An Elementary Proof of Goldbach's Conjecture
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Abstract
Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Human is very easily cheated by 2 words, infinite and anything, because we never see infinite and anything, so we always make simple thing complex. Goldbach’s conjecture is about all very simple numbers, the pattern of prime numbers likes a “kaleidoscope” of numbers, we divided any even numbers into 5 groups and primes into 4 groups, Goldbach’s conjecture becomes much simpler. Here we give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic, the prime number theorem, and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture, fundamental theorem of arithmetic, Euclid's proof of infinite primes, the prime number theorem

Introduction
Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many “advanced mathematics tools” are used to solve them, but they are still unsolved.

I believe that prime numbers are “basic building blocks” of the natural numbers and they must follow some very simple basic rules and do not need “advanced mathematics tools” to solve them. Two of the basic rules are the “fundamental theorem of arithmetic” and Euclid’s proof of endless prime numbers.

Fundamental theorem of arithmetic:
The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors. Primes can thus be considered the “basic building blocks” of the natural numbers.

**Euclid's proof** that the set of prime numbers is endless
The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that \( P_1 = 2, P_2 = 3, P_3 = 5 \) and so on. If we assume that there are just \( n \) primes, then the biggest prime will be labeled \( P_n \). Now we can form the number \( Q \) by multiplying together all these primes and adding 1, so

\[
Q = (P_1 \times P_2 \times P_3 \times P_4 \ldots \times P_n) + 1
\]

Now we can see that if we divide \( Q \) by any of our \( n \) primes there is always a remainder of 1, so \( Q \) is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either \( Q \) must be a prime or \( Q \) must be divisible by primes that are larger than \( P_n \).

Our assumption that \( P_n \) is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

**Discussions**
Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach’s conjecture is about all numbers, the pattern of prime numbers likes a “kaleidoscope” of numbers, if we divide all even numbers into 5 groups and primes into 4 groups, Goldbach’s conjecture will be much simpler.
If a number \((N>13)\) is not divisible by 3 or any prime which is smaller or equal to \(N/3\), it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 11, it have 1/3 chance is divisible by 3 and 1/7 chance is divisible by 7, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, and 1/11 chance to be divisible by 11, so on, so we have terms: 1/3, 1/7x2/3, 1/11x2/3x6/7, 1/13x2/3x6/7x10/11…,

Let \(N_o\) represent any odd number, the chance of \(N_o\) to be a non-prime is:

\[
\left\{(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17x18/19) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + \ldots \right\} \quad \text{Formula 1}
\]

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5.

Let \(\sum\) represent the sum of the infinite terms and \(\Delta=1-\sum\), according to Euclid's proof\(^3\) that the set of prime numbers is endless. \(\Delta\) is the chance of any odd number to be a prime. \(\sum\) may be very close to 1 when \(N\) is growing to \(\infty\), but always less than 1. Let \(\Delta=1-\sum\), when \(N\) is growing to \(\infty\), \(\Delta\) may be very close to 0, but always more than 0 according to Euclid's proof that the set of prime numbers is endless. If \(\Delta\) is 0, then there is no prime, that is not true.

The sum of first 20 terms = \(\{(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17x18/19) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + \ldots \} \)}

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Formula 1
\[
\frac{1}{59}x/3x6/7x10/11x12/13x16/17x18/19x22/3x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + \\
\frac{1}{61}x2/3x6/7x10/11x12/13x16/17x18/19x22/3x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) + \\
\frac{1}{67}x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + \\
\frac{1}{71}x2/3x6/7x10/11x12/13x16/17x18/19x22/3x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) + \\
\frac{1}{73}x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \\
\frac{1}{79}x2/3x6/7x10/11x12/13x16/17x18/19x22/3x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) = \\
[0.333333 + 0.095238 + 0.051948 + 0.039960 + 0.028207 + 0.02753 + 0.018590 + 0.014102 + 0.012738 + 0.010328 + 0.009069 + 0.008436 + 0.007538 + 0.006543 + 0.005766 + 0.005483 + 0.004910 + 0.004564 + 0.004377 + 0.003989] = 0.688872
\]

