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Integrated Framework of Optimization Technique and Information Theory Measures for Modeling Neutrosophic Variables

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Abstract

Uncertainty and indeterminacy are two major problems in data analysis these days. Neutrosophy is a generalization of the fuzzy theory. Neutrosophic system is based on indeterminism and falsity of concepts in addition to truth degrees. Any neutrosophy variable or concept is defined by membership, indeterminacy and non-membership functions. Finding efficient and accurate definition for neutrosophic variables is a challenging process. This paper presents a framework of Ant Colony Optimization and entropy theory to define a neutrosophic variable from concrete data. Ant Colony Optimization is an efficient search algorithm presented to define parameters of membership, indeterminacy and non-membership functions. The integrated framework of information theory measures and Ant Colony Optimization is proposed. Experimental results contain graphical representation of the membership, indeterminacy and non-membership functions for the temperature variable of the forest fires data set. The graphs demonstrate the effectiveness of the proposed framework.

Keywords

Neutrosophic set, Ant Colony Optimization, Information Theory Measures, Entropy function.

1. Introduction

These days, Indeterminacy is the key idea of the information in reality issues. This term alludes to the obscure some portion of the information representation. The fuzzy logic[1][2][3]] serves the piece of information participation degree. Thus, the indeterminacy and non-participation ideas of the information ought to be fittingly characterized and served. The neutrosophic [4][16] theory characterizes the informational index in mix with their membership, indeterminacy and non-membership degrees. Thus, the decisions could be practically figured out from these well defined information.

Smarandache in [5][13][14], and Salama et al. in[4], [9],[10][11][12][12][16] present the mathematical base of neutrosophic system and principles of neutrosophic data. Neutrosophy creates the main basics for a new mathematics field through adding indeterminacy concept to traditional and fuzzy theories[1][2][3][15].

Handling neutrosophic system is a new, moving and appealing field for scientists. In literature, neutrosophic toolbox implementation using object oriented programming operations and formulation is introduced in[18]. Moreover, a data warehouse utilizing neutrosophic methodologies and sets is applied in [17]. Also, the problem of optimizing membership functions using Particle Swarm Optimization was introduced in [24]. This same mechanism could be generalized to model neutrosophic variable.

The neutrosophic framework depends actually on the factors or variables as basics. The neutrosophic variable definition is without a doubt the base in building a precise and productive framework. The neutroshophic variable is made out of a tuple of value, membership, indeterminacy and non-membership. Pronouncing the elements of participation, indeterminacy and non-enrollment and map those to the variable values would be an attainable arrangement or solution for neutroshophic variable formulation.

Finding the subsets boundary points of membership and non-membership functions within a variable data would be an interesting optimization problem. Ant Colony Optimization (ACO)[19][20] is a meta-heuristic optimization and search procedure[22] inspired by ants lifestyle in searching for food. ACO initializes a population of ants in the search space traversing for their food according to some probabilistic transition rule. Ants follow each other basing on rode pheromone level and ant desirability to go through a specific path. The main issue is finding suitable heuristic desirability which should be based on the information conveyed from the variable itself. Information theory measures [6][20][21], [23] collect information from concrete data. The entropy definition is the measure of information conveyed in a variable. Whereas, the mutual information is the measure of data inside a crossing point between two nearby subsets of a variable. These definitions may help in finding limits of a membership function of neutrosophic variable subsets depending on the probability distribution of the data as the heuristic desirability of ants.

In a similar philosophy, the non- membership of a neutrosophic variable might be characterized utilizing the entropy and mutual information basing on the data probability distribution complement. Taking the upsides of the neutrosophic set definition; the indeterminacy capacity could be characterized from the membership and non-membership capacities.

This paper exhibits an incorporated hybrid search model amongst ACO and information theory measures to demonstrate a neutrosophic variable. The rest of this paper is organized as follows. Section 2 shows the hypotheses and algorithms. Section 3 announces the proposed integrated framework. Section 4 talks about the exploratory outcomes of applying the framework on a general variable and demonstrating the membership, indeterminacy and non-membership capacities. Conclusion and future work is displayed in section 5.

