "cardinality of the natural numbers". The set of natural numbers has cardinality.
(of the natural numbers) that is strictly less than the set of real numbers having cardinality (of the continuum) as it can be shown that there does not exist a bijective function from natural numbers to real numbers using Cantor’s diagonal argument or Cantor’s first uncountability proof.

Irrational numbers (UIS) = (I) Transcendental numbers (UIS) + (II) Algebraic numbers (CIS) with (I) >> (II). Almost all real and complex numbers are transcendental. All irrational numbers can imperatively be depicted as numbers with non-repeating decimal point digits of infinite length, with those decimal point digits being Completely Predictable. An algebraic number is any real or complex number that is a root of a non-zero polynomial in one variable with rational coefficients (or equivalently – by clearing denominators – with integer coefficients). All integers and rational numbers are algebraic, as are all roots of integers. Thus a transcendental number is a real or complex number that is not algebraic [associated with the criterion as just stated]; and it "transcends" the power of algebra to display it in its totality.

Real numbers (UIS) = (I) Irrational numbers (UIS) + (II) Rational numbers (CIS) with (I) > (II). If real numbers are to be the union of two countable sets, they would have to be [incorrectly] countable; so the irrational numbers must be [correctly] uncountable by following this 'proof by contradiction' argument.

Rational numbers (CIS) = (I) Fractions (CIS) + (II) Integer numbers (CIS) with (I) > (II). A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q. Since q may be equal to 1, every integer is a rational number. Fractions can imperatively be depicted as numbers with non-repeating decimal point digits of finite length type or repeating decimal point digits of infinite length type, with both sets of decimal point digits being Completely Predictable. Thus integers can specially be depicted either as the integer number itself followed by a (redundant) non-repeating decimal point digit ‘0’ or as fractions with numerator given by the integer number itself and denominator given by the (redundant) number ‘1’.

Integers (CIS): -∞,...,-3,-2,-1,0,1,2,3,...,+∞. Whole numbers (CIS): 0, 1, 2, 3,..., ∞. Natural numbers (CIS): 1, 2, 3, 4,..., ∞. Let x be the set consisting of either one or two number(s) such that x ∈ natural numbers, whole numbers or integers. Whenever relevant in this paper, we consider x for the relevant upper & / or lower boundary(ies) of interest in the study on a chosen set of numbers (such as even, odd, prime, and composite numbers).

Lemma 7.1. Natural numbers (CIS): 1, 2, 3, 4,..., ∞. The natural counting function Natural-π(x), defined as the number of natural numbers ≤ x, is Completely Predictable to be simply = x.

Proof. The formula for generating natural numbers with 100% certainty is \( N_i = i \) whereby \( N_i \) is the \( i^{th} \) natural number and \( i = 1, 2, 3,..., \infty \). For a given \( N_i \) number, its \( i^{th} \) position is simply i. Natural gap (\( G_{N_i} \)) = \( N_{i+1} - N_i \), with \( G_{N_i} \) always = 1. Thus there are x natural numbers ≤ x. The (coined) natural counting function, denoted here by Natural-π(x), is defined as the number of natural numbers ≤ x – this is Completely Predictable to be simply = x. The proof is now complete for Lemma 7.1.\]

Lemma 7.2. The even counting function Even-π(x), defined as the number of even numbers ≤ x, is Completely Predictable to be simply = \( \text{floor}(x/2) \).

Proof. Even numbers (CIS): 2, 4, 6, 8,..., ∞. The formula for generating even numbers with 100% certainty is \( E_i = i \times 2 \) whereby \( E_i \) is the \( i^{th} \) even number and
Lemma 7.3. The odd counting function \(\text{Odd-}\pi(x)\), defined as the number of odd numbers \(\leq x\), is Completely Predictable to be simply \(\lceil x/2 \rceil\).

Proof. Odd numbers (CIS): 1, 3, 5, 7, ..., \(\infty\). The formula for generating odd numbers with 100% certainty is \(O_i = (i \times 2) - 1\) whereby \(O_i\) is the \(i^{th}\) odd number and \(i = 1, 2, 3, \ldots, \infty\) abiding to the mathematical label "All natural numbers always ending with a digit 1, 3, 5, 7, or 9". For a given \(O_i\) number, its \(i^{th}\) position is calculated as \(i = (O_i + 1)/2\). Odd gap \((G_{O_i}) = O_{i+1} - O_i\), with \(G_{O_i}\) always \(= 2\). Thus there are \(\lceil x/2 \rceil\) odd numbers \(\leq x\). The (coined) odd counting function, denoted here by Odd-\(\pi(x)\), is defined as the number of odd numbers \(\leq x\) – this is Completely Predictable to be simply \(\lceil x/2 \rceil\). The proof is now complete for Lemma 7.3 \(\sqsubset \sqcup\).

Lemma 7.4. The prime counting function \(\text{Prime-}\pi(x)\), defined as the number of prime numbers \(\leq x\), is Incompletely Predictable and always need to be calculated using the Sieve of Eratosthenes algorithm.

Proof. Prime numbers (CIS): 2, 3, 5, 7, 11, 13, 17, ..., \(\infty\). The algorithm for generating all prime numbers \(P_i\) whereby \(P_1 = 2\), \(P_2 = 3\), \(P_3 = 5\), \(P_4 = 7\), ..., \(\infty\) with 100% certainty is based on the Sieve of Eratosthenes abiding to the mathematical label "All natural numbers apart from 1 that are evenly divisible by itself and by 1". Suffice to state here that although we can check the primality of a given odd number [check whether a given odd number is a prime number or not] by trial division, we can never determine its position without knowing the positions of preceding prime numbers. All prime numbers must be odd numbers and the only even prime number is 2. Prime gap \((G_{P_i}) = P_{i+1} - P_i\), with \(G_{P_1}\) constituted by all even numbers except the \(1^{st}\) \(G_{P_1}\) = \(3 - 2\) = 1. The prime counting function, denoted here by Prime-\(\pi(x)\) [which is traditionally denoted simply by \(\pi(x)\)], is defined as the number of prime numbers \(\leq x\) – this is Incompletely Predictable and always need to be calculated via the mentioned algorithm. We notice that by the very definition of prime gap above, every prime number [represented here with the aid of ‘\(n\)’ notation instead the usual ‘\(i\)’ notation] can be written as \(P_{n+1} = 2 + \sum_{i=1}^{n} G_{P_i}\), with ‘2’ denoting \(P_1\). Here \(i \& n = 1, 2, 3, 4, 5, \ldots, \infty\). The proof is now complete for Lemma 7.4 \(\sqsubset \sqcup\).

Lemma 7.5. The composite counting function Composite-\(\pi(x)\), defined as the number of composite numbers \(\leq x\), is Incompletely Predictable and always need to be calculated indirectly as the set of natural numbers minus the set of prime numbers [obtained using the Sieve of Eratosthenes algorithm].

Proof. Composite numbers (CIS): 1, 4, 6, 8, 9, 10, 12, ..., \(\infty\). Composite numbers have the mathematical label "All natural numbers other than that are evenly divisible by itself and by 1". The algorithm for generating all composite numbers \(C_i\) whereby \(C_1 = 1\), \(C_2 = 4\), \(C_3 = 6\), \(C_4 = 8\), ..., \(\infty\) with 100% certainty is also based on the Sieve of Eratosthenes albeit in an indirect manner by simply selecting [the excluded] non-prime natural numbers to be composite numbers. We
define the (coined) term Composite gap $G_{Ci}$ as $C_{i+1} - C_i$ with $G_{C1}$ constituted by 1 & 2 except the 1$\text{st}$ $G_{C1} = 4 - 1 = 3$. The (coined) composite counting function, denoted by Composite-$\pi(x)$, is defined as the number of composite numbers $\leq x$ – this is Incompletely Predictable and always need to be [indirectly] calculated via the mentioned algorithm. Applying similar ideas from prime numbers, we notice that by the very definition of composite gap above, every composite number [represented here with the aid of ‘n’ notation instead the usual ‘i’ notation] can be written as $C_{n+1} = 1 + \sum_{i=1}^{n} G_{Ci}$ with ‘1’ denoting $C_1$. Here $i \& n = 1, 2, 3, 4, 5, \ldots, \infty$. We crucially mention at this point that, in stark contrast to the equation “containing” but not identifying all prime numbers [outlined in the proof for Lemma 3.4 above] with prime gaps constituted by all even numbers [thus dealing with ‘unfriendly’ CIS property] except the very 1$\text{st}$ $GP_1 = 3 - 2 = 1$; the equivalent equation “containing” but not identifying all composite numbers deals with ‘friendly’ CFS property for composite gaps which are constituted by 1 & 2 except the very 1$\text{st}$ $GC_1 = 4 - 1 = 3$. Also we reinforce from the contents associated with Remark 1.1 above that we could conceptually and usefully visualize both the varying Incompletely Predictable prime gaps and composite gaps respectively as the varying Incompletely Predictable prime "prime gradients" and "composite gradients". The proof is now complete for Lemma 7.5.

The following is a useful list of mathematical relationships amongst the various groups of rational numbers: [Positive] Integers (CIS) = Whole numbers (CIS) = (I) Number ‘0’ (CFS) + (II) Natural numbers (CIS) with (I) < (II). Natural numbers (CIS) = (I) Even numbers (CIS) + (II) Odd numbers (CIS) with (I) = (II). Natural numbers (CIS) = (I) Prime numbers (CIS) + (II) Composite numbers (CIS) with (I) < (II). Composite numbers (CIS) = (I) Even numbers (CIS) + (II) [Odd numbers (CIS) - Prime numbers (CIS)] with (I) > (II) and Odd numbers > Prime numbers. Prime numbers (CIS) = (I) Natural numbers (CIS) - (II) Composite numbers (CIS) with (I) > (II).

Prime numbers < composite numbers and they are (A) mutually exclusive to each other. In fact, (B) composite numbers are the exact complementary counterparts of prime numbers simply because the Incompletely Predictable composite numbers = Completely Predictable natural numbers - Incompletely Predictable prime numbers. Relationships (A) and (B) allow us to forge a useful mental picture of “monotonously, slowly and eternally increasing prime and composite numbers which are inseparable” with composite numbers doing so at a relatively faster rate than prime numbers. Better visualization of this picture could desirably be achieved by graphing various derived formulations of relevant counting functions for both elements.

The set of natural number can be visualized to consist of two 'mutually exclusive, complementary and inseparable' subset groupings of either (i) Completely Predictable even and odd numbers, or (ii) Incompletely Predictable prime and composite numbers. Denote ‘A’ to represent natural, even, odd, prime, and composite numbers. We define the relevant counting function $A-\pi(x)$ as the number of $A \leq x$ with $x$ belonging to the set of natural number. As a prelude to outlining the all-important Information-Complexity conservation concept, we can easily define and compute below in a progressive manner the entity 'Grand-Total Gaps for A at x' (Grand-Total $\Sigma A_n$-Gaps) and their associated properties.
Proposition 7.6. For any given \( x \) value, the designated Information or Input is always validly represented by \( \Sigma_{\text{Natural}} x \)-Gaps = \( x - 1 \) for the (one) set of Completely Predictable natural numbers.

Proof. INPUT: Set of natural numbers (for \( x = 10 \)): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Natural-\( \pi(x) \) = 10. There are \( x - 1 = 9 \) Natural-Gaps each of ‘1’ magnitude: 1, 1, 1, 1, 1, 1, 1, 1, 1. \( \Sigma_{\text{Natural}} x \)-Gaps = \( 9 \times 1 = 9 \). This equates to "\( x - 1 \)" which we can regard as INFORMATION for Completely Predictable numbers. The proof is now complete for Proposition 7.6 \( \square \).

Proposition 7.7. For any given \( x \) value, the designated Complexity or Output is always validly represented by \( \Sigma_{\text{EvenOdd}} x \)-Gaps = \( 2x - 4 \) for the (two) sets of Completely Predictable even and odd numbers.

Proof. OUTPUT: Set of even and odd numbers (for \( x = 10 \)): 2, 4, 6, 8, 10 and 1, 3, 5, 7, 9. Even-\( \pi(x) \) = 5 and Odd-\( \pi(x) \) = 5. There are predictably \( \lceil \frac{x}{2} \rceil - 1 \) = 4 Even-Gaps each of ‘2’ magnitude: 2, 2, 2, 2. \( \Sigma_{\text{Even}} x \)-Gaps = \( 4 \times 2 = 8 \), and \( \lceil \frac{x}{2} \rceil - 1 \) = 4 Odd-Gaps each of ‘2’ magnitude: 2, 2, 2, 2. \( \Sigma_{\text{Odd}} x \)-Gaps = \( 4 \times 2 = 8 \). Grand-Total \( \Sigma_{\text{EvenOdd}} x \)-Gaps = \( 8 + 8 = 16 \). This equates to "\( 2x - 4 \)" which we can regard as COMPLEXITY for Completely Predictable numbers. The proof is now complete for Proposition 7.7 \( \square \).

Proposition 7.8. For any given \( x \) value, the designated Complexity or Output is always validly represented by \( \Sigma_{\text{PrimeComposite}} x \)-Gaps = \( 2x - 4 \) for the (two) sets of the Incompletely Predictable prime and composite numbers.

