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Optimal Design of Truss Structures Using a Neutrosophic Number Optimization Model under an Indeterminate Environment

Wenzhong Jiang & Jun Ye

Department of Electrical and Information Engineering and Department of Civil Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing, Zhejiang Province 312000, P.R. China.
Corresponding author: Jun Ye, E-mail: yehjun@aliyun.com

Abstract. This paper defines basic operations of neutrosophic numbers and neutrosophic number functions for objective functions and constraints in optimization models. Then, we propose a general neutrosophic number optimization model for the optimal design of truss structures. The application and effectiveness of the neutrosophic number optimization method are demonstrated through the design example of a two-bar truss structure under indeterminate environment to achieve the minimum weight objective under stress and stability constraints. The comparison of the neutrosophic number optimal design method with traditional optimal design methods proves the usability and suitability of the presented neutrosophic number optimization design method under an indeterminate/neutrosophic number environment.

Keywords: Neutrosophic number, neutrosophic number function, neutrosophic number optimization model, neutrosophic number optimal solution, truss structure design.

1 Introduction

In the real-world, there is incomplete, unknown, and indeterminate information. How to express incomplete, unknown, and indeterminate information is an important problem. Hence, Smarandache [1-3] firstly introduced a concept of indeterminacy, which is denoted by the symbol "I" as the imaginary value, and defined a neutrosophic number as $N = a + bI$ for $a, b \in R$ (all real numbers), which consists of both the determinate part a and the indeter-minate part bI. So it can express determinate and/or inde-terminate information in incomplete, uncertain, and inde-terminate problems. After that, Ye [4, 5] applied neutro-sophic numbers to decision making problems. Then, Kong et al. [6] and Ye [7] applied neutrosophic numbers to fault diagnosis problems under indeterminate environments. Further, Smarandache [8] introduced an interval function (so-called neutrosophic function/thick function $g(x) = [g_1(x), g_2(x)]$ for $x \in R$) to describe indeterminate problems by the interval functions. And also, Ye et al. [9] introduced neutrosophic/interval functions of the joint roughness coef-ficient and the shear strength in rock mechanics under in-determinate environments.

It is obvious that neutrosophic numbers are very suitable for the expression of determinate and/or indeterminate information. Unfortunately, existing optimization design methods [10-13] cannot express and deal with indeterminate optimization design problems of engineering structures under neutrosophic number environments. Furthermore, the Smarandache's neutrosophic function [8] cannot also express such an indeterminate function involving neutrosophic numbers. Till now, there are no concepts of neutrosophic number functions and neutrosophic number optimization designs in all existing literature. Therefore, one has to define new functions containing NNs to handle indeterminate optimization problems of engineering designs under a neutrosophic number environment. To handle this issue, this paper firstly defines a new concept of neutrosophic number functions for the neutrosophic number objective functions and constraints in engineering optimization design problems with determinate and indeterminate information, and then proposes a general neutrosophic number optimization model and a solution method to realize neutrosophic number optimization problems of truss structure design, where the obtained neutrosophic number optimal solution can satisfy the design requirements in indeterminate situations.

The remainder of this paper is structured as follows. Section 2 defines some new concepts of neutrosophic number functions to establish the neutrosophic number objective functions and constraints in indeterminate optimization design problems, and proposes a general neutrosophic number optimization model for truss structure designs. In Section 3, the neutrosophic number optimal design of a two-bar truss structure is presented under a neutrosophic number environment to illustrate the application and effectiveness of the proposed neutrosophic number optimization design method. Section 4 contains some conclusions and future research directions.

2 Neutrosophic numbers and optimization models

2.1 Some basic operations of neutrosophic numbers

It is well known that there are some indeterminate design parameters and applied forces in engineering design problems. For example, the allowable compressive stress of some metal material is given in design handbooks by a possible range between 420 MPa and 460 MPa, denoted by σ_p = [420, 460], which reveals the value of σ_p is an indeterminate range within the interval [420, 460]. Then a neutrosophic number $N = a + bI$ for $a, b \in R$ (all real numbers) can effectively express the determinate and/or indeterminate information as $N = 420 + 40I$ for $I \in [0, 1]$, where its determinate part is $a = 420$, its indeterminate part $bI = 40I$, and the symbol "I" denotes indeterminacy and belongs to the indeterminate interval [inf I, sup I] = [0, 1]. For another example, if some external force is within [2000, 2500] kN, then it can be expressed as the neutrosophic number $N = 2000 + 50I$ kN for $I \in [0, 10]$ or $N = 2000 + 5I$ kN for $I \in [0, 100]$ corresponding to some actual requirement.