2. Theory overview

2.1 Parameters of a neutrosophic variable

In the neutrosophy theory[5][13][14], every concept is determined by rates of truth $\mu_A(x)$, indeterminacy $\sigma_A(x)$, and negation $\nu_A(x)$ in various partitions. Neutrosophy is a generalization of the fuzzy hypothesis[1][2][3]] and an extension of the regular set. Neutrosophic is connected to concepts identified with indeterminacy. Neutrosophic data is defined by three main concepts to manage uncertainty. These concepts are joined together in the triple:

$$A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle \qquad (1)$$

Where

$\mu_A(x)$ is the membership degree,

$\sigma_A(x)$ is the indeterminacy degree,

$\nu_A(x)$ is the falsity degree.

These three terms form the fundamental concepts and they are independent and explicitly quantified. In neutrosophic set [7], each value $x \in X$ in set A defined by Eq. 1 is constrained by the following conditions:

$$0^- \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^+ \qquad (2)$$

$$0^- \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+ \quad (3)$$

Whereas, Neutrosophic intuitionistic set of type 1 [8] is subjected to the following:

$$0^- \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^+ \quad (4)$$

$$\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \leq 0.5 \quad (5)$$

$$0^- \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+ \quad (6)$$

Neutrosophic intuitionistic set of type 2 [5] is obliged by to the following conditions:

$$0.5 \leq \mu_A(x), \sigma_A(x), \nu_A(x) \quad (7)$$

$$\mu_A(x) \wedge \sigma_A(x) \leq 0.5, \ \mu_A(x) \wedge \nu_A(x) \leq 0.5, \ \sigma_A(x) \wedge \nu_A(x) \leq 0.5 \quad (8)$$

$$0^- \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 2^+ \quad (9)$$

2.2 Ant Colony Optimization (ACO)

The ACO [19][20]is an efficient search algorithm used to find feasible solutions for complex and high dimension problems. The intelligence of the ACO is based on a population of ants traversing the search workspace for their food. Each ant follows a specific path depending on information left previously from other ants. This information is characterized by the probabilistic transition rule Eq. 10.

$$p_j^m(t) = \frac{[\eta_j] \times [\tau_{ij}(t)]}{\sum_{i \in I_m} [\eta_i] \times [\tau_{ij}(t)]} \quad (10)$$

Where

η_j is the heuristic desirability of choosing node j and

τ_{ij} is the amount of virtual pheromone on edge (i, j)

The pheromone level guides the ant through its journey. This guide is a hint of the significance level of a node (exhibited by the ants went to the nodes some time recently). The pheromone level is updated by the algorithm using the fitness function.

$$\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \Delta\tau_{ij}(t) \quad (11)$$

Where 0< ρ <1 is a decay constant used to estimate the evaporation of the pheromone from the edges. $\Delta\tau_{ij}(t)$ is the amount of pheromone deposited by the ant.

The heuristic desirability η_j describes the association between a node j and the problem solution or the fitness function of the search. If a node has a heuristic value for a certain path then the ACO will use this node in the solution of the problem. The algorithm of ACO is illustrated in figure 1.

$$\eta_j = objective\ function \quad (12)$$

ACO Algorithm
Input :pd, N
%%%% pd number of decision variables in ant, N iterations, Present position (ant) in the search universe X_{id} , ρ evaporation rate, %%%%%%%
Output: Best_Solution
1: Initianlize_Node_Graph();
2: Initialize_Phermoni_Node();
3: While (num_of_Iterations>0) do
4: foreach Ant
5: η_j ***← objective function of the search space***
6: TRANSITION_RULE[j]= $p_j^m(t) = \frac{[\eta_j] \times [\tau_{ij}(t)]}{\sum_{i \in I_m} [\eta_i] \times [\tau_{ij}(t)]}$
7: Select node with the highest $p_j^m(t)$
8: Update Pheromone level $\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \Delta\tau_{ij}(t)$
9: num_of_Iterations--;
10: end While
11:Best_sol← solution with best η_j
12: output(Best_sol)

Figure 1: Pseudo code of ant colony optimization Algorithm

2.3 Entropy and Mutual Information

Information theory measures [6][20][23] collect information from raw data. The entropy of a random variable is a function which characterizes the unexpected events of a random variable. Consider a random variable X expressing the number on a roulette wheel or the number on a fair 6-sided die.

$$H(X) = \sum_{x \in X} -P(x) \log P(x) \qquad (13)$$

Joint entropy is the entropy of a joint probability distribution, or a multi-valued random variable. For example, consider the joint entropy of a distribution of mankind (X) defined by a characteristic (Y) like age or race or health status of a disease.