Proof. OUTPUT: Set of prime and composite numbers (for \( x = 12 \)): 2, 3, 5, 7, 11 and 1, 4, 6, 8, 9, 10, 12. Prime-\( \pi(x) \) = 5 and Composite-\( \pi(x) \) = 7. There are four Prime-Gaps of 1, 2, 2, 4 magnitude and six Composite-Gaps of 3, 2, 2, 1, 1, 2 magnitude. \( \Sigma_{\text{Prime}} x \)-Gaps = \( 1 + 2 + 2 + 4 = 9 \). \( \Sigma_{\text{Composite}} x \)-Gaps = \( 3 + 2 + 1 + 1 + 2 = 11 \). Grand-Total \( \Sigma_{\text{PrimeComposite}} x \)-Gaps = \( 9 + 11 = 20 \). This equates to "\( 2x - 4 \)" which we can regard as COMPLEXITY for Incompletely Predictable numbers. The proof is now complete for Proposition 7.8 \( \square \).

Incredibly, this (defacto) baseline "\( 2x - 4 \)" Grand-Total Gaps for Incompletely Predictable numbers output is identical to that for Completely Predictable numbers output. This common Grand-Total Gaps ingredient present in two outputs representing two vastly different groups of number is part of what we regard as fulfilling Information-Complexity conservation.

Let both \( x \) & \( N \) \( \in \mathbb{N} \), \( 1, 2, 3, ... \), \( \infty \). We utilize the word ‘Dimension’ here to contextually denote the relevant Dimension \( 2x - N \) whereby (i) the allocated [infinite] \( N \) integer values will result in Dimensions of the types \( 2x - 4 \), \( 2x - 5 \), \( 2x - 6 \), ..., \( 2x - \infty \) for the Prime-Composite mathematical landscape below and (ii) the allocated [finite] \( N \) integer values for the Even-Odd mathematical landscape in Appendix 2 below will result in Dimensions of the type \( 2x - 4 \). For both Prime-Composite and Even-Odd groupings, we have not included the very first (one-off) Dimension \( 2x - 2 \). [The term "mathematical landscape" is self-explanatorily employed in this paper to denote tabulated and graphed data showing specific mathematical patterns and features.]

Using the relevant data, we have now painstakingly tabulate (in Table 6) and graphically map (in Figure 7) the all-important [Incompletely Predictable] Prime-Composite mathematical landscape for a relatively larger \( x = 64 \) as demonstrated below (and ditto for the [Completely Predictable] Even-Odd mathematical landscape as demonstrated in Appendix 2 at the end of this paper). Legend: \( C = \) composite, \( P = \) prime, \( Y = \) Dimension \( 2x - 4 \) (for visual clarity). Of utmost im-
portance, we note that this Prime-Composite mathematical landscape made up of the relevant Dimensions will intrinsically incorporate prime and composite numbers in an integrated manner; and that there will be infinite times whereby relevant Dimensions will deviate away from the 'baseline' Dimension $2x - 4$ simply because prime and composite numbers are infinite in magnitude. For comparison, we have repeated this whole exercise for the [Completely Predictable] Even-Odd mathematical landscape in Appendix 2 and note the complete lack of deviation away from the 'baseline' Dimension $2x - 4$ apart from the one-off point of deviation as manifested by the initial Dimension $2x - 2$.

In Figure 7, Dimensions $2x - 4$, $2x - 5$, $2x - 6$, ..., $2x - \infty$ are symbolically represented by -4, -5, -6, ..., $\infty$ with $2x - 4$ displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this (Grand-Total Gaps) 'baseline' Dimension on a perpetual basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by ever larger negative integers will correspond to prime numbers associated with ever larger prime gaps and this phenomenon will generally happen at ever larger $x$ values. In other words, at ever larger $x$ values, Prime-$\pi(x)$ will overall become larger but with a decelerating trend whereas Composite-$\pi(x)$ will overall become larger but with an accelerating trend. This highlights the inevitable mathematical event of ever larger prime gaps occurring at ever larger $x$ values. We note that there is a complete presence of Chaos & Fractals phenomena being manifested in our graph.

The definitive derivation of the data in Table 6 is given next and this is clearly illustrated by two examples given for position $x = 31$ & 32. For $i \& x \in 1, 2, 3, ..., \infty$; $\Sigma PC_2$-Gap = $\Sigma PC_{x-1}$-Gap + Gap value at $P_{x-1}$ or Gap value at $C_{x-1}$ whereby (i) $P_i$ or $C_i$ at position $x$ is determined by whether the relevant $x$ value belongs to a prime (P) or composite (C) number, and (ii) both $\Sigma PC_1$-Gap and $\Sigma PC_2$-Gap = 0. Example for position $x = 31$: 31 is a prime number (P11). Our desired Gap value at $P_{10} = 2$. Thus $\Sigma PC_{31}$-Gap (58) = $\Sigma PC_{30}$-Gap (56) + Gap value at $P_{10}$ (2). Example for position $x = 32$: 32 is a composite number (C21). Our desired Gap value at $C_{20} = 2$. Thus $\Sigma PC_{32}$-Gap (60) = $\Sigma PC_{31}$-Gap (58) + Gap value at $C_{20}$ (2).
Table 6 Prime-Composite mathematical (tabulated) landscape using data obtained for \( x = 64 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P_i ) or ( C_i ), Gaps</th>
<th>( \sum \text{PC}_i )-Gaps</th>
<th>Dimension</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>( C_1, 3 )</td>
<td>0</td>
<td>2( x )-2</td>
</tr>
<tr>
<td>2</td>
<td>( P_1, 1 )</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>3</td>
<td>( P_2, 2 )</td>
<td>1</td>
<td>2( x )-5</td>
</tr>
<tr>
<td>4</td>
<td>( C_2, 2 )</td>
<td>4</td>
<td>Y</td>
</tr>
<tr>
<td>5</td>
<td>( P_5, 2 )</td>
<td>6</td>
<td>Y</td>
</tr>
<tr>
<td>6</td>
<td>( C_3, 2 )</td>
<td>8</td>
<td>Y</td>
</tr>
<tr>
<td>7</td>
<td>( P_4, 4 )</td>
<td>10</td>
<td>Y</td>
</tr>
<tr>
<td>8</td>
<td>( C_4, 1 )</td>
<td>12</td>
<td>Y</td>
</tr>
<tr>
<td>9</td>
<td>( C_5, 1 )</td>
<td>13</td>
<td>2( x )-5</td>
</tr>
<tr>
<td>10</td>
<td>( C_6, 2 )</td>
<td>14</td>
<td>2( x )-6</td>
</tr>
<tr>
<td>11</td>
<td>( P_5, 2 )</td>
<td>18</td>
<td>Y</td>
</tr>
<tr>
<td>12</td>
<td>( C_7, 2 )</td>
<td>20</td>
<td>Y</td>
</tr>
<tr>
<td>13</td>
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<td>Y</td>
</tr>
<tr>
<td>14</td>
<td>( C_8, 1 )</td>
<td>24</td>
<td>Y</td>
</tr>
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<td>15</td>
<td>( C_9, 1 )</td>
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<tr>
<td>16</td>
<td>( P_{10}, 1 )</td>
<td>26</td>
<td>2( x )-6</td>
</tr>
<tr>
<td>17</td>
<td>( P_7, 2 )</td>
<td>26</td>
<td>Y</td>
</tr>
<tr>
<td>18</td>
<td>( C_{11}, 2 )</td>
<td>32</td>
<td>Y</td>
</tr>
<tr>
<td>19</td>
<td>( P_8, 4 )</td>
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<td>Y</td>
</tr>
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<td>Y</td>
</tr>
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</tr>
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</tr>
<tr>
<td>23</td>
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</tr>
<tr>
<td>24</td>
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</tr>
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</tr>
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</tr>
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<td>47</td>
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</tr>
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<td>48</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
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</tr>
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<td>( C_{21}, 1 )</td>
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<td>Y</td>
</tr>
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<td>( C_{22}, 1 )</td>
<td>61</td>
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</tr>
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<td>( C_{23}, 1 )</td>
<td>62</td>
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<td>( C_{24}, 1 )</td>
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<td>( C_{25}, 2 )</td>
<td>64</td>
<td>2( x )-8</td>
</tr>
<tr>
<td>37</td>
<td>( P_{12}, 4 )</td>
<td>70</td>
<td>Y</td>
</tr>
<tr>
<td>38</td>
<td>( C_{26}, 1 )</td>
<td>72</td>
<td>Y</td>
</tr>
<tr>
<td>39</td>
<td>( C_{27}, 1 )</td>
<td>73</td>
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</tr>
<tr>
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<td>( C_{28}, 1 )</td>
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</tr>
<tr>
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<td>78</td>
<td>Y</td>
</tr>
<tr>
<td>42</td>
<td>( C_{29}, 2 )</td>
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</tr>
<tr>
<td>43</td>
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<tr>
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<td>( C_{30}, 1 )</td>
<td>84</td>
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</tr>
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<td>( P_{15}, 6 )</td>
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<td>Y</td>
</tr>
<tr>
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<td>( C_{33}, 1 )</td>
<td>92</td>
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<td>54</td>
<td>( C_{38}, 1 )</td>
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<td>( C_{44}, 1 )</td>
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</tr>
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<td>( C_{45}, 1 )</td>
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<td>64</td>
<td>( C_{46}, 1 )</td>
<td>122</td>
<td>2( x )-6</td>
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</table>
Finally, we easily observe the 'overall magnitude of composite numbers to be always greater than that of prime numbers' criterion to hold true from $x = 8$ onwards. For instance, position $x = 61$ corresponds to prime number 61 which is the $18^{th}$ prime number, whereas [the one lower] position $x = 60$ corresponds to composite number 60 which is the [much higher] $43^{rd}$ composite number.

8 Polignac's and Twin prime conjectures

We have already established that prime and composite numbers are mutually exclusive, complementary, inseparable, and infinite in magnitude. With the letter 'Y' symbolizing (baseline) Dimension $2x - 4$ and prime gap at $P_i = P_{i+1} - P_i$ with $P_i$ & $P_{i+1}$ respectively symbolizing consecutive "first" & "second" prime number in any $P_i$-$P_{i+1}$ pairings, we can conveniently denote (i) Dimensions YY grouping [depicted by $2x - 4$ initially appearing twice in (iii) below] as representing the signal for appearances of prime number pairings other than twin primes (with prime gap = 2), etc; (ii) Dimension YYYY grouping as representing the signal for appearances of prime number pairings as twin primes (with prime gap = 2); and (iii) Dimension $(2x - 4)$-Progressive-Grouping allocated to the $2x - 4, 2x - 4, 2x - 5, 2x - 6, 2x - 7, 2x - 8,..., 2x - \infty$ as elements of the precise and proportionate countable finite set (CFS) Dimensions representation of an individual prime number $P_i$ with its associated prime gap namely, Dimensions $2x - 4$ & $2x - 4$ pairing = twin prime (with both of its prime gap & CFS cardinality = 2); $2x - 4, 2x - 4, 2x - 5$ & $2x - 6$ pairing = cousin prime (with both of its prime gap & CFS cardinality = 4); $2x - 4, 2x - 4, 2x - 5, 2x - 6, 2x - 7 & 2x - 8$ pairing = sexy prime (with both of its prime gap & CFS cardinality = 6); and so on. Then the higher order [which is traditionally defined as closest possible] prime groupings of three prime numbers as prime triplets, of four prime numbers as prime quadruplets, of five prime numbers as prime quintuplets, etc can each be mathematically deemed to consist of relevant serendipitous groupings required-by-law to always respect the following unwritten mathematical rule: With the exception of the three 'outlier' prime numbers 3, 5, & 7; groupings of any three prime numbers as the P, P+2, P+4 combination (viz. manifesting two consecutive twin primes with prime gap = 2) is a mathematical impossibility. The 'anomaly' that one of every three sequential odd numbers is a multiple of three, and hence this particular number cannot be prime, would clearly explain this mathematical impossibility. Then the closest possible prime grouping must be of either P, P+2, P+6 format or P, P+4, P+2 format.

Note that prime groupings not respecting the traditional closest-possible-prime groupings above are also the norm [occurring infinitely often], and they simply indicate the continual presence of prime gaps $\geq 6$ [by which we tentatively propose here to the wider scientific community to arbitrarily represent 'large gaps']. As prime numbers become sparser at larger range; the perpetual presence of prime gaps $\geq 6$ of progressive greater magnitude will, in a general and gentle manner, occur ever more frequently.

Based on not dissimilar rationale to above, we can deduce that as prime numbers become sparser at larger range; the permanent presence of prime gaps 2 & 4 [by which we tentatively propose here to the wider scientific community to arbitrarily represent 'small gaps'] will, in a general and gentle manner, occur ever less
frequently. Thus nature seems to dictate that in order to comply with Information-Complexity conservation, the permanent requirement, at larger range, of intermittently resetting to baseline Dimension \(2x - 4\) occurring four times in a row as denoted by Dimension YYYY grouping [indicating the occurrence of twin primes] is inevitable.

We can now insightfully understand the Dimension YYYY unique signal of twin prime appearances in full details. The initial two CFS Dimensions YY components of YYYY fully represent the "first" prime number component of the twin prime number pairing. The last two Dimensions YY components of YYYY signifying the appearance of the "second" prime number component of the twin prime number pairing is also the initial first-two-element component of the full CFS Dimensions representation for the "first" prime number component of the following non-twin prime number pairing. The seemingly "bizarre" uniqueness of twin primes (with prime gap = 2) is that they are represented by repeating the single type Dimension \(2x - 4\) twice whereas in all other 'higher order' prime number pairings (with prime gaps \(\geq 4\)), they will always require multiple types Dimension representation.