It is noteworthy that there are $N = a$ for $bI = 0$ and $N = bI$ for $a = 0$ in two special cases. Clearly, the neutrosophic number can easily express its determinate and/or indeterminate information, where I is usually specified as a possible interval range [inf I, sup I] in actual applications. Therefore, neutrosophic numbers can easily and effectively express determinate and/or indeterminate information under indeterminate environments.

For convenience, let Z be all neutrosophic numbers (Z domain), then a neutrosophic number is denoted by $N = a + bI = [a + b(\inf I), a + b(\sup I)]$ for $I \in [\inf I, \sup I]$ and $N \in Z$. For any two neutrosophic numbers $N_1, N_2 \in Z$, we can define the following operations:

$$\begin{aligned} N_1 + N_2 &= a_1 + a_2 + (b_1 + b_2)I \\ &= [a_1 + a_2 + b_1(\inf I) + b_2(\inf I), \\ &\quad a_1 + a_2 + b_1(\sup I) + b_2(\sup I)] \end{aligned};\tag{1}$$

$$\begin{aligned} N_1 - N_2 &= a_1 - a_2 + (b_1 - b_2)I \\ &= [a_1 - a_2 + b_1(\inf I) - b_2(\inf I), \\ &\quad a_1 - a_2 + b_1(\sup I) - b_2(\sup I)] \end{aligned};\tag{2}$$

$$\begin{aligned} N_1 \times N_2 &= a_1a_2 + (a_1b_2 + a_2b_1)I + b_1b_2I^2 \\ &= \begin{bmatrix} \min\begin{pmatrix} (a_1 + b_1(\inf I))(a_2 + b_2(\inf I)), \\ (a_1 + b_1(\inf I))(a_2 + b_2(\sup I)), \\ (a_1 + b_1(\sup I))(a_2 + b_2(\inf I)), \\ (a_1 + b_1(\sup I))(a_2 + b_2(\sup I)) \end{pmatrix}, \\ \max\begin{pmatrix} (a_1 + b_1(\inf I))(a_2 + b_2(\inf I)), \\ (a_1 + b_1(\inf I))(a_2 + b_2(\sup I)), \\ (a_1 + b_1(\sup I))(a_2 + b_2(\inf I)), \\ (a_1 + b_1(\sup I))(a_2 + b_2(\sup I)) \end{pmatrix} \end{bmatrix} \end{aligned};\tag{3}$$

$$\begin{aligned} \frac{N_1}{N_2} &= \frac{a_1 + b_1I}{a_2 + b_2I} = \frac{[a_1 + b_1(\inf I), a_1 + b_1(\sup I)]}{[a_2 + b_2(\inf I), a_2 + b_2(\sup I)]} \\ &= \begin{bmatrix} \min\begin{pmatrix} \frac{a_1 + b_1(\inf I)}{a_2 + b_2(\sup I)}, \frac{a_1 + b_1(\inf I)}{a_2 + b_2(\inf I)}, \\ \frac{a_1 + b_1(\sup I)}{a_2 + b_2(\sup I)}, \frac{a_1 + b_1(\sup I)}{a_2 + b_2(\inf I)} \end{pmatrix}, \\ \max\begin{pmatrix} \frac{a_1 + b_1(\inf I)}{a_2 + b_2(\sup I)}, \frac{a_1 + b_1(\inf I)}{a_2 + b_2(\inf I)}, \\ \frac{a_1 + b_1(\sup I)}{a_2 + b_2(\sup I)}, \frac{a_1 + b_1(\sup I)}{a_2 + b_2(\inf I)} \end{pmatrix} \end{bmatrix} \end{aligned}.\tag{4}$$

2.2 Neutrosophic number functions and neutrosophic number optimization model

In engineering optimal design problems, a general optimization model consists of the objective function and constrained functions. In indeterminate optimization problems of engineering designs, then, objective functions and constrained functions may contain indeterminate information. To establish an indeterminate optimization model in a neutrosophic number environment, we need to define neutrosophic number functions in Z domain.