$$I(X;\ Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \qquad (14)$$

3. The proposed frame work

An Integrated hybrid model of ACO and information theory measures (entropy and mutual information) as the objective function is presented. The ACO[19][20] is a heuristic searching algorithm used to locate the ideal segments of the membership and non-membership functions of a neutrosophic variable. The indeterminacy function is calculated by the membership and non-membership functions basing on the definitions of neutrosophic set illustrated in section 2. The objective function is the amount of information conveyed from various partitions in the workspace. Therefore, the total entropy [21] is used as the objective function on the variables workspace. Total entropy calculates amount of information of various partitions and intersections between these partitions. Best points in declaring the membership function are the boundaries of the partitions. The ants are designed to form the membership and non-membership partitions as illustrated in figure 2. A typical triangle membership function would take the shape of figure 2.

The triangle function of a variable partition is represented by parameters (L, (L+U)/2, U). Finding best values of L and U for all partitions would optimize the membership (non-membership) function definition. Figure 3 give a view of the ant with n partitions for each fuzzy variable.

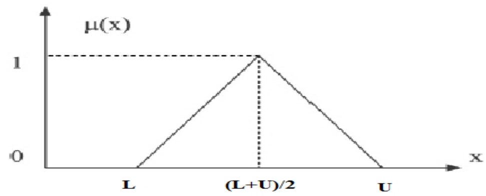

Figure 2 : corresponding to triangle fuzzy membership and its boundary parameters

Individual	L_1	U_1	L_2	U_2	……	L_n	U_n

Figure 3: Individual in ACO for Triangle function

One of the main difficulties in designing optimization problem using ACO is finding the heuristic desirability which formulates the transition rule. The amount of information deposited by neutrosophic variable inspires the ACO to calculate the transition rule and find parameters of membership, indeterminism and non-membership declarations. The membership function subsets are declared by ant parameters in figure 2. The histogram of a variable shows the data distribution of the different values. Therefore, the set of parameters are mapped to the histogram of a given variable data (Fig. 4).

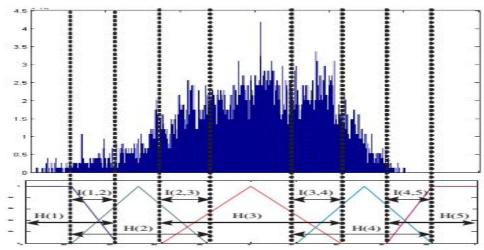

Figure 4: Fuzzy discretizing of the histogram into n joint subsets and m-1 intersections

The objective function is set as the total entropy of partitions[23]. By enhancing partition's parameters to optimize the total entropy of the histogram subsets, the optimal membership design of the variable is found.