We now conveniently carry out the valid procedure of endowing all Dimensions with exponent / power / index of 1 for subsequent perusal in our on-going mathematical arguments. \(P_1=2\) is represented by CFS as Dimension \((2x - 4)^1\) (with both of its prime gap & CFS cardinality = 1); \(P_2=3\) is represented by CFS as Dimensions \((2x - 5)^1\) & \((2x - 4)^1\) (with both of its prime gap & CFS cardinality = 2); \(P_3=5\) is represented by CFS Dimension \((2x - 4)^1\) & \((2x - 4)^1\) (with both of its prime gap & CFS cardinality = 2), etc.

**Proposition 8.1.** For any given \(x\) value in Case 1 Completely Predictable even and odd numbers pairing and in Case 2 Incompletely Predictable prime and composite numbers pairing, and with Case 1 and Case 2 totally independent of each other; the grand total number of Dimensions [Complexity] must exactly equal to either the two combined subtotal number of Dimensions [Complexity] to precisely represent each of the Completely Predictable even and odd numbers in Case 1, or the two combined subtotal number of Dimensions [Complexity] to precisely represent each of the Incompletely Predictable prime and composite numbers in Case 2.

**Proof.** Natural numbers can directly be constituted from either the combined even & odd numbers in Case 1 or the combined prime & composite numbers in Case 2. The correctly designated infinitely many CFS of Dimensions that can be used to precisely represent both the combined even & odd numbers in Case 1 and the combined prime & composite numbers in Case 2 must also directly and proportionately be representative of the relevant natural numbers arising from the combined subtotal of even & odd numbers in Case 1 and the combined subtotal of prime & composite numbers in Case 2. The proof is now complete for Proposition 8.1.

**Proposition 8.2.** For any given \(x\) value apart from the \(x = 1\) value for both cases, in Case 1 pairing for Completely Predictable even and odd numbers and Case 2 pairing for Incompletely Predictable prime and composite numbers, and with Case 1 and Case 2 totally independent of each other; the Dimension \((2x - N)^1\) [Complexity] representations of all Completely Predictable even and odd numbers in Case 1 and all Incompletely Predictable prime and composite numbers in Case 2, must respectively be given by \(N = 4\) in Case 1 and by \(N \geq 4\) in Case 2. Furthermore, prime numbers will obey the 'Plus-Minus Composite Gap 2 Number

**Proof.** Apart from the very first Dimension $(2x - 2)^1$ representation in groupings of even & odd numbers in Case 1 and prime & composite numbers in Case 2; the smallest possible N value in Dimension $(2x - N)^1$ representation for both groupings must be 4. This Dimension $(2x - 4)^1$ simply represent the maximum possible (defacto) baseline "2x - 4" Grand-Total Gaps as per Proposition 7.7 for Case 1 & Proposition 7.8 for Case 2, thus intrinsically complying in full with Information-Complexity [Input-Output] conservation. Note that all the CFS of Dimensions that can be used to precisely represent the combined even & odd numbers in Case 1 will persistently consist of the same [solitary] Dimension $(2x - 4)^1$ after the very first Dimension $(2x - 2)^1$. The perpetual repeated deviation of N values away from the $N = 4$ (minimum) in Case 2 is simply representative of the infinite magnitude of both prime & composite numbers. The proof is now complete for the first part of Proposition 8.2 $\Box$.

The Information-Complexity of each prime number is conserved because it must always remain constant as explained using prime number '61'. At Position $x = 61$ equating to $P_{18} = 61$, it is exactly represented by CFS Dimensions $(2x - 4)^1, (2x - 4)^1, (2x - 5)^1, (2x - 6)^1, (2x - 7)^1 & (2x - 8)^1$ (with both its prime gap & CFS cardinality = 6). This Virtual Container CFS Dimensions style of representation at that particular Position $x = 61$ seems to indicate an "unknown but correct" prime number with prime gap = 6 [without revealing its associated full information consisting of '61' = 31$^{st}$ odd number = $P_{18}$ at Position $x = 61$ with prime gap = 6] if the Position $x = 61$’s associated full information [which can only be completely determined by calculating all preceding CFS Dimensions / prime gaps prior to this particular CFS Dimensions / prime gap] is with-held from us. Put in a different manner, we can always confirm that ‘61’ is prime by primality tests such as trial division but we will not glean the prime gap of 6 information associated with ‘61’ unless it is displayed in the unique CFS Dimensions representation at Position $x = 61$ whereby we have now seemingly gained the extra "prime gap of 6 information". However on closer inspection, in order to ultimately arrive at this unique CFS Dimensions representation containing the extra "prime gap of 6 information" in prime number '61' at Position $x = 61$, will still require prior computing of all preceding CFS Dimensions / prime gaps – this is simply manifesting the hallmark property of Incompletely Predictable entities by which prime numbers (or their equivalent CFS Dimensions / prime gaps representation) belong.

By invoking certain broad principles such as expressed through the Universality of Physical & Mathematical Laws, Pigeonhole principle and Proof by contradiction technique, we can categorically make the following valid statements using sound mathematical judgment. The total number of individual CFS Dimensions required to represent each and every known prime numbers will have to be infinite in magnitude simply because prime numbers are [overall] infinite in magnitude. This is equivalent to the exact mathematical statement that the standalone Dimensions YY groupings [representing the signals for "higher order" non-twin primes appearances] &/or as the front Dimensions YY (sub)groupings [which by itself is fully representative of twin primes] from the Dimensions YYYY appearances, must always recur on an indefinite basis. Common sense alone would suggest that twin primes and the "higher order" cousin primes, sexy primes, etc should aesthetically all be infinite in magnitude simply because they should regularly and universally
arise as part of the components in Dimensions YY and Dimensions YYYYY appearances. We provide the proof for this statement in the following paragraphs.

An isolated prime is defined as a prime number P such that neither P-2 nor P+2 is prime. In other words, P is not part of a twin prime pair. For example, 23 is an isolated prime, since 21 and 25 are both composite. We note that the repeated inevitable presence of Dimension YY grouping is nothing more than indicating the repeated occurrences of isolated prime. This constitutes yet another view on Dimension YY.

As general principles which are fully applicable except right at the beginning of prime & composite number integer sequences, prime gaps = 2, 4, 6,... are CIS and composite gaps = 1 & 2 are CFS. Composite numbers with composite gap = 1 are the "defacto" basic numbers needing to eternally recur simply because they are present in any two consecutive natural numbers [which themselves are also fundamentally and eternally present in the Prime-Composite mathematical landscape representing the 'composite gap = 1 signatures' to signify the actual prime gaps per se for non-twin prime numbers]. Composite numbers with composite gap = 2 can then be considered as the "default" basic numbers needing to eternally recur simply because they must be present as 'composite gap = 2 signatures' to signify the appearances of any prime numbers per se.

An alternative and advantageous view on prime numbers would stem from the perspective of this "manageable" CFS composite gaps [instead of the "unmanageable" CIS prime gaps] with various observable clear-cut intrinsic patterns involving ALTERNATING PRESENCE and ABSENCE of composite numbers with composite gap = 2 in association with every CFS Dimensions representations of prime numbers with prime gaps ≥ 4. This all-important observation in the context of our Prime-Composite mathematical landscape can be classified as a mathematical law needing to be abided by all non-twin prime numbers. Twin primes with CFS Dimensions YY representations are always associated with a composite number with composite gap = 2, and are thus exempted from this law [now designated with the conveniently shortened name "Plus-Minus Composite Gap 2 Number Alternating Law"]. We coined the law for the prime gap = 2 situation as "Plus Composite Gap 2 Number Continuous Law". Two illustrative examples for both laws: A twin prime (with prime gap = 2) in its unique CFS Dimensions format is always followed by a composite number with composite gap = 2 [constant] pattern. A cousin prime (with prime gap = 4) in its unique CFS Dimensions format is always followed by two composite numbers with composite gap = 1 & then one composite number with composite gap = 2 [combined] pattern ALTERNATING with three consecutive composite numbers with composite gap = 1 [non-combined] pattern. From this simple observation alone, one can rigorously deduce that we can already/always generate an infinite magnitude of composite numbers from each of the composite gaps of 1 & 2 [automatically endowed with the same composite gaps of 1 & 2 respectively]. We can see that this composite gap = 2 ALTERNATING pattern behavior in cousin primes will not hold true unless twin primes & all other non-cousin primes are infinite in magnitude and integratedly supplying essential "driving mechanism" to eternally sustain this composite gap = 2 ALTERNATING pattern behavior in cousin primes. Thus we have already discussed and established that (i) except for the very first CFS composite number 1 with associated composite gap = 3, composite numbers with composite gaps = 1 & 2 must both belong to CIS and (ii) twin primes and cousin primes in their CFS Dimensions formats
are CIS closely intertwined together when depicted using composite numbers with composite gaps $= 1$ & $2$ with each supplying their own peculiar (infinite) share of associated composite numbers with composite gap $= 2$ [thus contributing to the overall pool of composite numbers with composite gap $= 2$].

A inevitable mathematical statement in relation to "composite gap $= 2$ pool contribution" based on mathematical reasoning above is that, at the bare minimum, EITHER twin prime numbers OR at least one of the non-twin prime numbers must be infinite in magnitude. A beautiful natural question that follows is: Why then should all the generated sets of prime numbers from 'small gaps' [of $2$ & $4$] and 'large gaps' [of $\geq 6$] alike not all belong to CIS thus allowing true uniformity in prime number distribution? Again we can see in Table 6 above depicting the Prime-Composite data for $x = 64$ that, for instance, prime numbers with prime gap $= 6$ must also persistently have this 'last-place' composite numbers endowed with composite gap $= 2$ intermittently appearing in certain rhythmic ALTERNATING patterns, thus complying with the Plus-Minus Composite Gap 2 Number Alternating Law. This CFS Dimensions representation for prime numbers with prime gap $= 6$ will again generate their infinite share to the pool of associated composite numbers with composite gap $= 2$. The presence of this last-place composite numbers with composite gap $= 2$ in various alternating pattern in their appearances & non-appearances must SELF-GENERATINGLY be similarly extended in a mathematically consistent fashion ad infinitum to all remaining infinite number of prime gaps [which were not discussed in details above]. The proof is now complete for the second part of Proposition 8.2 $\Box$.

The preceding few paragraphs above then provide the rigorous proofs for Polignac's and Twin prime conjectures in that we have mathematically shown in a self-consistent manner that prime gaps are [necessarily] infinite (and arbitrarily large) in magnitude with each individual prime gap [necessarily] generating prime numbers which are again infinite in magnitude. The Plus-Minus Composite Gap 2 Number Alternating Law, this CFS Dimensions representation for prime numbers with prime gap $= 6$ will again generate their infinite share to the pool of associated composite numbers with composite gap $= 2$. The presence of this last-place composite numbers with composite gap $= 2$ in various alternating pattern in their appearances & non-appearances must SELF-GENERATINGLY be similarly extended in a mathematically consistent fashion ad infinitum to all remaining infinite number of prime gaps [which were not discussed in details above]. The proof is now complete for the second part of Proposition 8.2 $\Box$.

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Incorrect/incomplete recurrences in any of those mentioned Dimensions or in their exponents [e.g. using exponents $\frac{2}{3}$ or $\frac{3}{2}$ instead of exponent 1] would have the dire consequence of "Dimensional analysis non-homogeneity" resulting in drastically incorrect or incomplete representation of all known prime numbers. The fixed mathematical landscape "pages" for prime numbers will have to permanently display Chaos [sensitivity to initial conditions viz. positions of subsequent prime numbers are "sensitive" to positions of initial prime numbers] and Fractals [manifesting fractal dimensions with self-similarity viz. those aforementioned Dimensions for prime numbers must always be present, albeit in a non-identical
manner, for all ranges of \( x \). Advocated in another manner, the Chaos and Fractals phenomena of those Dimensions for prime numbers above must always be correctly present signifying the accurate composition of prime and composite numbers for different (predetermined) mathematical landscape "pages" for prime numbers that are self-similar but never identical.

As previously mentioned in this paper, we regard a 'conjecture' to become a 'hypothesis' when that particular conjecture has been rigorously proven to be true. Abiding to this notational use for those terms, we should now call Polignac’s and Twin prime conjectures as Polignac’s and Twin prime hypotheses.

9 Polignac’s and Twin prime hypotheses

The lemmas and propositions from the preceding section above should now provide all necessary evidences to support the following Theorem (Polignac-Twin prime) I to IV (Virtual Container) which will be seen to further contribute towards fully strengthening the rigorous proofs for Polignac’s and Twin prime conjectures in a succinct manner. Only after successfully procuring those rigorous proofs are we finally permitted to term Polignac’s and Twin prime conjectures more appropriately as Polignac’s and Twin prime hypotheses.