Definition 1. A neutrosophic number function with n design variables in Z domain is defined as

$$F(\boldsymbol{X}, \boldsymbol{I}): Z^n \to Z. \tag{1}$$

where $\boldsymbol{X} = [x_1, x_2, \ldots, x_n]^T$ for $\boldsymbol{X} \in Z^n$ is a n-dimensional vector and $F(\boldsymbol{X}, I)$ is either a neutrosophic number linear function or a neutrosophic number nonlinear function.

For example, $F_1(\boldsymbol{X}, I) = (1+2I)x_1 + x_2 + (2+3)I$ for $\boldsymbol{X} = [x_1, x_2]^T \in Z^2$ is a neutrosophic number linear function, then $F_2(\boldsymbol{X}, I) = Ix_1^2 + (3+I)x_2^2$ for $\boldsymbol{X} = [x_1, x_2]^T \in Z^2$ is a neutrosophic number nonlinear function.

2.3 General neutrosophic number optimization model

Generally speaking, neutrosophic number optimization design problems with n design variables in Z domain can be defined as the general form of a neutrosophic number optimization model:

$$\begin{aligned} &\min F(\boldsymbol{X}, I) \\ &\text{s.t. } G_k(\boldsymbol{X}, I) \le 0,\ k = 1, 2, \dots, m \\ &H_j(\boldsymbol{X}, I) = 0,\ j = 1, 2, \dots, s \\ &\boldsymbol{X} \in Z^n,\ I \in [\inf I, \sup I], \end{aligned} \qquad (2)$$

where $F(\boldsymbol{X}, I)$ is a neutrosophic number objective function and $G_1(\boldsymbol{x})$, $G_2(\boldsymbol{x})$, …, $G_m(\boldsymbol{x})$ and $H_1(\boldsymbol{x})$, $H_2(\boldsymbol{x})$, …, $H_s(\boldsymbol{x})$: $Z^n \to Z$ are neutrosophic number inequality constraints and neutrosophic number equality constraints, respectively, for $\boldsymbol{X} \in Z^n$ and $I \in [\inf I, \sup I]$.

However, if the neutrosophic number optimal solution of design variables satisfies all these constrained conditions in a neutrosophic number optimization model, the optimal solution is feasible and otherwise is unfeasible. Generally speaking, the optimal solution of design variables and the value of the neutrosophic number objective function usually are neutrosophic numbers/interval ranges (but not always).

To solve the neutrosophic number optimization model (2), we use the Lagrangian multipliers for the neutrosophic number optimization model. Then the Lagrangian function that one minimizes is structured as the following form:

$$\begin{aligned} &L(\boldsymbol{X}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = F(\boldsymbol{X}, I) + \\ &\sum_{k=1}^{m} \mu_k G_k(\boldsymbol{X}, I) + \sum_{j=1}^{s} \lambda_j H_j(\boldsymbol{X}, I), \\ &\boldsymbol{\mu} \in Z^m,\ \boldsymbol{\lambda} \in Z^s,\ \boldsymbol{X} \in Z^n,\ I \in [\inf I, \sup I]. \end{aligned} \qquad (3)$$

The common Karush-Kuhn-Tucker (KKT) necessary conditions are introduced as follows:

$$\nabla F(\boldsymbol{X}, I) + \sum_{k=1}^{m} \{\mu_k \nabla G_k(\boldsymbol{X}, I)\} + \sum_{j=1}^{s} \{\lambda_j \nabla H_j(\boldsymbol{X}, I)\} = 0 \qquad (4)$$

combined with the original constraints, complementary slackness for the inequality constraints and $\mu_k \ge 0$ for $k = 1, 2, \dots, m$.

However, it may be difficult to solve neutrosophic nonlinear optimization models in indeterminate nonlinear optimization design problems, such as multiple-bar truss structure designs under neutrosophic number environments, by the Karush-Kuhn-Tucker (KKT) necessary conditions. Hence, this paper will research on the neutrosophic number optimization design problem of a simple two-bar truss structure in the following section to realize the primal investigation of the truss structure optimal design in a neutrosophic number environment.

3 Optimal design of a two-bar truss structure under a neutrosophic number environment

To demonstrate the neutrosophic number optimal design of a truss structure in an indeterminate environment, a simply two-bar truss structure is considered as an illustrative design example and showed in Fig.1. In this example, the two bars use two steel tubes with the length L, in which the wall thick is T=25mm. The optimal design is performed in a vertically external loading case. The vertical applied force is $2F = (3+0.4I)\times10^5$N, the material Young's modulus and density $E=2.1\times10^5$ MPa and ρ = 7800 kg/m^3, respectively, and the allowable compressive stress is $\sigma_p = 420 + 40I$.