For the first 20 term: \( \sum = 0.688872, \Delta = 1-\sum = 0.311128 \)

The chance of \( N_0 \) to be a prime is: \( \Delta = 1 - [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + \\
(1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + \\
(1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + \\
(1/41x2/3x6/7x10/11x12/13x16/17x18/22x23x28/29x30/31x36/37) + \\
(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + \\
(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + \ldots] \} \text{----------Formula 2}

Let \( S1 \) represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,...; \( S3 \) represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193,...; \( S7 \) represents a prime with 7 as its last digit, such as 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197,...; and \( S9 \) represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,...

Let \( O1 \) represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,...; \( O3 \) represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,....; \( O7 \) represents an odd number with 7 as its last digit, such as 7, 17, 27, 37, 47, 57, 67, 77,...; and \( O9 \) represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,...

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.
Every odd number (O1) with 1 as its last digit is a product of $1 \times 1$, $3 \times 7$, and $9 \times 9$. The first $9$ is 19, but the odd number $9 = 3 \times 3$, so 3 is the smallest prime for $9$.

The chance of any odd number O1 to be a prime is: $\Delta_1 = 1 - \sum_1 = 1 - [\left(1/3\right) + \left(1/11x2/3\right) + \left(1/13x2/3x10/11\right) + \left(1/19x2/3x10/11x12/13\right) + \left(1/23x2/3x10/11x12/13x18/19\right) + \left(1/29x2/3x10/11x12/13x18/19x22/23\right) + \left(1/31x2/3x10/11x12/13x18/19x22/23x28/29\right) + \left(1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31\right)] + \left(1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41\right) + \ldots$$\text{-------------------------}

Formula 3.

The sum of first 20 terms of $\sum_1 = \left[\left(1/3\right) + \left(1/11x2/3\right) + \left(1/13x2/3x10/11\right) + \left(1/19x2/3x10/11x12/13\right) + \left(1/23x2/3x10/11x12/13x18/19\right) + \left(1/29x2/3x10/11x12/13x18/19x22/23\right) + \left(1/31x2/3x10/11x12/13x18/19x22/23x28/29\right) + \left(1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31\right)] + \left(1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41\right) + \ldots$ $\text{-------------------------}

For the first 20 term: $\sum_1 = 0.63254$, $\Delta_1 = 1 - \sum_1 = 0.36746$, for any number N, $\Delta_1 > 0$.

Every odd number with 3 as its last digit is a product of $3x$1 or $7x$9; $1$ is decided by $3$ or $3$ is decided by $1$ and $9$ is decided by $7$ or $7$ is decided by 9, so we need to consider only $3$ and $7$, $1$ and $9$, $3$ and $9$, or $1$ and $7$. 
The chance of any odd number $O_3 ($3 \times 1$ or $7 \times 9$) to be a prime is: $\Delta_3 = 1 - \sum_{3} = 1 - \left[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x12/13x16/17x22/23x36/37) + (1/47x2/3x6/7x12/13x16/17x22/23x36/37x42/43) + \ldots \right] \text{---------Formula 4}

The chance of any odd number $O_7 ($7 \times 1$, or $3 \times 9$) to be a prime is: $\Delta_7 = 1 - \sum_{7} = 1 - \left[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23) + (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + \ldots \right] \text{---------Formula 5}

The formula 4 is same as formula 5, so there is almost same number of primes with 3 or 7 as their last digit.