The complete set of even number prime gaps are traditionally & conveniently divided into 'small gaps' and 'large gaps'. In this paper, we arbitrarily denote prime numbers with 'small gaps' as having the finite number of prime gaps = 2 & 4 and prime numbers with 'large gaps' as having the infinite number of prime gaps \( \geq 6 \). We have already established in the previous section above that (i) composite numbers with composite gap = 1 are involved in representing the infinite magnitude of all possible prime gaps except that for twin primes, viz. for prime gaps = 4, 6, 8,... [whereby twin primes whose prime gap = 2 will always involve composite numbers with composite gap = 2 representations] and (ii) composite numbers with composite gap = 2 represent the appearances of relevant prime numbers which must compulsorily be present on an indefinite basis. Furthermore, we ingeniously establish through the Plus-Minus Composite Gap 2 Number Alternating Law that composite numbers with composite gaps = 2 present in each of the prime numbers with prime gaps \( \geq 4 \) situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence for relevant composite numbers with composite gap = 2. The prime gap = 2 situation obeys the Plus Composite Gap 2 Number Continuous Law. These are the dominant underlying driving mechanisms for the infinite magnitude of prime numbers generated by each of the prime gaps \( \geq 4 \) and prime gap = 2 scenarios in a mathematically consistent manner. The case for twin primes with prime gap = 2 scenario can best be understood as the special situation of "rhythmic patterns with CONTINUAL presence" for relevant composite numbers with composite gap = 2. All these prime number INTERLINKED driving mechanisms must be perpetually present (viz. must be "self-generating") in every single prime gap in order to contribute towards generating the [complete] infinite size pool of composite numbers with composite gap = 2.

Alphonse de Polignac (1826 - 1863) was a French mathematician. In 1849, the year he was admitted to Polytechnique, he made what is known as Polignac’s conjecture which relates the complete set of prime numbers to all prime gaps = 2, 4, 6,..., \( \infty \) [viz. all the even numbers]. We reiterate here again that Twin prime
conjecture, which relates twin prime numbers to prime gap = 2, is nothing more than a subset of Polignac’s conjecture.

**Theorem {Polignac-Twin prime} I.** The set of prime numbers \( P_n = 2, 3, 5, 7, 11, ..., \infty \) or the proxy set of composite numbers \( C_n = 1, 4, 6, 8, 9, ..., \infty \) is infinite in magnitude with each and every conceivable prime or composite number [but not its actual identity] irrefutably, accurately and completely represented by the following formula involving prime gaps \( G_{P_i} \) viz. \( P_{n+1} = 2 + \sum_{i=1}^{n} G_{P_i} \) or involving composite gaps viz. \( C_{n+1} = 1 + \sum_{i=1}^{n} G_{C_i} \) whereby prime and composite numbers are symbolically represented here with the aid of ‘n’ notation instead of the usual ‘i’ notation used in this research paper; and \( i \& n = 1, 2, 3, 4, 5, ..., \infty \). The number ‘2’ in the first formula represent \( P_1 \), the very first (and only even) prime number. The number ‘1’ in the second formula represent \( C_1 \), the very first (and only odd) composite number. Note that the natural numbers required to represent prime gaps must be infinite in magnitude but the natural numbers required to represent composite gaps must be finite in magnitude.

**Proof.** We treat and closely analyze the above formulae as unique mathematical objects looking for key intrinsic properties and behaviors. By definition, each prime or composite number is assigned a unique prime or composite gap. The absolute number of prime or composite numbers and (thus) prime or composite gaps are known to be infinite in magnitude. As original true formulae containing all possible prime or composite numbers by themselves (viz. without computationally supplying prime or composite gaps as "input information" to generate the necessary prime or composite numbers as "output complexity"), these formulae will intrinsically incorporate the actual presence [but not the actual locations] of the complete set of prime or composite numbers. See Proposition 9.1 below, based on the language using cardinality and pigeonhole principle, for further supporting materials. The proof is now complete for Theorem {Polignac-Twin prime} I ⊓ ⊔.

**Theorem {Polignac-Twin prime} II.** The set of prime gaps \( G_{P_i} = 2, 4, 6, 8, 10, ..., \infty \) is infinite (and arbitrarily large) in magnitude with each and every conceivable prime gap irrefutably, accurately and completely represented by Dimensions \( (2x - 4)^1, (2x - 5)^1, (2x - 6)^1, ..., (2x - \infty)^1 \) which must satisfy Information-Complexity conservation in a self-consistent manner. Furthermore, this nominated method of prime gap representation using these Dimensions is purportedly the only (solitary) way to achieve the necessary conservation.

**Proof.** The relevant part of the proof from Proposition 8.2 stated that all prime numbers can be represented by the Dimension \( (2x - N)^1 \) with \( N \geq 4 \) for any given \( x \) value (except for the \( x = 1 \) value). If each prime number is endowed with a specific prime gap value, then each such prime gap must [via logical mathematical deduction] be able to be represented by the mentioned Dimension \( (2x - N)^1 \). The preceding mathematical statement is absolutely correct as there is, by definition, a unique prime gap value associated with each prime number. Proposition 9.1 below predominantly based on cardinality language provides further supporting materials that prime gaps are infinite (and arbitrarily large) in magnitude. The proof is now complete for Theorem {Polignac-Twin prime} II ⊓ ⊔.

**Theorem {Polignac-Twin prime} III.** To maintain Dimensional analysis (DA) homogeneity, those aforementioned Dimensions \( (2x - N)^1 \) endowed with
Riemann-Polignac-Twin prime

exponent 1] from Theorem \{Polignac-Twin prime\} II must repeat themselves on an indefinite basis in the following specific combinations – (i) Dimension \((2x - 4)^1\) only appearing as twin \([two-times-in-a-row]\) and quadruplet \([four-times-in-a-row]\) sequences, and (ii) Dimensions \((2x - 5)^1\), \((2x - 6)^1\), \((2x - 7)^1\), \((2x - 8)^1\), ..., \((2x - \infty)^1\) appearing as progressive groupings of \([even numbers]\) \(2, 4, 6, 8, 10, ..., \infty\).

To accommodate the \((only)\) even prime number \('2'\), the exceptions to this DA homogeneity compliance will expectedly occur right at the beginning of the prime number sequence – (i) the one-off appearance of Dimension \((2x - 2)^1\), (ii) the one-off appearances of Dimension \((2x - 4)^1\) and Dimension \((2x - 5)^1\) in a solitary \([isolated]\) manner, and (iii) the one-off appearance of Dimension \((2x - 4)^1\) as a quintuplet \([five-times-in-a-row]\) sequence. Theorem \{Polignac-Twin prime\} III can be more succinctly stated as the eternal repetitions of well-ordered sets constituted by Dimensions \((2x - 4)^1\), \((2x - 5)^1\), \((2x - 6)^1\), \((2x - 7)^1\), \((2x - 8)^1\), ..., \((2x - \infty)^1\). These sequentially arranged sets consist of countable finite sets \([CFS]\) whereby from \(x = 11\) onwards, each set will always commence initially as \('baseline'\) Dimension \((2x - 4)^1\) at \(x = odd\) number values and will always end with its last Dimension at \(x = even\) number values. Each set will also have varying cardinality with the value derived from \(2, 4, 6, 8, 10, 12, ..., \infty\); and the correctly combined sets will always intrinsically generate the two infinite sets of prime and, by default, composite numbers in an integrated manner whereby at ever larger \(x\) values, Prime-\(\pi(x)\) will overall become larger but with a decelerating trend and Composite-\(\pi(x)\) will overall become larger but with an accelerating trend.

**Proof.** Theorem \{Polignac-Twin prime\} III simply represent a mathematical summary of all the expressed characteristics of Dimension \((2x - N)^1\) when used to represent prime numbers with intrinsic display of Dimensional analysis homogeneity. This summary has been rigorously derived in Section 7 & 8 above. See Proposition 9.2 below for supporting details on the Dimensional analysis aspect. *The proof is now complete for Theorem \{Polignac-Twin prime\} III.*

**Theorem \{Polignac-Twin prime\} IV.** Aspect 1. The "quantitive" aspect to the existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by simultaneously utilizing concepts derived from Set theory, and incorporating arguments based on 'pigeonhole principle'. Aspect 2. The "qualitative" aspect to the existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by the 'Plus-Minus Composite Gap 2 Number Alternating Law' and 'Plus Composite Gap 2 Number Continuous Law'. Aspect 1 and Aspect 2 are mutually inclusive of each other.

**Proof.** The required concepts derived from Set theory will mainly involve the cardinality of a set together with its 'well-ordering principle' application. The supporting materials for these concepts and for arguments based on 'pigeonhole principle' in relation to Aspect 1 are found in Proposition 9.1 below. The 'Plus-Minus Composite Gap 2 Number Alternating Law' is applicable to all even number prime gaps \([apart from the special case of first even number prime gap = 2 for twin primes]\). The prime gap = 2 situation will obey the 'Plus Composite Gap 2 Number Continuous Law'. These Laws are essentially "descriptive" laws inferring underlying intrinsic driving mechanisms that enables the infinity magnitude association for both prime gaps and prime numbers to co-exist. By the same token, these Laws have the important implication that they must be applicable to those relevant prime gaps on a perpetual time scale. The supporting materials in re-
lation to Aspect 2 are found in Proposition 8.2 above. The proof is now complete for Theorem \{Polignac-Twin prime\} IVI.

We now generate parallel insightful arguments to that obtained in the process of proving Riemann hypothesis. These powerful arguments, based similarly on utilizing Dimensional analysis (DA), are equally valid despite the desired [conjure up] exponent ‘1’ here in prime-composite number setting (for Polignac’s and Twin prime conjectures) being totally different to the desired [natural occurring] exponent ‘\(\frac{1}{2}\)’ in Riemann zeta function setting (for Riemann hypothesis). We obtain two mutually inclusive conditions from these arguments: Condition 1. The presence of any Dimension(s) that do not repeat itself (themselves) on an indefinite basis or with exponent other than 1 will give rise to the incomplete set of prime numbers or incorrect set of non-prime numbers ["the DA-wise mathematical impossibility argument" associated with inevitable de novo DA non-homogeneity], together with Condition 2. The presence of all Dimensions that do repeat themselves on an indefinite basis and with exponent of 1 will give rise to the complete set of prime numbers ["the DA-wise one and only one mathematical possibility argument" associated with inevitable de novo DA homogeneity] from Theorem \{Polignac-Twin prime\} III above, fully support the rather mute but whole point of this study in that the CFS Dimensions format Virtual Container representations of prime and composite numbers [and their respective prime and composite gaps] are proven to be completely accurate when these two (mutually inclusive) conditions are met. We see that Condition 1 simply reflect the proof from Theorem \{Polignac-Twin prime\} III above in that all prime numbers will be associated with Dimensional analysis homogeneity. In addition, Condition 2 include the corollary on the inevitable appearance of incomplete prime numbers or non-prime numbers [which will always be associated with Dimensional analysis non-homogeneity] being tightly incorporated into this mathematical framework. See Propositions 9.1 & 9.2, and Corollary 9.3 below for further supporting materials.

Ignoring the glitch caused by the (only) even prime number ‘2’ at the commencement of prime number sequence, we can further analyze the two components prime numbers and composite numbers in terms of (i) measurements based on cardinality of countable infinite set (CIS) and (ii) the pigeonhole principle which states that if \(n\) items are put into \(m\) containers, with \(n > m\), then at least one container must contain more than one item. The composite gaps can only be "finitely" constituted by the numerical values 1 & 2 (except the first composite gap = 3) and the prime gaps can only be "infinitely" constituted by the numerical values 2, 4, 6, ..., \(\infty\) (except the first prime gap = 1). We note that the ordinality of all infinite prime (and infinite composite) numbers is "fixed" implying that each one of the infinite well-ordered Dimension sets conforming to the countable finite set (CFS) type as constituted by Dimensions \((2x - 4)^1, (2x - 5)^1, (2x - 6)^1, (2x - 7)^1, (2x - 8)^1, ..., (2x - \infty)^1\) on the respective gaps for prime (and composite) numbers, must also be "fixed". Only by this method alone can we then accommodate each and every one of the infinite prime (and composite) numbers.

**Proposition 9.1.** Even number prime gaps are infinite (and arbitrarily large) in magnitude with each individual even number prime gap generating odd prime numbers which are again infinite in magnitude.

**Proof.** In this proof, we ignore the first and only known even prime number ‘2’ associated with the first and only known odd number prime gap = 1 and assume the complete set of all prime numbers to only consist of all known odd prime numbers.
Let the cardinality of (i) all prime numbers (CIS) derived from all prime gaps 2, 4, 6, ..., ∞ sets (CIS) = T, (ii) all prime numbers (CIS) derived from prime gap 2 set (CIS) = T_2, all prime numbers (CIS) derived from prime gap 4 set (CIS) = T_4, all prime numbers (CIS) derived from prime gap 6 set (CIS) = T_6, etc. Paradoxically \( T = T_2 + T_4 + T_6 + ... + T_\infty \) is mathematically valid despite \( T = T_2 = T_4 = T_6 = ... = T_\infty \) (when defined in terms of the *well-ordering principle* applied to the cardinality of each set). But if prime numbers derived from one or more prime gap(s) are finite in magnitude of the CFS variety, this will breach the CIS "uniformity" property resulting in (i) DA non-homogeneity and (ii) the inequality \( T > T_2 + T_4 + T_6 + ... + T_\infty \). In the language of pigeonhole principle, residual prime numbers (still CIS in magnitude) not accounted for by the CFS-type prime gap(s) will have to be [incorrectly] contained in one (or more) of the other prime gap(s). Ditto for composite numbers with a similar argument able to be construed up for the case if composite numbers from one of the composite gaps are finite in magnitude and of the CFS variety. Furthermore, the Plus-Minus Composite Gap 2 Number Alternating Law has an underlying built-in intrinsic mechanism to automatically apply to [and further generate] all prime gaps \( \geq 4 \) in a mathematically consistent *ad infinitum* manner. Twin primes with prime gap = 2 obeys the Plus Composite Gap 2 Number Continuous Law which is endowed with its own unique underlying built-in intrinsic mechanism to automatically apply to [and further generate] prime gap = 2 appearances in a mathematically consistent *ad infinitum* manner. The above arguments using the cardinality of number property [with "quantitative" features/patterns] thus constitute the rigorous proof for Proposition 9.1 as it has been shown that (i) prime gaps and (ii) prime numbers generated from each of the prime gaps, must all consist of CIS. The proof on "quantitative" aspect is now complete for Proposition 9.1 ⊓ ⊔.

Proposition 9.1 fully encompass Polignac’s and Twin prime conjectures. When Proposition 9.1 is rigorously proven to be correct, it will be the overall mathematical "quantitative" statement to fully describe the complete set of prime numbers as generated by the Sieve of Erastosthenes algorithm. This complete set of prime numbers derived from all even number prime gaps can be fully represented by the Dimensions \( (2x - N)^1 \) concept as rigorously stated in Theorem \{Polignac-Twin prime\} I - IV above. As Theorem \{Polignac-Twin prime\} I - IV is not falsifiable, our respectful opinion is that they must act as valid Virtual Container for all prime & composite numbers, and their respective gaps. Table 6 and Figure 7 on Prime-Composite mathematical landscape clearly depict perpetual "qualitative" features/patterns by which overall mathematical "qualitative" statements can be made supporting (i) the Plus-Minus Composite Gap 2 Number Alternating Law (which literally can be stated as composite numbers with composite gaps = 2 present in each of the prime numbers with prime gaps \( \geq 4 \) situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence for relevant composite numbers with composite gap = 2), and (ii) the Plus Composite Gap 2 Number Continuous Law (which literally can be stated as composite numbers with composite gaps = 2 continual appearances in each of the (twin) prime numbers with prime gap = 2 situation). The proof on "qualitative" aspect is now complete for Proposition 9.1 ⊓ ⊔.

From all the above tedious mathematical reasoning, Polignac’s and Twin prime conjectures have now been proven to be true thus becoming Polignac’s and Twin prime hypotheses with the overall implication that all prime numbers generated.
from each of the infinite (and arbitrarily large) in magnitude prime gaps 2, 4, 6, ..., \( \infty \) are again infinite in magnitude. The four steps (‘mathematical footprints’) in specific sequence required to prove Theorem (Polignac-Twin prime) I - IV can be outlined next as:

**Step 1:** Use the 2-variable formula with 'prime number' variable & 'prime gap' variable to "contain" all prime numbers without knowing their true identities (in a virtual manner). **Step 2:** Use Dimensions \((2x - 4)^1, (2x - 5)^1, (2x - 6)^1, ..., (2x - \infty)^1\) to "contain" all prime gaps without knowing their true identities (in a virtual manner). **Step 3:** Define DA homogeneity as the perpetual recurrences of specific groupings of those Dimensions with exponent 1 for all ranges of \(x\). **Step 4:** Supporting mathematical arguments for the "quantitative" aspect will represent all prime numbers in a complete manner, and that for the "qualitative" aspect will also represent all prime numbers in a complete manner.

**Proposition 9.2.** Only the defined Dimensional analysis homogeneity will always result in the correct \& complete set of prime numbers.

**Proof.** The DA definition is completely dependent on these Dimensions. As all prime (and composite) numbers are "fixed", we can deduce from Figure 7 and Table 6 above that there is one (and only one) way to represent Information-Complexity conservation using our defined Dimensions. Thus, there is one (and only one) way to depict all prime numbers using these Dimensions in a self-consistent manner and this can only be achieved with the one (and only one) DA homogeneity possibility. *The proof is now complete for Proposition 9.2.*

**Corollary 9.3.** The defined Dimensional analysis non-homogeneity will always result in the incorrect \&/ or incomplete set of prime numbers.

**Proof.** Proposition 9.2 equates DA homogeneity with the correct \& complete set of prime numbers with full mathematical consistency. There are "more than one" DA non-homogeneity possibilities. For instance, if a particular \((2x - 4)^1\) Dimension derived from \((2x - 4)^1, (2x - 5)^1, (2x - 6)^1, ..., (2x - \infty)^1\) terminates prematurely and does not perpetually repeat (resulting in loss of continuity and thus depicting one DA non-homogeneity possibility); then there are intuitively two 'broad' DA possibilities here; namely, (one) DA homogeneity possibility and "all others" endowed with DA non-homogeneity possibilities. This meant that the mathematical consistency of Dimensions \((2x - 5)^1, (2x - 6)^1, (2x - 7)^1, (2x - 8)^1, ..., (2x - \infty)^1\) appearing as progressive groupings of [even numbers] 2, 4, 6, 8, 10, ..., \( \infty \) will be halted without justification. For optimal clarity, we have treated all those Dimensions above using exponents and depict them as \((2x - 4)^1, (2x - 5)^1, (2x - 6)^1, (2x - 7)^1, (2x - 8)^1, ..., (2x - \infty)^1\). Then a particular Dimension, using the \((2x - 4)^1\) example (endowed with exponent 1), that stop recurring at some point in the prime number sequence would have DA non-homogeneity and be depicted against-all-trends as \((2x - 4)^0\) when endowed with a totally different exponent – which is arbitrarily set as 0 in this case. Thus a Dimension that stop recurring will result in the well-ordered CFS sets from the progressive groupings of [even numbers] 2, 4, 6, 8, 10, ..., \( \infty \) for Dimensions \((2x - 5)^1, (2x - 6)^1, (2x - 7)^1, (2x - 8)^1, ..., (2x - \infty)^1\) to stop existing (and ultimately for sequential prime numbers to stop appearing) at that point using this grouping method – with the likely ensuing outcome that prime and composite numbers are overall [incorrectly] finite in magnitude. Finally, a Dimension with fractional exponent values (other than 1) will always result in non-prime and non-composite (fractional) numbers. *The proof is now complete for Corollary 9.3.*
Thus the seemingly small but utterly essential sequential mathematical steps in (i) representing all prime numbers using a '2-variable function' (made up of prime number variable and prime gap variable) and (ii) then further representing all prime gaps with the defined Dimensions, will crucially allow proper DA process to happen in the absolute correct way. The 'strong' principle argument mathematical end-result is that DA homogeneity will equate to the complete set of prime numbers whereas DA non-homogeneity will not equate to the complete set of prime numbers. One could additionally advocate for a 'weak' principle argument supporting DA homogeneity for prime numbers in that nature should not "favor" any particular Dimension(s) to terminate and therefore DA non-homogeneity does not, and cannot, exist for prime numbers.

10 Composite counting function, L-function, LMFDB, Moonshine theory, GUT, TOE and Fundamental Laws

There are many deep-seated connections between prime numbers, prime counting function, Sieve of Eratosthenes, Polignac’s & Twin prime conjectures, Riemann zeta (or its proxy Dirichlet eta) function, and Riemann hypothesis. Being Incompletely Predictable entities or problems, we now highlight the notable near-identical occurrence of exception-to-the-rule event in prime numbers in that the very first & solitary even prime number is '2'; and in Riemann zeta function in that the very first & solitary negative Gram[y=0] intercept at the designated $\sigma = \frac{1}{2}$ critical line occur only when parameter $t = 0$ giving rise to the resulting $\zeta(\frac{1}{2}) = -1.4603545$ (rounded off to seven decimal places). Note that $\zeta(\frac{1}{2})$ can be calculated as a limit similar to the limit for the Euler-Mascheroni constant or Euler gamma.

It is commonly advocated that the rigorous proof for Riemann hypothesis would be instrumental in proving the efficacy of techniques that estimate prime counting function (traditionally denoted by $\pi(x)$ but in this paper by Prime-$\pi(x)$) efficiently and reasonably well.

The sets of prime number and composite number are complementary to each other because they are both stipulated directly and indirectly by the same Sieve of Eratosthenes. The prime counting function is the function counting the number of prime numbers less than or equal to some real number $x$. One can easily deduce that inventing a complementary and equivalent 'composite counting function' is absolutely a valid mathematical exercise. This proxy composite counting function, denoted in this paper by Composite-$\pi(x)$, can then be categorically stated as the function counting the number of composite numbers less than or equal to some real number $x$. It must also enjoy the exact same mathematical privileges that are extended to prime counting function.

The sets of nontrivial zeros, Gram[y=0] points and Gram[x=0] points in Riemann zeta function are complementary to each other. This can only happen when all three sets are stipulated by the same Riemann zeta function.

By an L-function, we generally refer to a Dirichlet series with a functional equation and an Euler product. Contextually, the simplest example of an L-function is Riemann zeta function on which the 1859 Riemann hypothesis is based upon. L-functions are ubiquitous in number theory and hence have applications to mathematical physics and cryptography. They arise from and encode information about a number of mathematical objects and it is necessary to exhibit these objects along
with the L-functions themselves since typically we need these objects to compute L-functions. For examples, L-functions can come from modular forms, elliptic curves, number fields, and Dirichlet characters, as well as more generally from automorphic forms, algebraic varieties, and Artin representations. Broadly based on these examples, the mammoth ‘L-functions and Modular Forms Database’ (LMFDB) creation was conducted with massive team-effort collaboration from an international group of more than 80 researchers from 12 countries which included prominent mathematicians such as from American Institute of Mathematics in United States, University of Bristol in United Kingdom, and Dartmouth College in United States. The LMFDB idea was first conceived at an American Institute of Mathematics workshop in 2007. Six years after commencing the LMFDB project [website address http://www.lmfdb.org/], its launching was celebrated on May 10, 2016.

In effect, LMFDB can be considered an uncharted mathematical terrain providing a detailed atlas of mathematical objects that highlights deep relationships and serves as a guide to latest research happening in physics, computer science and mathematics. Elliptic curves arise naturally in many parts of mathematics and can be described by a simple cubic equation. They also form the basis of cryptographic protocols used by most of the major internet companies including Google, Facebook and Amazon. Modular forms are more mysterious objects constituted by complex functions with an almost unbelievable degree of symmetry. The two mathematical worlds of elliptic curves and modular forms are remarkably connected via their L-functions. It is this deep connection that was in essence required in the late 20th century by famous British number theorist Andrew John Wiles to successfully achieve his proof of Fermat’s Last Theorem. To put into perspective the importance of LMFDB in relation to active research areas such as involving Monstrous moonshine (Moonshine theory), Mathieu moonshine, and Umbral moonshine with their conjectured roles in Quantum gravity and String theory; we think that most physicists would have a positive opinion or consensus on the potential role of these research areas in successfully merging gravity with Grand Unified Theory (GUT) – consisting of the unification of electromagnetism, weak nuclear force and strong nuclear force – thus giving rise to the holy grail Theory of Everything (TOE).

We briefly divert here to mention that the name ‘Standard Model of particle physics’, commenced in the 1970s, denotes the theory describing three of the four known fundamental forces in the universe (viz. the electromagnetic, weak, and strong interactions of GUT), as well as classifying all known elementary particles. Despite all its predictive power, it is not ”perfect” in that it can’t explain gravity, dark matter or dark energy.

String theories assume that fundamental building blocks of the universe are strings instead of point particles. String duality is a class of symmetries in physics that link different String theories, with K3 surfaces appearing almost ubiquitously in string duality. A K3 surface is a complex or algebraic smooth minimal complete surface that is regular and has trivial canonical bundle. Not least because of this issue of multiple String theories (and hence multiple possibilities), an alternative view is that all four fundamental forces of nature will always exist as the current status quo with gravity obeying laws [perhaps endowed with certain ”continuous” Completely Predictable properties] derived from Einstein’s Theory of General Relativity and the three forces of GUT obeying laws [perhaps endowed with certain ”discrete” Incompletely Predictable properties] based on Quantum mechanics. Al-
ternatively stated, nature will intrinsically never allow the mathematical merging together of those two totally incompatible situations; namely, the "continuous" property on the one hand and "discrete" property on the other hand. Despite this issue, LMFDB with one of its crucial features acting as "intricate catalog of mathematical objects" will, metaphorically speaking, be the source supplying the required mathematical objects in those mentioned research areas.

In the grand scheme of things, this paper manifests the classically encountered phenomenon that pure and applied mathematics during, and resulting from, the derivation of many mathematical proofs are largely inseparable. Some of the less conventional aspects of the resulting applied mathematics in regards to the following (depicted from biologist-to-physicist point of view with highest-to-lowest decreasing hierarchical order) are intuitively useful:

[I. 'Living Things' obeying Complex Emergent Fundamental Laws]
[II. 'Living Things' obeying Simple Emergent Fundamental Laws]
[III. 'Nonliving Things' obeying Complex Elementary Fundamental Laws]
[IV. 'Nonliving Things' obeying Simple Elementary Fundamental Laws]

In this context, our Incompletely Predictable problems of Riemann hypothesis, Polignac's and Twin prime conjectures are 'Nonliving Things' obeying Complex Elementary Fundamental Laws. People have often strived to obtain pivotal scientific answers on whether 'Living Things' arise from 'Nonliving Things' via the Evolution process [as per atheist belief] or 'Living Things' arise from 'Nonliving Things' via the Creation process [as per religious belief]. We speculatively hope and selfishly dream that the applied mathematics pathway resulting from solving Riemann hypothesis, Polignac's and Twin prime conjectures could at least one day lead to answering the following question: Could the concocted expression "Living Things seem to exist at the edge of Chaos and Fractals" be mathematically equivalent to the statement "Living Things must be made up of a combination of Completely Predictable entities, Incompletely Predictable entities, and/or Completely Unpredictable entities"?

11 Conclusions

In this research paper, we have provided rigorous proofs for the three open problems of Riemann hypothesis, Polignac’s and Twin prime conjectures by predominantly using the Virtual Container Research Method (VCRM). The way we achieve these proofs is succinctly outlined by our 'Executive Summary' in the Introduction section above. One could envisage our VCRM employed in this paper to anticipatedly be accepted as [futuristic] applied mathematics especially for solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' (or simply the 'Incompletely Predictable problems') containing Incompletely Predictable entities of infinite magnitude. The VCRM can schematically be depicted below.

\[
\begin{array}{c}
\text{Completely Predictable problems} \\
\rightarrow \text{Completely Predictable entities with Completely Predictable properties} \\
\rightarrow \text{Solutions for Completely Predictable problems}
\end{array}
\]

\[
\begin{array}{c}
\text{Incompletely Predictable problems} \\
\rightarrow \text{Incompletely Predictable entities with Incompletely Predictable properties} \\
\rightarrow \text{Completely Predictable meta-properties} \\
\rightarrow \text{Solutions for Incompletely Predictable problems}
\end{array}
\]
The following overall property has been harnessed to obtain the successful proof on Riemann hypothesis. The countable finite set (CFS) of exactly three types of axes intercepts occurring in Riemann zeta function only when $\sigma = \frac{1}{2}$ as opposed to the CFS of exactly two types of axes intercepts occurring in Riemann zeta function when $\sigma \neq \frac{1}{2}$. The three types of axes intercepts in Riemann zeta function occurring only when $\sigma = \frac{1}{2}$ are completely constituted by the three complementary countable infinite sets (CIS) of Gram[x=0,y=0] points (or nontrivial zeros), Gram[y=0] points (‘usual’ Gram points), and Gram[x=0] points.

The following overall property has been harnessed to obtain the successful proofs on Polignac’s and Twin prime conjectures. The CIS of [Completely Predictable] natural numbers 1, 2, 3, 4, 5,... with their associated CIS of [Completely Predictable] natural gap consisting of the numbers 1, 1, 1, 1, 1, 1, 1,... are completely constituted by two complementary sets of numbers, namely (i) the CIS of [Incompletely Predictable] prime numbers 2, 3, 5, 7, 11, 13, 17,... with their associated CIS of [Incompletely Predictable] prime gaps consisting of the numbers 1, 2, 2, 4, 2, 4,... and (ii) the CIS of [Incompletely Predictable] composite numbers 1, 4, 6, 8, 9, 10, 12,... with their associated CIS of [Incompletely Predictable] composite gaps consisting of the numbers 3, 2, 2, 1, 1, 2,... The overall properties “prime numbers are odd numbers” and “composite numbers are even numbers” are applicable to all prime and composite numbers alike except right at the beginning of their number sequences due to the one-off exception that the very first (and only) prime number ‘2’ is an even number and the very first (and only) composite number ’1’ is an odd number. This last statement can reciprocally be expressed as: the even number ‘2’ is not a composite number despite the general rule stating that “all even numbers must in general be composite numbers”.

“Prime numbers can be described as atoms. What mathematicians have been missing is a kind of mathematical number spectrometer. Chemists have an atomic spectrometer machine that, if given a molecule, will tell us the atoms that it is built from. Mathematicians have never invented a mathematical version of this. The proof of Riemann hypothesis would have given us perfect understanding on how prime numbers work, and translating this into essential knowledge allowing construction of prime number spectrometer. Suddenly all cryptic codes are breakable. No internet transaction would be safe as the whole of e-commerce depends on the integrity of humongous [non-prime] numbers (molecules) to be anonymously or secretly constituted from its basic prime numbers (atoms). In other words, breaching this integrity by identifying prime numbers constituents of relevant humongous numbers using prime number spectrometer would have massive implication in that it has now brought the whole of e-commerce to its knees overnight.”

The truthfulness of the preceding narrative paragraph can now be beautifully refuted by us here as follows. Having solved Riemann hypothesis, Polignac’s and Twin prime conjectures is simply irrelevant because the CIS of nontrivial zeros and prime numbers must be treated as Incompletely Predictable entities abiding by Complex Elementary Fundamental Laws that usually involve Incompletely Predictable Laws [and not Simple Elementary Fundamental Laws that usually involve Completely Predictable Laws]. Thus the construction of prime number spectrometer is intuitively and literally a mathematical impossibility. We have thus dispelled the doom-and-gloom prophecy that financial disaster might follow when successful proof of Riemann hypothesis occur.
Without going into finer details using number theory, the irrationality measure (or irrationality exponent or approximation exponent or Liouville-Roth constant) of any real number is a measure of how "closely" it can be approximated by rationals. For a rational number, the irrationality measure is 1. The Thue-Siegel-Roth theorem states that for an algebraic irrational number, viz. real but not rational number, then the irrationality measure is 2. Transcendental irrational numbers have irrationality measure 2 or greater; for instance, the transcendental Euler's number e (= 2.718281828459...) has irrationality measure equal to 2. The [seemingly] simplistic-looking Liouville numbers is typified by Liouville's constant, sometimes also called Liouville's number, a real number defined by \[ L = \sum_{n=1}^{\infty} 10^{-n!} = 0.110001000000000000000001... \] with '!' denoting factorial. These numbers are irrational numbers of [the relatively more "complex"] transcendental types instead of [the relatively less "complex"] algebraic types; and their numerical make-up consist of just '0' and '1' digits. Despite this apparently simple-looking numerical make-up of Liouville numbers (as opposed to more complicated-looking numerical make-up of e), they are precisely those numbers [paradoxically] having infinite irrationality measure. For the above, we would assign all [Completely Predictable] rational numbers to obeying Simple Elementary Fundamental Laws, and all [Incompletely Predictable] irrational numbers to obeying Complex Elementary Fundamental Laws.

We now endeavor to compare, contrast and reconcile the two entities 'Living Things' and 'Nonliving Things'. Rigorous mathematical proofs must obviously be associated with 100% certainty. This can only apply to Simple and Complex Elementary Fundamental Laws on 'Nonliving Things'. Diverging onto proofs for Simple and Complex Emergent Fundamental Laws on 'Living Things', one observe that they can never be associated with perfect 100% certainty simply because we are dealing with "ALIVE" Living Things with dynamic spatial and temporal properties that could not be totally predictable. In this setting, the proofs for the Simple cases [e.g. physiologically modeling Cardiac Output (CO) equals to Heart Rate (HR) multiplied by Stroke Volume (SV) in the Cardiovascular System (CVS)] will comparatively be less challenging to derive than the Complex cases [e.g. physiologically modeling complex Human Brain functions using Neural Networks in the Central Nervous System (CNS)].

Note that the terms 'Elementary' and 'Emergent' are used here in the preceding and subsequent paragraphs to, respectively, denote 'Nonliving Things' and 'Living Things'. In real life situation for 'Living Things', there will always be the perpetual presence of infinitesimally tiny and unpredictable "Chaos and Fractals physiological variability", for instance, in the Simple Emergent Fundamental Law CO = HR X SR. This variability phenomenon will inevitably occur even in the most relaxed state of a person in deep sleep whereby dynamic processes such as intrinsic neuro-endocrine continuous signal input to the heart must occur on a permanent basis thus giving rise to this variability.

For the medically oriented readers, we finish off this topic by touching on Evidence based Medicine (EBM) and Evidence based Practice (EBP). Both could comply with either Simple or Complex Emergent Fundamental Laws on Living Things (namely, Human Beings in this scenario). EBM is typically depicted pictorially as a 'Pyramidal hierarchy of Literature Review' classifying available medical
research materials into [the most powerful] Systematic Reviews down to [the least powerful] Expert Opinion.

Then EBP = Clinician Experience + Patient Expectation + Best Practice; with Best Practice being roughly equated with EBM. For doctors and medical researchers confronted daily with responsibly following and improving up-to-date EBP and EBM, they must be familiar with most statistical tools employed in medical research with the classic example being research hypothesis expressed as a null hypothesis [the "devil’s advocate" position] and alternative hypothesis. The level of statistical significance for hypothesis testing is often expressed as the so-called p-value. Whilst there is relatively little justification why a [cut-off] significance level of 0.05 is widely used in academic research [rather than 0.01 or 0.10]; we could be particularly more confident in our results by setting a more stringent level of (say) 0.01 [a 1% chance or less; 1 in 100 chance or less]. Despite this experimental / research tactic, we could strive to, but never, achieve perfect or 100% confidence in our results by setting ever more stringent levels.

We outline the overlapping pure and applied mathematics on our rigorous proof for Riemann hypothesis in this paper only after more than 150 years from its initial proposal in 1859. Conforming to all logical arguments above, one can firmly believe that this lengthy delay is simply because Riemann zeta function contains an infinite number of Incompletely Predictable intercepts demonstrating Supraminimal Simplicity, or alternatively stated, contains none of the Completely Predictable infinite intercepts demonstrating Supramaximal Simplicity [whereby Supramaximal Simplicity does allow multiple type solutions to prove a particular conjecture]. This will then require a proviso that there is only one [solitary] way using the VCRM to solve this ‘Incompletely Predictable problem’ which belongs to the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution'.

Named after Danish mathematician Jørgen Pedersen Gram (June 27, 1850 – April 29, 1916), the ‘usual’ Gram points are the other conjugate pairs values on the critical line defined by Im{ζ(1/2 ± it)} = 0 whereby they obey Gram’s Rule and Rosser’s Rule with many other interesting characteristics. As a bonus, we additionally provide detailed explanations below on this Gram points (or Gram[y=0] points) and its closely related Gram[x=0] points.

The Z function is a function used for studying the Riemann zeta function along the critical line. It is also called the Riemann-Siegel Z function, the Riemann-Siegel zeta function, the Hardy function, the Hardy Z function, and the Hardy zeta function. It can be defined in terms of the Riemann-Siegel theta function and the Riemann zeta function by

\[ Z(t) = e^{i\theta(t)} \zeta(\frac{1}{2} + it) \]

whereby \( \theta(t) = \arg(\Gamma(\frac{1+it}{2})) - \frac{\log \pi}{4} t \). In the next paragraph, we will only outline a brief exposition of some of the useful properties of Gram points.

The algorithm used to compute \( Z(t) \) is called the Riemann-Siegel formula. The zeta function on the critical line, \( \zeta(\frac{1}{2} + it) \), will be real when \( \sin(\theta(t)) = 0 \). Positive real values of \( t \) where this occurs are called Gram points and can also be described as the points where \( \theta(t) \) is an integer. The real part of zeta function on the critical line tends to be positive, while the imaginary part alternates more regularly between positive and negative values. That means that the sign of \( Z(t) \) must be opposite to that of the sine function most of the time, so one would expect the nontrivial zeros of \( Z(t) \) to alternate with zeros of the sine term, i.e. when \( \theta \) takes on integer multiples of \( \pi \). This turns out to hold most of the time
and is known as Gram’s Rule (Law) - a law which is violated infinitely often though. Thus Gram’s Law is the statement that nontrivial zeros of \( Z(t) \) alternate with Gram points. Gram points which satisfy Gram’s Law are called 'good', while those that do not are called 'bad'. A Gram block is an interval such that its very first and last points are good Gram points and all Gram points inside this interval are bad. The exercise of counting nontrivial zeros then reduces to that of counting all Gram points where Gram’s Law is satisfied, and adding to that the count of nontrivial zeros inside each Gram block. With this process we do not have to locate nontrivial zeros exactly, and we just have to compute \( Z(t) \) accurately enough to show that it changes sign.

Up to this point, a crucial observation to note from the above is that Riemann zeta function (and its proxy Dirichlet eta function) will always generate an infinite number of relevant spirals/loops on which will be located all nontrivial zeros and 'usual' Gram points (and our Gram \([x=0]\) points) in a fixed relationship manner. Thus the afore-mentioned Gram's Law and its violation, Gram block, etc will predictably and periodically occur an infinite number of times.

Hadamard product:

\[
\zeta(s) = e^{\left(\log(2\pi) - \frac{1}{2} - \gamma\right) s} \prod_{\rho} \left(1 - \frac{s}{\rho}\right) e^{s/\rho}
\]

\[
= \pi^{\frac{s}{2}} \frac{\prod_{\rho} \left(1 - \frac{s}{\rho}\right)}{2(s-1) \Gamma\left(1 + \frac{s}{2}\right)}
\]

Euler product formula:

\[
\sum_{n=1}^{\infty} \frac{1}{n^s} \quad \{\text{which is } \zeta(s)\}
\]

\[
= \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}
\]

\[
= \frac{1}{(1 - 2^{-s}) \cdot (1 - 3^{-s}) \cdot (1 - 5^{-s}) \cdot (1 - 7^{-s}) \cdot (1 - 11^{-s}) \cdots} \frac{1}{(1 - p^{-s}) \cdots}
\]

The beautiful Hadamard product above is the infinite product expansion of Riemann zeta function, \( \zeta(s) \), based on Weierstrass’s factorization theorem – this product simultaneously contains both trivial and nontrivial zeros. The beautiful Euler product formula above connects Riemann zeta function and prime numbers and was discovered by Euler – this identity has, by definition, the left hand side being \( \zeta(s) \) and the infinite product on the right hand side extends over all prime numbers \( p \). The form of the Hadamard product clearly displays the simple pole at \( s = 1 \), the trivial zeros at all even negative integers due to the gamma function term in the denominator, and the nontrivial zeros at \( s = \rho \); with the letter \( \gamma \) in the expansion here specifically denoting the Euler-Mascheroni constant. Note that with the second simpler infinite product expansion formula of Hadamard, to ensure convergence, the product should be taken over "matching pairs" of zeroes, i.e. the factors for a pair of zeroes of the form \( \rho \) and \( 1 - \rho \) should be combined.