The optimal design objective of the truss structure is to minimize the weight of the truss structure in satisfying the constraints of stress and stability. In this class of optimization problems, the average diameter D of the tube and the truss height H are taken into account as two design variables, denoted by the design vector $\boldsymbol{X} = [x_1, x_2]^{\mathrm{T}} = [D, H]$.

Due to the geometric structure symmetry of the two-bar truss, we only consider the optimal model of one bar of both.

First, the total weight of the tube is expressed by the following formula:

$$M = 2\rho AL = 2\pi\rho T x_1 (\mathrm{B}^2 + x_2^2)^{1/2},$$

where A is the cross-sectional area $A = \pi T x_1$ and $2B$ is the distance between two supporting points.

Then, the compressive force of the steel tube is

$$F_1 = \frac{FL}{x_2} = \frac{F(B^2 + x_2^2)^{1/2}}{x_2},$$

where L is the length of the tube and F_1 is the compressive force of the tube. Thus, the compressive stress of the tube is represented as the following form:

$$\sigma = \frac{F_1}{A} = \frac{F(B^2 + x_2^2)^{1/2}}{\pi T x_1 x_2}.$$

Hence, the constrained condition of the strength for the tube is written as

$$\frac{F(B^2 + x_2^2)^{1/2}}{\pi T x_1 x_2} \le \sigma_p.$$

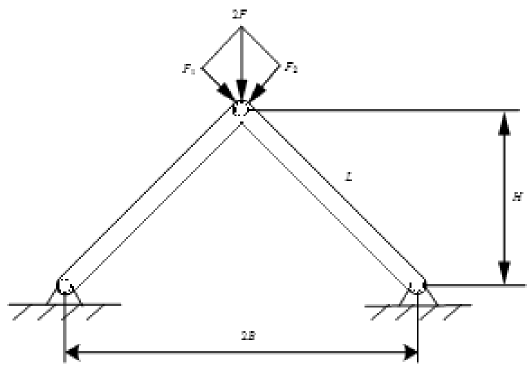

Fig. 1 Two-bar truss structure

For the stability of the compressive bar, the critical force of the tube is given as follows:

$$F_c = \frac{\pi^2 E W_I}{L^2} = \frac{\pi^2 EA(T^2 + x_1^2)}{8(B^2 + x_2^2)} ,$$

where W_I is the inertia moment of the cross-section of the tube.

The critical stress of the tube is given as

$$\sigma_c = \frac{F_c}{A} = \frac{\pi^2 E(T^2 + x_1^2)}{8(B^2 + x_2^2)} .$$

Thus, the constrained condition of the stability for the tube is written as

$$\frac{F(B^2 + x_2^2)^{1/2}}{\pi T x_1 x_2} \le \frac{\pi^2 E(T^2 + x_1^2)}{8(B^2 + x_2^2)} .$$

Finally, the neutrosophic optimization model of the truss structure can be formulated as:

$$\begin{aligned} &\min M(\boldsymbol{X}, I) = 2\pi\rho T x_1 (B^2 + x_2^2)^{1/2} \\ &s.t.\quad G_1(\boldsymbol{X}, I) = \frac{F(B^2 + x_2^2)^{1/2}}{\pi T x_1 x_2} - \sigma_p \le 0 \\ &\qquad G_2(\boldsymbol{X}, I) = \frac{F(B^2 + x_2^2)^{1/2}}{\pi T x_1 x_2} - \frac{\pi^2 E(T^2 + x_1^2)}{8(B^2 + x_2^2)} \le 0 \end{aligned} .$$

By solving the neutrosophic optimization model, the neutrosophic number optimal solution of the two design variables is given as follows:

$$X^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} \dfrac{\sqrt{2}F}{\pi T(420 + 40I)} \\ B \end{bmatrix} , = \begin{bmatrix} \dfrac{1.414(1.5 + 0.2I) \times 10^5}{7.85(420 + 40I)} \\ 760 \end{bmatrix}$$

In this case, the neutrosophic number optimal value of the objective function is obtained as follows:

$$M(X^*, I) = \frac{4\rho FB}{\sigma_p} = \frac{2371.2(1.5 + 0.2I)}{(420 + 40I)} .$$

Since there exists the indeterminacy I in these neutrosophic number optimal values, it is necessary that we discuss them when the indeterminacy I is specified as possible ranges according to actual indeterminate requirements in the actual application.