The sum of first 20 terms of $\sum_{3} = [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x12/13x16/17x36/37x12/13x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) + (1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) + (1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83)

+ (1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) + (1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) + (1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) + (1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + (1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + (1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137) + (1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157)

$= N/[13+1/10.5+1/22.75+1/32.23+1/46.33+1/77.92+1/93.07+1/104.15+1/120+1/164.61+1/171.01+1/197.14+1/233.2+1/250.2+1/262.47+1/279.8+1/317.27+1/344.97+1/398.24+1/416.11+\ldots=0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.008333+0.005848+0.005073+0.004288+0.003997+0.003810+0.003574+0.003152+0.002899+0.002511+0.002403+0.002331] = 0.6102883$

For the first 20 term: $\sum_{3} = \sum_{7} = 0.6103$, $\Delta_3 = \Delta_7 = 1 - \sum_{3} = 0.3897$, $\Delta_3 = \Delta_7$. 
For a large number \( N \), \( \Delta_3 \) should be very close to \( \Delta_7 \). For any number \( N \), \( \Delta_3 \) or \( \Delta_7 >0 \)

Every odd number (O9) with 9 as its last digit is a product terms of $1 x9$, $3x3$ or $7x7$. We need to consider only $1, 3,$ and $7$.

The chance of any odd number O9 to be a prime is: \( \Delta_9 = 1 - \sum_{9} = 1 - \left[ \frac{1}{3} + \left( \frac{1}{7x2} / \frac{3x6}{7} \right) + \left( \frac{1}{11x2} / \frac{3x6}{7x10} \right) + \left( \frac{1}{13x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} + \frac{1}{1/37x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} \right) + \left( \frac{1}{41x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37} \right) + \left( \frac{1}{43x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41}{41} \right) + \left( \frac{1}{47x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43} \right) + \left( \frac{1}{53x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47}{47} \right) + \left( \frac{1}{61x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53} \right) + \left( \frac{1}{67x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53x60} / \frac{61}{61} \right) + \left( \frac{1}{71x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53x60} / \frac{61x66}{61x66} / \frac{67}{67} \right) + \left( \frac{1}{73x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53x60} / \frac{61x66}{67x70} / \frac{71x72}{70} / \frac{71x72}{73} \right) + \left( \frac{1}{97x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53x60} / \frac{61x66}{67x70} / \frac{71x72}{73} / \frac{73x82}{83} \right) + \left( \frac{1}{101x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53x60} / \frac{61x66}{67x70} / \frac{71x72}{73} / \frac{73x82}{83x96} / \frac{97}{97} \right) + \left( \frac{1}{103x2} / \frac{3x6}{7x10} / \frac{11x12}{13x16} / \frac{17x22}{23x30} / \frac{31x36}{37x40} / \frac{41x42}{43x46} / \frac{47x52}{53x60} / \frac{61x66}{67x70} / \frac{71x72}{73} / \frac{73x82}{83x96} / \frac{97x100}{10} \right) = 0.333333 + 0.095238 + 0.051948 + 0.039960 + 0.028208 + 0.019622 + 0.013926 + 0.01129 + 0.009914 + 0.009222 + 0.008242 + 0.007152 + 0.006097 + 0.005462 + 0.005004 + 0.004868 + 0.004222 + 0.003424 + 0.003393 + 0.003294 = 0.660395

For the first 20 term: \( \sum_{9} = \sum_{9} = 0.660395, \Delta_9 = 1 - \sum_{9} = 0.339605 \), for any number \( N \), \( \Delta_9 >0 \)

Euclid's proof that the set of prime numbers is endless, from these formulas, the number of primes $1$, $3$, $7$, $9$ are very close and they are endless.
For N=600, the smallest prime $1 is 11, it decided the possible largest prime $3 is 53, the smallest prime $9 is 19, but 3 x 3 is 9, so the possible largest prime $7 is 47, 47 x 3 x3=423 (the next will 67 x 3 x3=603>600), so we have: Prime number with 3 as its last digit=600/10 –{600/10[(1/3)+ (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47)]} = 600/10-600/10[0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333 = 60-60x0.566654 = 60-34 = 26, it is same as the real number. For N=600, we have Δ3=1-Σ3=1-0.566654 = 0.433346, every odd number with 3 as its last digit has almost 43% chance to be bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime. a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime.