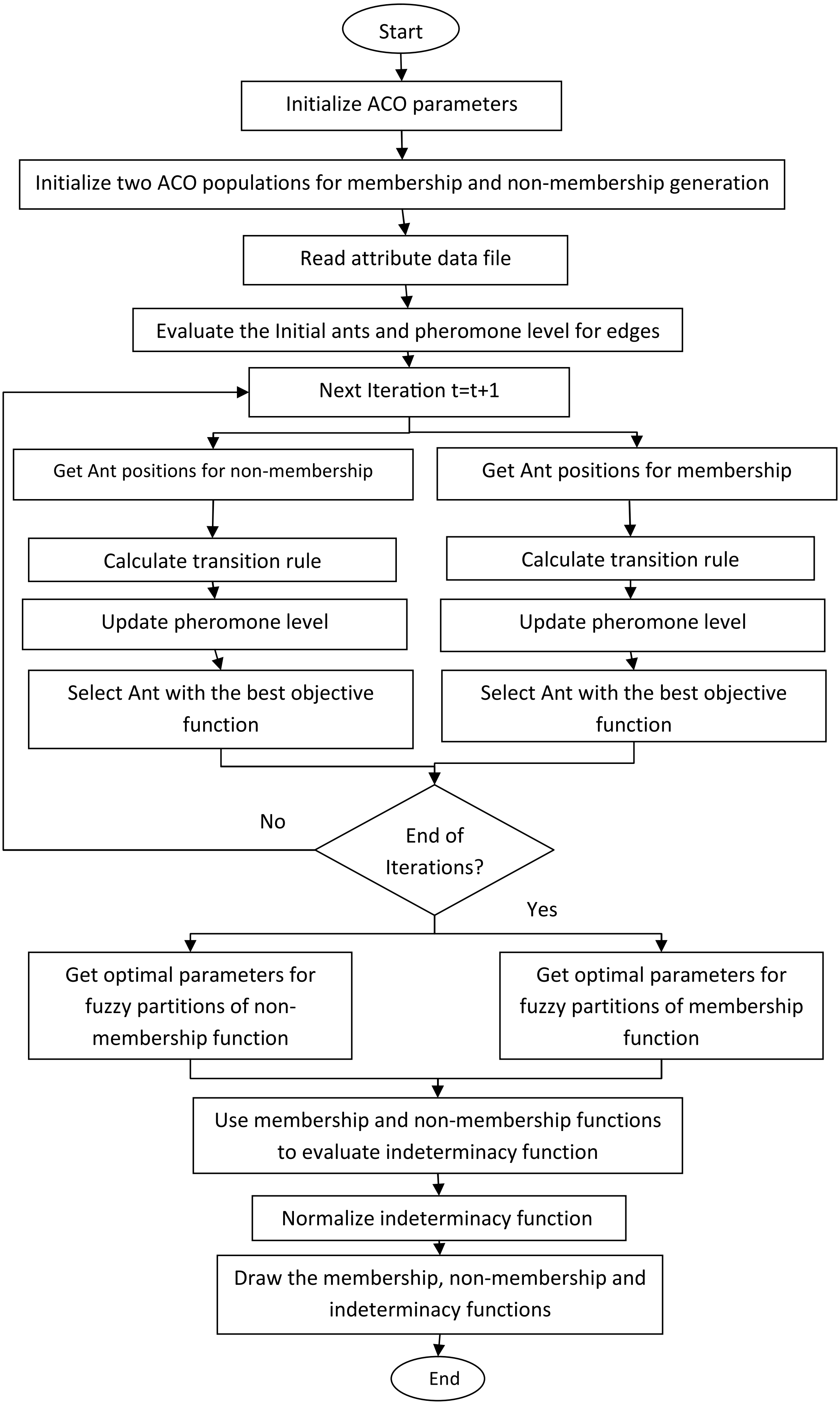

Figure 4: Flow chart for the modelling neuotrosophic variable using ACO

Input :pd, N, variable_datafile
%%%% pd number of decision variables in particle, N ieteration, Present position in the search universe X_{id} , ρ is the decay rate of phermone. %%%%%%
Output: membership, non-membership and indeterminacy function, conversion rate.
1: X←Initianlize_Ants(); % Each ant is composed of **pd** decision variables for fuzzy partitions
2:Att←Read_data(variable_datafile)
3:Objective_mem_ ← Evaluate _ Objective_of_Particles (X, P(Att)); % According to entropy and Mutual information
4: Objective_non_mem ← Evaluate _ Objective_of_Particles (X, 1-P(Att)); % According to entropy and Mutual information

5: **While** (num_of_Iterations<Max_iter)
% membership generation
6: **foreach** Ant
 7: $\eta_j \leftarrow H = \sum_{i=1}^{n} H(i) - \sum_{j=1}^{n-1} I(j, j+1)$
 8: $p_j^m(t) \leftarrow \frac{[\eta_j]\times[\tau_{ij}(t)]}{\sum_{i\in I_m}[\eta_i]\times[\tau_{ij}(t)]}$
 9: $\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \Delta\tau_{ij}(t)$
10:**end foreach**
11: Best_sol_mem← **max**(η_j) % Best found value until iteration t
% non-membership generation
12: **foreach** Ant
 13 $\eta_j \leftarrow H = \sum_{i=1}^{n} H(i) - \sum_{j=1}^{n-1} I(j, j+1)$
 14: $p_j^m(t) \leftarrow \frac{[\eta_j]\times[\tau_{ij}(t)]}{\sum_{i\in I_m}[\eta_i]\times[\tau_{ij}(t)]}$
 15: $\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \Delta\tau_{ij}(t)$
16:**end foreach**
17: Best_sol_non-mem← **max**(η_j) % Best found value until iteration t
18: **End While**
18: Best _mem ← Best_sol_mem
19: Best _non-mem ← Best_sol_non-mem
20: indeterminacy← calculate-ind(Best _mem, Best _non-mem);
21: Draw(Best _mem, Best _non-mem, indetrminancy)
22: Draw_conversions_rate()
23: **Output** membership, non-membership and indeterminacy function, conversion rate.