Obviously, the neutrosophic number optimization problem reveals indeterminate optimal results (usually neutrosophic number optimal solutions, but not always). If the indeterminacy I is specified as different possible ranges of I =0, $I \in [0, 1]$, $I \in [1, 3]$, $I \in [3, 5]$, $I \in [5, 7]$, and $I \in [7, 10]$ for convenient analyses, then all the results are shown in Table 1.

Table 1. Optimal results of two-bar truss structure design in different specified ranges of $I \in$ [inf I, sup I]

$I \in$ [inf I, sup I]	$D = x_1^*$ (mm)	$H = x_2^*$ (mm)	$M(\boldsymbol{X}^*, I)$ (kg)
$I = 0$	64.3312	760	8.4686
$I \in [0, 1]$	[58.7372, 72.9087]	760	[7.7322, 9.5977]
$I \in [1, 3]$	[56.7068, 82.2321]	760	[7.4649, 10.8250]
$I \in [3, 5]$	[61.0109, 83.3923]	760	[8.0315, 10.9778]
$I \in [5, 7]$	[64.3312, 84.2531]	760	[8.4686, 11.0911]
$I \in [7, 10]$	[63.7036, 90.0637]	760	[8.3860, 11.8560]

In Table 1, if I = 0, it is clear that the neutrosophic number optimization problem is degenerated to the crisp optimization problem (i.e., traditional determinate optimization problem). Then under a neutrosophic number environment, neutrosophic number optimal results are changed as the indeterminate ranges are changed. Therefore, one will take some interval range of the indeterminacy I in actual applications to satisfy actual indeterminate requirements of the truss structure design. For example, if we take the indeterminate range of $I \in [0, 1]$, then the neutrosophic number optimal solution is $D = x_1^*$ = [58.7372, 72.9087] mm and $H = x_2^*$ = 760mm. In actual design, we need the de-neutrosophication in the neutrosophic optimal solution to determinate the suitable optimal design values of the design variables to satisfy some indeterminate requirement. For example, if we take the maximum values of the optimal solution for $I \in [0, 1]$, we can obtain D = 73mm and H = 760mm for the two-bar truss structure design to satisfy this indeterminate requirement.

However, traditional optimization design methods [10-13] cannot express and handle the optimization design problems with neutrosophic number information and are special cases of the neutrosophic number optimization design method in some cases. The comparison of the proposed neutrosophic number optimization design method with traditional optimization design methods demonstrates the usability and suitability of this neutrosophic number optimization design method under a neutrosophic number environment.

4 Conclusion

Based on the concepts of neutrosophic numbers, this paper defined the operations of neutrosophic numbers and neutrosophic number functions to establish the neutrosophic number objective function and constraints in neutrosophic number optimization design problems. Then, we proposed a general neutrosophic number optimization model with constrained optimizations for truss structure design problems. Next, a two-bar truss structure design example was provided to illustrate the application and effectiveness of the proposed neutrosophic number optimization design method.

However, the indeterminate (neutrosophic number) optimization problems may contain indeterminate (neutrosophic number) optimal solutions (usually neutrosophic numbers, but not always), which can indicate possible optimal ranges of the design variables and objective function when indeterminacy *I* is specified as a possible interval ranges in actual applications.

In general, indeterminate designs usually imply indeterminate optimal solutions from an indeterminate viewpoint. Then in the de-neutrosophication satisfying actual engineering design requirements we can determinate the suitable optimal design values of design variables in the obtained optimal interval solution corresponding to designers' attitudes and/or some risk situations to be suitable for actual indeterminate requirements.

It is obvious that the neutrosophic number optimization design method in a neutrosophic number environment is more useful and more suitable than existing optimization design methods of truss structures since the traditional determinate/indeterminate optimization design methods cannot express and handle the neutrosophic number optimization design problems under an indeterminate environment. Therefore, the neutrosophic number optimization design method provides a new effective way for the optimal design of truss structures under indeterminate/neutrosophic number environments.

Nonetheless, due to existing indeterminacy "*I*" in the neutrosophic number optimization model, it may be difficult to solve complex neutrosophic number optimization models. In the future, therefore, we shall further study solving algorithms/methods for neutrosophic number optimization design problems and apply them to mechanical and civil engineering designs under indeterminate / neutrosophic number environments.

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