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120,…,

N-$3= O7, if we can prove at least one of these O7 is $7, then N is the sum of 2 primes; or N-$7= O3, if we can prove at least one of these O3 is $3, then N is the sum of 2 primes as table 1.

Table 1.

<table>
<thead>
<tr>
<th>$7</th>
<th>…</th>
<th>137</th>
<th>127</th>
<th>107</th>
<th>97</th>
<th>67</th>
<th>47</th>
<th>37</th>
<th>17</th>
<th>7</th>
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<td>N-127</td>
<td>N-107</td>
<td>N-97</td>
<td>N-67</td>
<td>N-47</td>
<td>N-37</td>
<td>N-17</td>
<td>N-7</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$3</th>
<th>…</th>
<th>113</th>
<th>103</th>
<th>83</th>
<th>73</th>
<th>53</th>
<th>43</th>
<th>23</th>
<th>13</th>
<th>3</th>
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<td>N-113</td>
<td>N-103</td>
<td>N-83</td>
<td>N-73</td>
<td>N-53</td>
<td>N-43</td>
<td>N-23</td>
<td>N-13</td>
<td>N-3</td>
</tr>
</tbody>
</table>
From Formula 3, 4, 5, and 6, we know there are similar numbers of primes with 1, 3, 7, or 9 as their last digit.

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N, there is approximately $N/[4\times\ln(N)]$ of primes with 7 as their last digit (the first line of table 1, 4 x is due to there are only $1/4$ of total primes with 7 as its last digit), correspondingly, we have $N/[4\times\ln(N)]$ of O3(N-$7$, the second line of table 1) in which there should be approximately $\{N/[4\times\ln(N)]\}/\ln\{N/[4\times\ln(N)]\}$. For any number N>100, $\{N/[4\times\ln(N)]\}/\ln\{N/[4\times\ln(N)]\}>4$, so for any number N>200, there are more than 4 pairs of primes in which one prime ($7$) has 7 as its last digit and another prime (N-$7$) has 3 as its last digit. When N with 0 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true. For $3$ and N-$3$ pairs (the bottom 2 lines of table 1) is also true.

Table 2.

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>151</th>
<th>131</th>
<th>101</th>
<th>71</th>
<th>61</th>
<th>41</th>
<th>31</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-S1</td>
<td>...</td>
<td>N-151</td>
<td>N-131</td>
<td>N-101</td>
<td>N-71</td>
<td>N-61</td>
<td>N-41</td>
<td>N-31</td>
<td>N-11</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>S9</th>
<th>149</th>
<th>139</th>
<th>109</th>
<th>89</th>
<th>79</th>
<th>59</th>
<th>29</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
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<td>...</td>
<td>N-149</td>
<td>N-139</td>
<td>N-109</td>
<td>N-89</td>
<td>N-79</td>
<td>N-59</td>
<td>N-29</td>
<td>N-19</td>
</tr>
</tbody>
</table>

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N, there is approximately $N/[4\times\ln(N)]$ of primes with 1 as their last digit (the first line of table 2), correspondingly, we have
The number of pairs of primes in which one prime has 1 as its last digit and another prime has 9 as its last digit is approximately \(\frac{N}{4 \ln(N)}\). For any number \(N > 200\), \(\frac{N}{4 \ln(N)}\) is greater than 4, so for any number \(N\), there are more than 4 pairs of primes in which one prime (\(N - 1\)) has 1 as its last digit and another prime (\(N - 9\)) has 9 as its last digit. When \(N\) with 0 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true. For \(N - 7\) and \(N - 3\) pairs (the bottom 2 lines of table 2) is also true.