The usual primary by-products arising out of the rigorous proof for Riemann hypothesis are often stated as "With this one solution, we have proven five hundred
theorems or more at once”. This apply to the many important theorems in number theory (mostly about prime numbers) that rely on properties of Riemann zeta or Dirichlet eta functions such as where trivial and nontrivial zeros are, and are not, located. A classical example of this primary by-product is the resulting absolute and full delineation of the prime number theorem, which relates to prime counting function. This function is usually denoted by \( \pi(x) \) and is defined as the number of prime numbers less than or equal to \( x \). In mathematics, the logarithmic integral function or integral logarithm \( \text{li}(x) \) is a special function. It is relevant in problems of physics and has number theoretic significance, occurring in prime number theorem as an estimate of the number of prime numbers less than a given value. In prime number theorem, the form of this function is defined so that \( \text{li}(2) = 0 \); viz. \( \text{li}(x) = \int_2^x \frac{du}{\ln u} = \text{li}(x) - \text{li}(2) \). The symbol ‘\( \ln \)’ here denotes natural logarithm. Thus the rigorous proof for Riemann hypothesis on nontrivial zeros location at \( \sigma = \frac{1}{2} \), together with the negative even number locations for trivial zeros, is instrumental in proving the efficacy of techniques that estimate \( \pi(x) \) efficiently and reasonably well. In particular, our rigorous proof of Riemann hypothesis will now confirm the ”best possible” bound for the error (the ”smallest possible” error) of prime number theorem.

We mention here that there are other less accurate ways of estimating \( \pi(x) \) such as that conjectured by Gauss and Legendre at the end of the 18th century. This is approximately \( x/\ln x \) in the sense \( \lim_{x \to \infty} \frac{\pi(x)}{x/\ln x} = 1 \). For prime counting function, other functions more convenient to work with can also be utilized and they open up a whole new world of marvelous mathematical relationships. An example is Riemann prime counting function (aka prime power counting function), commonly denoted by \( J(x) \). This non-infinite series function has jumps of \( 1/n \) for prime powers \( p^n \), and with it taking a value halfway between the two sides at discontinuities. Amazingly, the prime counting function \( \pi(x) \) is related to \( J(x) \) by the Mobius transform. More amazingly still, \( J(x) \) is related to Riemann zeta function through the Mellin transform (which is an integral transform).

In number theory, Skewes’ number is any of several extremely large numbers used by South African mathematician Stanley Skewes as upper bounds for the smallest natural number \( x \) for which \( \text{li}(x) < \pi(x) \). These bounds have since been improved by others; there is a crossing near \( e^{727.95133} \). It is not known whether it is the smallest. John Edensor Littlewood, who was Skewes’ research supervisor, had proved in 1914 [10] that there is such a number (and so, a first such number); and indeed found that the sign of the difference \( \pi(x) - \text{li}(x) \) changes infinitely often. This then refute all prior numerical evidence available that seem to suggest \( \text{li}(x) \) was always more than \( \pi(x) \). The massive key point here is that the [100% accurate] \( \pi(x) \) mathematical tool being ”wrapped around” by the [less-than-100% accurate] approximate mathematical tool \( \text{li}(x) \) infinitely often via this ‘sign of difference’ changes meant that \( \text{li}(x) \) must be the most efficient approximate mathematical tool. Contrast this with the ”crude” \( x/\ln x \) approximate mathematical tool where values obtained diverge away from \( \pi(x) \) at increasingly greater rate when larger range of prime numbers are being studied.

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References


Appendix 1. Prerequisite lemma, corollary and propositions for Gram[x=0] and Gram[y=0] conjectures

For the mathematical treatment of our two cases on Gram[x=0] and Gram[y=0] points here, we will follow a similar procedure carried out for the above case on nontrivial zeros (Gram[x=0,y=0] points).

Lemma [Appendix] 1.1. The \{Modified-for-Gram[x=0] & [y=0] points\}-Riemann-Dirichlet Ratio can be derived from Riemann zeta or Dirichlet eta function with absolute ability to incorporate the actual presence [but not the actual locations] of the complete set of Gram[x=0] and Gram[y=0] points.

Proof. We hereby depict the case on Gram[y=0] points (which is the usual ‘Gram points’) to obtain the relevant [simply named] \{Modified-for-Gram points\}-Riemann-Dirichlet Ratio. Apply \(n^2\) (Euler) to Eq. (3), we have \(\zeta(s) = \gamma.\eta(s) = \gamma.[Re\{\eta(s)\} + i.\text{Im}\{\eta(s)\}]\) whereby

\[
\text{Re}\{\eta(s)\} = \sum_{n=1}^{\infty} \left( (2n-1)^{-\sigma}.\cos(t.\log(2n-1)) - (2n-1)^{-\sigma}.\cos(t.\log(2n)) \right) \quad \text{and}
\]

\[
\text{Im}\{\eta(s)\} = t. \sum_{n=1}^{\infty} \left( (2n)^{-\sigma}.\sin(t.\log(2n)) - (2n-1)^{-\sigma}.\sin(t.\log(2n)) \right) \quad (25)
\]

Here \(\gamma\) is the proportionality factor \(\frac{1}{\pi^2}\).

As Gram[y=0] points based on \(\zeta(s)\) is identical to that based on its proxy \(\eta(s)\), then Gram[y=0] conjecture is satisfied when

\[
\sum \text{Re}\text{Im}\{\eta(s)\} = \text{Re}\{\eta(s)\} + 0, \text{ or simply } \text{Im}\{\eta(s)\} = 0 \quad (26)
\]
Applying Eq. (26) to Eq. (25), this equation can be simplified and be reduced to

\[
\sum_{n=1}^{\infty} ((2n)^{-\sigma} \sin(t \log(2n)) - (2n-1)^{-\sigma} \sin(t \log(2n))) = 0
\]

\[
\sum_{n=1}^{\infty} (2n)^{\sigma} \sin(t \log(2n)) = \sum_{n=1}^{\infty} ((2n-1)^{-\sigma} \sin(t \log(2n)))
\]  

(27)

We note from the above sequential mathematical derivation of Eq. (27) that this equation will completely and intrinsically fulfills the ‘presence of the complete set of Gram[y=0] points without knowing their actual location’ criteria.

\[
\sum_{n=1}^{\infty} \sin(t \log(2n)) = \sum_{n=1}^{\infty} \sin(t \log(2n-1))
\]

(28)

Eq. (28) above will also abide to this specified criteria as it is simply the result of rearranging the terms in Eq. (27) thus giving rise to our desired {Modified-for-Gram points}-Riemann-Dirichlet Ratio. This proof is now complete for Lemma [Appendix] 1.1.

Denote the left hand side ratio as Ratio R1 (of a ‘cyclical’ nature) and the right hand side ratio as Ratio R2 (of a ‘non-cyclical’ nature). The {Modified-for-Gram points}-Riemann-Dirichlet Ratio calculations, valid for all continuous real number values of t, would theoretically result in infinitely many non-Hybrid integer sequences [here arbitrarily] for the 0 < \sigma < 1 critical strip region of interest with n = 1, 2, 3, ..., \infty being discrete integer number values, or n being continuous real numbers from 1 to \infty with Riemann integral applied in the interval from 1 to \infty. This infinitely many integer sequences can geometrically be interpreted to representatively cover the entire plane of the critical strip bounded by \sigma values of 0 and 1, thus (at least) allowing our proposed proof on Gram[y=0] conjecture to be of a ‘complete’ nature.

**Proposition [Appendix] 1.2.** The equivalent Sigma-Power Laws can be rigorously derived from {Modified-for-Gram[x=0] & [y=0] points}-Riemann-Dirichlet Ratio.

**Proof.** We hereby depict the case on Gram[y=0] points (which is the usual ‘Gram points’) to obtain its equivalent Sigma-Power Laws. We apply Riemann integral to the four continuous functions of Ratio R1 and Ratio R2 in Eq. (28) thus depicting the {Modified-for-Gram points}-Riemann-Dirichlet Ratio in the integral forms – see the subsequent Eq. (33) below.

Thereafter, step-by-step we derive the closely related {Modified-for-Gram points}-Dirichlet \(\sigma\)-Power Law [expressed in real numbers] and the {Modified-for-Gram points}-Riemann \(\sigma\)-Power Law [expressed in real and complex numbers] – these two laws are further elaborated below. The {Modified-for-Gram points}-Sigma-Power Law has its Dirichlet and Riemann versions directly related to each other via Dirichlet \(\eta(s)\) being the equivalence of Riemann \(\zeta(s)\) but without the \(\frac{1}{1-2^s}\) proportionality factor. We stress that it is the main underlying mathematically-consistent properties of symmetry and constraints arising from this power law that also allowed our most direct, basic and elementary proof for the Gram[y=0] conjecture to mature. An important characteristic to note of {Modified-for-Gram points}- \(\sigma\)-Power Law is that its exact formula expression in the usual mathematical language \(y = f(x_1, x_2)\) format description for a 2-variable function] consists
of \( y = \{2n\} \) or \( \{2n-1\} = f(t, \sigma) \) with \( n = 1, 2, 3, \ldots, \infty \) or \( n = 1 \) to \( \infty \) with Riemann integral application; \(-\infty < t < +\infty; \) and \( \sigma \) being of real number values \( 0 < \sigma < 1 \) corresponding to the [arbitrarily defined] critical strip of interest in this particular case scenario.

For the, initially, \( \{2n\} \) parameter integration of R1, \( \int_{1}^{\infty} \sin(t \cdot \log(2n)).dn \)

Use integration by u-substitution technique to obtain \( u = t \cdot \log(2n), \) \( n = \frac{1}{2}e^{\frac{1}{u}}, \) \( \frac{du}{dn} = \frac{2n}{t} = \frac{1}{t}, \) \( du = t \cdot \frac{dn}{n}, \) \( dn = 2n \cdot \frac{du}{2t} = n \cdot \frac{du}{t} \)

\[
\int_{1}^{\infty} \sin(u).\frac{n}{u}.du = \int_{1}^{\infty} \sin(u).\frac{1}{t} \cdot e^{\frac{1}{u}}.du = \frac{1}{2t} \int_{1}^{\infty} \sin(u).e^{\frac{1}{u}}.du
\]

Use the Products of functions proportional to their second derivatives, namely the indefinite integral \( \int \sin(a u).e^{b u} du = \frac{e^{b u}}{a^2 + b^2} (b \cdot \sin(a u) - a \cdot \cos(a u)) + C \) (Comparatively, we observe that \( \int \cos(a u).e^{b u} du = \frac{e^{b u}}{a^2 + b^2} (b \cdot \cos(a u) + a \cdot \sin(a u)) + C \).

Then \( a = 1, b = \frac{1}{t}, \) and temporarily ignore the \( \frac{1}{2t} \) term, we have

\[
\int_{1}^{\infty} \sin(u).e^{\frac{1}{u}}.du = \left[ \frac{e^{\frac{1}{u}}}{(1 + \frac{1}{t^2})} - \frac{1}{t} \cdot \sin(u) - \cos(u) \right]_{1}^{\infty} + C
\]

Now apply the non-linear combination of sine and cosine functions identity, namely \( a \cdot \sin(u) + b \cdot \cos(u) = c \cdot \sin(u + \varphi) \) where \( c = \sqrt{a^2 + b^2} \) and \( \varphi = a \tan(b, a) \).
Here \( a = 1, b = -1, c = \sqrt{(\frac{1}{2})^2 + 1} = \sqrt{(\frac{1}{4} + 1)} = \sqrt{(\frac{1}{4} + 1)}. \) Then we have

\[
\int_{1}^{\infty} \frac{\sin(u)\cdot e^{\frac{1}{u}}.du}{(t^2 + 1)} = \left[ \frac{(t^{2 + 1})}{(t^2 + 1)} \cdot \sin(u + atan2(b, a)) \right]_{1}^{\infty}
\]

But there was a \( \frac{1}{2t} \) term in front of this integral as can be seen above. Then after substituting this term and simplifying, the integral

\[
\int_{1}^{\infty} \sin(u).e^{\frac{1}{u}}.du = \left[ \frac{e^{\frac{1}{u}}}{2\sqrt{(t^2 + 1)}} \cdot \sin(u - arctan(t)) \right]_{1}^{\infty}
\]

But \( u = t \cdot \log(2n) \). Reverting back to the \( n \) variable, the equation for the \( \{2n\} \) parameter finally becomes

\[
\int_{1}^{\infty} \sin(t \cdot \log(2n)).dn = \left[ \frac{(2n) \cdot e^{\frac{1}{t}}}{2\sqrt{(t^2 + 1)}} \cdot \sin(t \cdot \log(2n) - arctan(t)) \right]_{1}^{\infty}
\]