Function calculate-ind($\mu_A(x)$, $\nu_A(x)$)
1: Input:($\mu_A(x)$, $\nu_A(x)$)
2: Output: indeterminacy
3: $0^- - [\mu_A(x) + \nu_A(x)] \leq \sigma_A(x) \leq 3^+ - [\mu_A(x) + \nu_A(x)]$
4: indeterminacy ←**Normalize**($\sigma_A(x)$)**;**
5: Return indeterminacy
5: End Fun

Figure 5: Algorithm for the modelling neuotrosophic variable using ACO

To model (n) membership functions, variable histogram is partitioned into n overlapped subsets that produce n-1 intersections. Every joint partition corresponds to joint entropy and each overlap is modelled by mutual information. Eq.15 shows the total entropy which is assigned to the heuristic desirability of ants.

$$\eta_j = H = \sum_{i=1}^{n} H(i) - \sum_{j=1}^{n-1} I(j, j+1) \quad (15)$$

Where n is the number of partitions or subsets in the fuzzy variable,

H is the total entropy,

H(i) is the entropy of subset i,

I is the mutual information between to intersecting partitions(i,j).

In membership function modelling, the total entropy function Eq. 13, 14 and 15 are calculated by the probability distribution P(x) of the variable data frequency in various partitions and the intersecting between them. The complement of probability distribution $1-$ P(x) is utilized to measure the non-membership of variable data in different partitions. Therefore, the non-membership objective function will compute Eq. 13, 14 and 15 with the variable data frequency complement in different partitions and overlapping.

According to Eq.3 & 6, the summation of the membership, non membership and indeterminacy values for the same instance is in the interval [$0^-, 3^+$]. Hence the indeterminacy function is declared by Eq. 16.

$$0^- - [\mu_A(x) + \nu_A(x)] \le \sigma_A(x) \le 3^+ - [\mu_A(x) + \nu_A(x)] \quad (16)$$

Where Eq. 9 states that the summation of the membership, non membership and indeterminacy values for the same instance is in the interval [$0^-, 2^+$]. Hence, the indeterminacy function is defined as Eq. 17.

$$0^- - [\mu_A(x) + \nu_A(x)] \le \sigma_A(x) \le 2^+ - [\mu_A(x) + \nu_A(x)] \quad (17)$$

By finding the membership and non-membership definition of x , the indeterminacy function $\sigma_A(x)$ could be driven easily from Eq. 15 or 16. The value of the indeterminacy function should be in the interval $[0^-\ 1^+]$, hence the $\sigma_A(x)$ function is normalized according to Eq. 18.

$$Normalized\_\sigma_A(x_i) = \frac{\sigma_A(x_i) - min(\sigma_A(x))}{max(\sigma_A(x)) - min(\sigma_A(x))} \quad (18)$$

Where $\sigma_A(x_i)$ is the indeterminacy function for the value x_i . The flow chart and algorithm of the integrated framework is illustrated in figure 5 and 6 respectively.

4. Experimental Results

The present reality issues are brimming with vulnerability and indeterminism. The neutrosophic field is worried by picking up information with degrees of enrollment, indeterminacy and non-participation. Neutrosophic framework depends on various neurtosophic factors or variables. Unfortunately, the vast majority of the informational indexes accessible are normal numeric qualities or unmitigated characteristics. Henceforth, creating approaches for characterizing a neutrosophic set from the current informational indexes is required.

The membership capacity function of a neutrosophy variable, similar to the fuzzy variable, can take a few sorts. Triangle membership is very popular due to its simplicity and accuracy. Triangle function is characterized by various overlapping partitions. These subsets are characterized by support, limit and core parameters. The most applicable parameter to a specific subset is the support which is the space of characterizing the membership degree. Finding the start and closure of a support over the universe of a variable could be an intriguing search issue suitable for optimization. Meta-heuristic search methodologies [22] give a intelligent procedure for finding ideal arrangement of solutions is any universe. ACO is a well defined search procedure that mimics ants in discovering their sustenance. Figure 3 presents the ant as an individual in a population for upgrading a triangle membership function through the ACO procedure. The ACO utilizes the initial ant population and emphasizes to achieve ideal arrangement.