For \(N = 600\) (see table 3), \(\frac{N}{4 \ln(N)} = \frac{600}{4 \ln(600)}\) is greater than 4, so for any number \(N\), there are more than 4 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit. That should be true for \(N - 7\) and \(N - 3\) pairs, in fact, 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime with 9 as its last digit.

Table 3.
2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 52, 62, 72, 82, 92, 102, 112, 122,…,

Table 4.
According to the prime number theorem, the number of prime numbers less than N is approximately given by \( N/\ln(N) \), so for any number N, there is approximately \( N/[4x\ln(N)] \) of primes with 3 as their last digit (the first line of table 4), correspondingly, we have \( N/[4x\ln(N)] \) of O9(N-$3$, the second line of table 4) in which there should be approximately \( \{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} \). For any number N>100, \( \{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} \)>4, so for any number N, there are more than 4 pair of primes in which one prime ($3$) has 3 as its last digit and another prime (N-$3$) has 9 as its last digit. When N with 2 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true. For $9$ and N-$9$ pairs (the bottom 2 lines of table 4) is also true.

Table 5.

<table>
<thead>
<tr>
<th>$1$</th>
<th>…</th>
<th>151</th>
<th>131</th>
<th>101</th>
<th>71</th>
<th>61</th>
<th>41</th>
<th>31</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-$1$</td>
<td>…</td>
<td>N-151</td>
<td>N-131</td>
<td>N-101</td>
<td>N-71</td>
<td>N-61</td>
<td>N-41</td>
<td>N-31</td>
<td>N-11</td>
</tr>
</tbody>
</table>

According to the prime number theorem, the number of prime numbers less than N is approximately given by \( N/\ln(N) \), so for any number N, there is approximately \( N/[4x\ln(N)] \) of primes with 1 as their last digit (the first line of table 5), correspondingly, we have \( N/[4x\ln(N)] \) of O1(N-$1$, the second line of table 5) in which there should be approximately \( \{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} \). For any number N>200, \( \{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} \)>4, so for any number N, there are more than 4 pairs of primes in which one prime ($1$) has 1 as its last digit and another prime (N-$1$) has 1 as its last digit. When N with 2 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true.

3. When any even integer (N) has 4 as its last digit, such as 14, 24, 34, 44, 54, 64, 74, 84, 94, 104, 114,…..

Table 6.
According to the prime number theorem, the number of prime numbers less than \( N \) is approximately given by \( \frac{N}{\ln(N)} \), so for any number \( N \), there is approximately \( \frac{N}{4\ln(N)} \) of primes with 7 as their last digit (the first line of table 6), correspondingly, we have \( \frac{N}{4\ln(N)} \) of \( O7(N-7) \), the second line of table 6) in which there should be approximately \( \frac{N}{4\ln(N)} \). For any number \( N>200 \), \( \frac{N}{4\ln(N)} / \ln\{N/[4\ln(N)]\}>4 \), so for any number \( N \), there are more than 4 pairs of primes in which one prime ($7$) has 7 as its last digit and another prime (N-$7$) has 7 as its last digit. When \( N \) with 4 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true.

Table 7.
According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 3 as their last digit (the first line of table 7), correspondingly, we have N/[4xln(N)] of O1(N-$3$, the second line of table 7) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}. For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($3$) has 3 as its last digit and another prime (N-$3$) has 1 as its last digit. When N with 4 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true. For $1$ and N-$1$ pairs (the bottom 2 lines of table 7) is also true.

3. When any even integer (N) has 6 as its last digit, such as 6,

4. 16, 26, 36, 46, 56, 66, 76, 86, 96, 106, 116,….

Table 8.