(29)

In a similar manner integration for the \( \{2n-1\} \) parameter, this equation becomes

\[
\int_{1}^{\infty} \sin(t \cdot \log(2n-1) - arctan(t)) + C \]_{1}^{\infty}
\]

(30)
In R2 using \(\{2n\}\) parameter,

\[
\int_1^\infty (2n)^\sigma dn = \frac{1}{3} (2n)^{\frac{\sigma}{2}} + C \bigg|_1^\infty \quad \text{when } \sigma = \frac{1}{2} \tag{31}
\]

For the equivalent R2 based on \(\{2n-1\}\) parameter,

\[
\int_1^\infty (2n-1)^\sigma dn = \frac{1}{3} (2n-1)^{\frac{\sigma}{2}} + C \bigg|_1^\infty \quad \text{when } \sigma = \frac{1}{2} \tag{32}
\]

The Ratio R1 and Ratio R2 of \(\{\text{Modified-for-Gram points}\}\)-Riemann-Dirichlet Ratio (for \(\sigma = \frac{1}{2}\)) is defined by the integral

\[
\frac{[(2n) \cdot e^{\frac{t}{2}}/2\sqrt{(t^2 + 1)} \cdot \sin(t \cdot \log(2n) - \arctan(t))]}_1^\infty}{[(2n-1) \cdot e^{\frac{t}{2}}/2\sqrt{(t^2 + 1)} \cdot \sin(t \cdot \log(2n-1) - \arctan(t))]}_1^\infty
= \frac{[\frac{1}{3} (2n)^{\frac{1}{2}}]}{[\frac{1}{3} (2n-1)^{\frac{1}{2}}]} \quad \text{← this is R1}
\]

Canceling out the common parameter \(\{2n\}\) and \(\{2n-1\}\) terms,

\[
\frac{[e^{\frac{t}{2}}/2\sqrt{(t^2 + 1)} \cdot \sin(t \cdot \log(2n) - \arctan(t))]}_1^\infty}{[(2n-1)^{\frac{1}{2}}]}_1^\infty
= \frac{[\frac{1}{3} (2n)^{\frac{1}{2}}]}{[\frac{1}{3} (2n-1)^{\frac{1}{2}}]} \quad \text{← this is R2} \tag{33}
\]

The \(\{\text{Modified-for-Gram points}\}\)-Dirichlet and \(\{\text{Modified-for-Gram points}\}\)-Riemann \(\sigma\)-Power Laws are given by the exact formulae in Eqs. (34) to (37) below with \(\psi\) being the same proportionality constant valid for both power laws. We can now dispense with the constant of integration C. Using Dimensional analysis (DA) approach we can easily conclude that the 'fundamental dimension' [Variable / Parameter / Number X to the power of Number Y] has to be represented by the particular 'unit of measure' [Variable / Parameter / Number X to the power of Number Y] whereby Number Y needs to be of the specific value \(\frac{1}{2}\) for DA homogeneity to occur. This de novo DA homogeneity equates to the location of the complete set of Gram[\(y=0\)] points and is crucially a fundamental property present in all laws of Physics. The 'unknown' \(\sigma\) variable, now endowed with the value of \(\frac{1}{2}\), is treated as Number Y.

\(\{\text{Modified-for-Gram points}\}\)-Dirichlet \(\sigma\)-Power Law using the \(\{2n\}\) parameter:

\[
[(2n) \cdot e^{\frac{t}{2}} / 2(t^2 + 1)^{\frac{1}{2}} \cdot \sin(t \cdot \log(2n) - \arctan(t))]}_1^\infty = \psi \cdot \frac{1}{3} (2n)^{\frac{1}{2}} \bigg|_1^\infty
\]
With the common parameter \{2n\} canceling out on both sides, the equation reduces to

\[
\frac{e^{\frac{t}{2}}}{{2(t^2 + 1)^\frac{1}{2}}} \cdot \sin(t \cdot \log(2n) - \arctan(t)) - \psi \frac{1}{3} (2n)^{\frac{1}{2}}_1 = 0
\]

Similarly for the \{2n-1\} parameter, this equivalent equation is

\[
\frac{e^{\frac{t}{2}}}{{2(t^2 + 1)^\frac{1}{2}}} \cdot \sin(t \cdot \log(2n-1) - \arctan(t)) - \psi \frac{1}{3} (2n-1)^{\frac{1}{2}}_1 = 0
\]

Finally, the \{Modified-for-Gram\} Riemann \(\sigma\)-Power Law is given by the exact formulae using \{2n\} and \{2n-1\} parameters with the \(\gamma = \frac{2\frac{t}{2} \cdot \cos(t \cdot \log(2) + i \cdot \sin(t \cdot \log(2)))}{(2\frac{t}{2} (\cos(t \cdot \log(2) + i \cdot \sin(t \cdot \log(2)) - 2) \cdot \frac{1}{3} (2n)^{\frac{1}{2}}_1)^\infty}

\[
\left[ \frac{e^{\frac{t}{2}}}{{2(t^2 + 1)^\frac{1}{2}}} \cdot \sin(t \cdot \log(2n) - \arctan(t)) - \psi \cdot \frac{1}{3} (2n)^{\frac{1}{2}}_1 \right]_{1}^{\infty} = 0
\]

\[
\left[ \frac{e^{\frac{t}{2}}}{{2(t^2 + 1)^\frac{1}{2}}} \cdot \sin(t \cdot \log(2n-1) - \arctan(t)) - \psi \cdot \frac{1}{3} (2n-1)^{\frac{1}{2}}_1 \right]_{1}^{\infty} = 0
\]

The proof is now complete for Proposition [Appendix] 1.2.

**Proposition [Appendix] 1.3.** Application of Dimensional analysis homogeneity to the equivalent Sigma-Power Laws will always be associated with the one specific \(\sigma = \frac{1}{2}\) value for Gram[x=0] & Gram[y=0] points and will enable the rigorous proofs for Gram[x=0] & Gram[y=0] conjectures to mature.

**Proof.** We again depict the case on Gram[y=0] points here. We note the \(\gamma\) proportionality factor given by Eq. (13) above when depicted with the \(2\frac{t}{2}\) constant numerical value (derived using \(\sigma = \frac{1}{2}\) as proposed in the original Gram[y=0] conjecture) further allowing, and enabling, de novo Dimensional analysis homogeneity compliance in the \{Modified-for-Gram points\} Riemann \(\sigma\)-Power Law in Eqs. (36) and (37) above. There is only one type of \(\frac{1}{2}\) exponent present in \{Modified-for-Gram points\} Riemann \(\sigma\)-Power Law indicating Dimensional analysis homogeneity. This two mathematical statements essentially complete the proof for Proposition [Appendix] 1.3 with complimentary demonstration below for the Dimensional analysis non-homogeneity case scenario.

**Corollary [Appendix] 1.4.** Application of Dimensional analysis non-homogeneity to Sigma-Power Laws will never be associated with the one specific \(\sigma = \frac{1}{2}\) value for Gram[x=0] & Gram[y=0] points and will enable the rigorous proofs for Gram[x=0] & Gram[y=0] conjectures to mature.
Proof. We again depict the case on Gram\([y=0]\) points here. We illustrate the Dimensional analysis non-homogeneity property for a \(\sigma = \frac{1}{4}\) arbitrarily chosen value [clear-cut case with \(\{2n\}\)-parameter] of \{Modified-for-Gram points\}-Riemann \(\sigma\)-Power Law lying on a non-critical line (with total absence of Gram\([y=0]\) points) in the following formula derived using Eqs. (13) and (36). As Ratio \(R1\) component of \{Modified-for-Gram points\}-Riemann-Dirichlet Ratio is independent of \(\sigma\) variable, unlike the Ratio \(R2\) component of \{Modified-for-Gram points\}-Riemann-Dirichlet Ratio and the \(\gamma\) proportionality factor which are dependent on \(\sigma\) variable, we now note the mixture of \(\frac{1}{4}\) and \(\frac{1}{2}\) exponents subtly, but nevertheless, present in this formula confirming Dimensional analysis non-homogeneity. Also the replacement of \(\frac{1}{3}\) fraction with \(\frac{2}{5}\) fraction [derived from substituting \(\sigma = \frac{1}{4}\) into \(\frac{1}{2}(\sigma+1)\)] has oc- curred. Mathematically, this Dimensional analysis non-homogeneity property for any real number value of \(\sigma\), when \(\sigma \neq \frac{1}{2}\) and \(0 < \sigma < 1\), will always be present indicative of the full presence of \{Non-critical lines\}-Gram\([y=0]\) points, or by the same token, indicative of total absence of Gram\([y=0]\) points.

\[
\left[ \frac{e^{\frac{1}{4}}}{2(t^2+1)^{\frac{1}{2}}}, \sin(t \log(2n) - \arctan(t)) - \psi, \frac{2^{\frac{1}{4}} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^{\frac{1}{4}} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2)) - 2)} \right]_{1}^{\infty} = 0
\]

The proof is now complete for Corollary \([Appendix\] 1.4.\[\Box\].

The \(\frac{1}{2}\) exponent in Eq. (38) only occur once in the denominator of the first term. The subtlety of Dimensional analysis non-homogeneity for \{Non-critical lines\}-Gram\([y=0]\) points is even more pronounced when compared to its closely related cousin Eq. (18) above for Riemann \(\sigma\)-Power Law [with easy clarification and confirmation of the \(\frac{1}{2}\) exponent occurring twice in the first term].

For Gram\([x=0]\) points, Gram\([x=0]\) conjecture is satisfied by Eqs. (39) to (41) below, whereby Eq. (39) is the equivalent of Eq. (26) above.

\[
\sum Re \{\eta(s)\} = 0 + Im \{\eta(s)\}, \text{ or simply } Re \{\eta(s)\} = 0 \quad (39)
\]

Not unexpectedly with only minor subtraction (-) operator to addition (+) operator sign change required, the equivalent to Eq. (36) and Eq. (38) above using \(\{2n\}\) parameter for Gram\([x=0]\) points can easily be derived to (respectively) be:

\[
\left[ \frac{e^{\frac{1}{4}}}{2(t^2+1)^{\frac{1}{2}}}, \sin(t \log(2n) + \arctan(t)) - \psi, \frac{2^{\frac{1}{4}} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^{\frac{1}{4}} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2)) - 2)} \right]_{1}^{\infty} = 0
\]

\[
\left[ \frac{e^{\frac{1}{4}}}{2(t^2+1)^{\frac{1}{2}}}, \sin(t \log(2n) + \arctan(t)) - \psi, \frac{2^{\frac{1}{4}} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2))) - 2)}{2^{\frac{1}{4}} \cdot (\cos(t \log(2) + i \cdot \sin(t \log(2)) - 2)} \right]_{1}^{\infty} = 0
\]

Dimensional analysis homogeneity and non-homogeneity are demonstrated once again by Eq. (40) and Eq. (41) respectively for the Gram\([x=0]\) points case scenario.
Fig. 8 Even-Odd mathematical (graphed) landscape using data obtained for x = 64.

Appendix 2. Tabulated and graphical depictions on Even-Odd mathematical landscape for x = 64

In Figure 8, Dimensions 2x - 2 & 2x - 4 are symbolically represented by -2 & -4 with 2x - 4 displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must reset itself onto this (Grand-Total Gaps) 'baseline' Dimension after the initial Dimension 2x - 2 on a permanent basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by the two negative integers are Completely Predictable with both Even-π(x) and Odd-π(x) becoming larger at a constant rate. We note that there is a complete absence of Chaos & Fractals phenomena being manifested in our graph.

The definitive derivation of the data in Table 7 is given next and this is clearly illustrated by two examples given for position x = 31 & 32. For i & x \(\in\) 1, 2, 3, ..., \(\infty\): \(\Sigma EO_i\)-Gap = \(\Sigma EO_{i-1}\)-Gap + Gap value at \(E_{i-1}\) or Gap value at \(O_{i-1}\) whereby (i) \(E_i\) or \(O_i\) at position \(x\) is determined by whether the relevant \(x\) value belongs to an even (E) or odd (O) number, and (ii) both \(\Sigma EO_1\)-Gap and \(\Sigma EO_2\)-Gap = 0. Example for position x = 31: 31 is an odd number (O16). Our desired Gap value at O15 = 2. Thus \(\Sigma EO_{31}\)-Gap (58) = \(\Sigma EO_{30}\)-Gap (56) + Gap value at O15 (2). Example 2 for position x = 32: 32 is an even number (E16). Our desired Gap value at E15 = 2. Thus \(\Sigma EO_{32}\)-Gap (60) = \(\Sigma EO_{31}\)-Gap (58) + Gap value at E15 (2).

Using the relevant data above, we have now painstakingly tabulate (in Table 7) and graphically map (in Figure 8) the [Completely Predictable] Even-Odd mathematical landscape for x = 64. Legend: E = even, O = odd. Involved Dimensions are 2x - 2 & 2x - 4 with Y denoting Dimension 2x - 4 for visual clarity. This Even-Odd mathematical landscape, made up of Dimension 2x - 4 (except for the very first and only Dimension 2x - 2), will intrinsically incorporate even and odd numbers in an integrated manner. Except for the very first odd number, we note that all Completely Predictable even and odd numbers, and all their identical gaps, can be represented by the countable finite set of [single] Dimension 2x - 4.
Table 7  Even-Odd mathematical (tabulated) landscape using data obtained for $x = 64$.  

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