Table 1:Parameters of ACO

Maximum Number of Iterations	50
Population Size (number of ants)	10
Decaying rate	0.1

The total entropy given by Eq. 15 characterizes the heuristic desirability which affects the probabilistic transition rule of ants in the ACO algorithm. The probability distribution $P(x)$ presented in Eq. 13, 14 and 15 is used to calculate the total entropy function. The ACO parameters like Maximum Number of Iterations, Population Size, and pheromone decaying rate are presented in table 1.

The non-membership function means the falsity degree in the variables values. Hence, the complement of a data probability distribution $1-P(x)$ is utilized to create the heuristic desirability of the ants in designing the non-membership function Eq. 13, 14 and 15.

The indeterminacy capacity of variable data is created by both membership and non-membership capacities of the same data using neutrosophic set declaration in section 2 and Eq. 16 or 17. Afterwards, Eq. 18 is used to normalize the indeterminacy capacity of the data. Through simulation, the ACO is applied by MATLAB , PC with Intel(R) Core (TM) CPU and 4 GB RAM. The simulation are implemented on the temperature variable from the Forest Fires data set created by: Paulo Cortez and Anbal Morais (Univ. Minho) [25]. The histogram of a random collection of the temperature data is shown in figure 7.

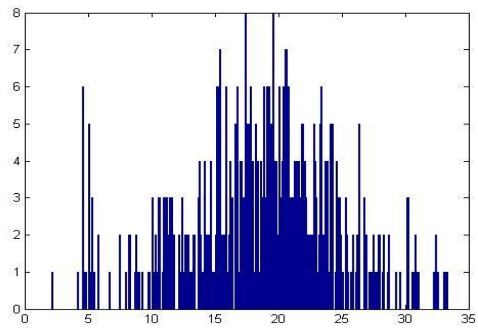

Figure 6: Temparature Variable Histogram

Figures 8: a, b and c presents the resulting membership, non-membership and indeterminacy capacities produced by applying the ACO on a random collection of the temperature data.

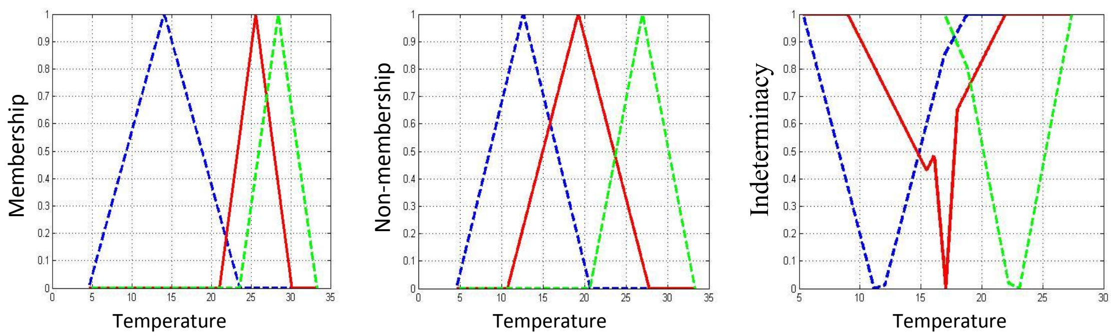

Figure 7: a. Membership Function b. Non-membership Function c. Inderminacy Function

5. Conclusion

A proposed framework utilizing the ant colony optimization and the total entropy measure for mechanizing the design of neutrosophic variable is exhibited. The membership, non-membership and indeterminacy capacities are utilized to represent the neutrosophy idea. The enrollment or truth of subset could be conjured from total entropy measure. The fundamental system aggregates the total entropy to the participation or truth subsets of a neutrosophic concept. The ant colony optimization is a meta-heuristic procedure which seeks the universe related to variable X to discover ideal segments or partitions parameters. The heuristic desirability of ants, for membership generation, is the total entropy based on the probability density function of random variable X. Thusly, the probability density complement is utilized to design non-membership capacity. The indeterminacy capacity is identified, as indicated by neutrosophic definition, by the membership and non-membership capacities. The results in light of ACO proposed system are satisfying. Therefore, the technique can be utilized as a part of data preprocessing stage within knowledge discovery system. Having sufficient data gathering, general neutrosophic variable outline for general data can be formulated.

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