<table>
<thead>
<tr>
<th>$3$</th>
<th>…</th>
<th>153</th>
<th>113</th>
<th>103</th>
<th>83</th>
<th>73</th>
<th>53</th>
<th>43</th>
<th>23</th>
<th>13</th>
<th>3</th>
</tr>
</thead>
</table>

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 3 as their last digit (the first line of table 8), correspondingly, we have N/[4xln(N)] of O3(N-$3$, the second line of table 8) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}. For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($3$) has 3 as its last digit and another prime (N-$3$) has 3 as its last digit. When N with 6 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true.
According to the prime number theorem, the number of prime numbers less than N is approximately given by \( \frac{N}{\ln(N)} \), so for any number N, there is approximately \( \frac{N}{4\ln(N)} \) of primes with 9 as their last digit (the first line of table 9), correspondingly, we have \( \frac{N}{4\ln(N)} \) of 07(N-S9, the second line of table 9) in which there should be approximately \( \frac{N}{4\ln(N)} \) pairs (the bottom 2 lines of table 9) is also true.

5. When any even integer (N) has 8 as its last digit, such as 8, 18, 28, 38, 48, 58, 68, 78, 88, 98, 108, 118…..

Table 9.

<table>
<thead>
<tr>
<th>N-S9</th>
<th>…</th>
<th>149</th>
<th>139</th>
<th>109</th>
<th>89</th>
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</tr>
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<tr>
<td>N-149</td>
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<td>N-109</td>
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<td>N-79</td>
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<tr>
<td>N-127</td>
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<td>N-107</td>
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<td>N-67</td>
<td>N-47</td>
<td>N-37</td>
<td>N-17</td>
<td>N-7</td>
<td></td>
</tr>
</tbody>
</table>
According to the prime number theorem, the number of prime numbers less than \(N\) is approximately given by \(N/\ln(N)\), so for any number \(N\), there is approximately \(N/\lfloor 4 \times \ln(N) \rfloor\) of primes with 9 as their last digit (the first line of table 10), correspondingly, we have \(N/\lfloor 4 \times \ln(N) \rfloor\) of \(O(1)\) primes with 9 as their last digit (the second line of table 10) in which there should be approximately \(\left\lfloor N/\lfloor 4 \times \ln(N) \rfloor \right\rfloor/\ln\left\lfloor N/\lfloor 4 \times \ln(N) \rfloor \right\rfloor\). For any number \(N > 200\), \(\left\lfloor N/\lfloor 4 \times \ln(N) \rfloor \right\rfloor/\ln\left\lfloor N/\lfloor 4 \times \ln(N) \rfloor \right\rfloor > 4\), so for any number \(N\), there are more than 4 pairs of primes in which one prime \(O(1)\) has 9 as its last digit and another prime \((N-O(1))\) has 9 as its last digit. When \(N\) with 8 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true.

Table 11.

<table>
<thead>
<tr>
<th>$7</th>
<th>...</th>
<th>127</th>
<th>107</th>
<th>97</th>
<th>67</th>
<th>47</th>
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<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-$$7</td>
<td>...</td>
<td>N-127</td>
<td>N-107</td>
<td>N-97</td>
<td>N-67</td>
<td>N-47</td>
<td>N-37</td>
<td>N-17</td>
<td>N-7</td>
</tr>
</tbody>
</table>

| \$1  | ... | 151 | 131 | 101 | 71  | 61  | 41  | 31  | 11  |
According to the prime number theorem, the number of prime numbers less than N is approximately given by \( N/\ln(N) \), so for any number N, there is approximately \( N/[4x\ln(N)] \) of primes with 7 as their last digit (the first line of table 11), correspondingly, we have \( N/[4x\ln(N)] \) of O1(N-$7$, the second line of table 11) in which there should be approximately \( \{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} \), For any number N>200, \( \{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} > 4 \), so for any number N, there are more than 4 pairs of primes in which one prime (7) has 7 as its last digit and another prime (N-$7$) has 1 as its last digit. When N with 8 as its last digit, we have proved Goldbach’s Conjecture is true if the prime number theorem is true. For $1 and N-$1 pairs (the bottom 2 lines of table 11) is also true.

So we have proved that Goldbach’s Conjecture is true if the prime number theorem is true for any even number N.

References: