# Complex Neutrosophic Soft Set 

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#### Abstract

In this paper, we propose the complex neutrosophic soft set model, which is a hybrid of complex fuzzy sets, neutrosophic sets and soft sets. The basic set theoretic operations and some concepts related to the structure of this model are introduced, and illustrated. An example related to a decision making problem involving uncertain and subjective information is presented, to demonstrate the utility of this model.


Keywords-complex fuzzy sets; soft sets; complex neutrosophic sets; complex neutrosophic soft sets; decision making

## I. Introduction

The neutrosophic setmodel (NS) proposed by Smarandache [1, 2] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in the real world. It is a generalization of the theory of fuzzy set [3], intuitionistic fuzzy sets [4, 5], interval-valued fuzzy sets [6] and interval-valued intuitionistic fuzzy sets [7]. A neutrosophic set is characterized by a truthmembership degree $(t)$, an indeterminacy-membership degree (i) and a falsity-membership degree ( $f$ ) independently, all of which lie in the real standard or nonstandard unit interval $]^{-0}, 1^{+}$. Since this interval is difficult to be used in real-life situations, Wang et al.[8]introduced single-valued neutrosophic sets (SVNSs) whose functions of truth, indeterminacy and falsity lie in [0, 1]. Neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, and intuitionistic neutrosophic setshave been applied in a wide variety of fields including decision making, computer science, engineering, and medicine [1-2, 8-27,36-37].

The study of complex fuzzy sets were initiated by Ramot et al. [28]. Among the well-known complex fuzzy based models in literature are the complex intuitionistic fuzzy set (CIFS) [29, 30], complex vague soft set (CVSS) [31, 32] and complex intuitionistic fuzzy soft set (CIFSS) [33].These models have been used to represent the uncertainty and periodicity aspects of an object simultaneously, in a single set. The complex-valued membership and non-membership functions in these modelshave the potential to be used to represent uncertainty in instances such as the wave function in quantum mechanics, impedance in electrical engineering, the changes in meteorological activities, and time-periodic decision making problems.
Recently, Ali and Smarandache[34]developed a hybrid model of complex fuzzy sets and neutrosophic sets, called the complex neutrosophic set. This model has the capability of handling the different aspects of uncertainty, such as incompleteness, indeterminacy and inconsistency, whilst simultaneously handling the periodicity aspect of the objects, all in a single set. The complex neutrosophic set is defined by complex-valued truth, indeterminacy and falsity membership functions. The complex-valued truth membership function is a combination of truth amplitude term (truth membership) and a phase term which represents the periodicity of the objects. Similarly, the complexvalued indeterminacy and falsity membership functions consists of indeterminacy amplitude and phase terms, and falsity amplitude and phase terms, respectively. The complex neutrosophic set is a generalized framework of all the other existing models in literature.

However, the complex neutrosophic set has one shortcoming: the lack of a suitable parameterization tool. To overcome this problem, we introduce the complex neutrosophic soft set (CNSS) model. This model refers to a complex neutrosophic set defined in a soft setting, and this provides the complex neutrosophic set with adequate parameterization capabilities. The rest of the paper is organized as follows. In section 2, we present an overview of some basic definitions and properties which serves as the background to our work in this paper. In section 3, the main definition of the CNSS and some related concepts are presented. In section 4, the basic set theoretic operations for this model are defined. The utility of this model is demonstrated by applying it in a decision making problem in section 5 . Concluding remarks are given in section 6 .

## II. Preliminaries

In this section, we recapitulate some important concepts related to neutrosophic sets (NSs), and complex neutrosophic sets(CNSs). We refer the readers to $[1,8,10,34]$ for further details pertaining to these models.

Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$.
Definition 1.[1] A neutrosophic set Ais an object having the form $A=\left\{\left\langle x, T_{A}(x), I_{A}(x), F_{A}(x)\right\rangle: x \in X\right\}$, where the functions $T, I, F: X \rightarrow]^{-} 0,1^{+}[$, denote the truth, indeterminacy, and falsitymembership functions, respectively, of the element $x \in X$ with respect to set $A$. These membership functions must satisfy the condition

$$
{ }^{-} 0 \leq T_{A}(x)+I_{A}(x)+F_{A}(x) \leq 3^{+} . \text {(1) }
$$

The functions $T_{A}(x), I_{A}(x)$ and $F_{A}(x)$ are real standard or nonstandard subsets of $]^{-} 0,1^{+}$. These intervals make it difficult to apply NSs to practical problems, and this led to the introduction of a single-valued neutrosophic set (SVNS) in [12]. This model is a special case of NSs and is better suited to handle real-life problems and applications.

Definition 2.[8]An SVNS Ais a neutrosophic set that is characterized by a truth-membership function $T_{A}(x)$, an indeterminacy-membership function $I_{A}(x)$, and a falsitymembership function $F_{A}(x)$, where $T_{A}(x), I_{A}(x), F_{A}(x) \in$ $[0,1]$. A SVNS A can thus be written as

$$
\begin{equation*}
\mathrm{A}=\left\{<\mathrm{x}: T_{A}(x), I_{A}(x), F_{A}(x)>, \mathrm{x} \in \mathrm{X}\right\} \tag{2}
\end{equation*}
$$

Definition 3.[8]The complement of a neutrosophic set $A$ denoted by $A^{c}$, is as defined below for all $x \in X$ :

$$
T_{A}^{c}(x)=F_{A}(x), I_{A}^{c}(x)=1-I_{A}(x), F_{A}^{c}(x)=T_{A}(x) .
$$

Definition 4.[8]Let $A$ and $B$ be two neutrosophic sets in a universe of discourse $X$. Then the union and intersection of $A$ and $B$ is as defined below for all $x \in X$ :
(i) The union of $A$ and $B$, denoted as $A \cup B$, is defined as:

$$
\left.A \cup B=\left\{\begin{array}{l}
x, T_{A}(x) \vee T_{B}(x),  \tag{3}\\
I_{A}(x) \wedge I_{B}(x), \\
F_{A}(x) \wedge F_{B}(x)
\end{array}\right): x \in X\right\}
$$

(ii) The intersection of $A$ and $B$, denoted as $A \cap B$, is defined as:

$$
A \cap B=\left\{\left(\begin{array}{l}
x, T_{A}(x) \wedge T_{B}(x),  \tag{4}\\
I_{A}(x) \vee I_{B}(x), \\
F_{A}(x) \vee F_{B}(x)
\end{array}\right): x \in X\right\}
$$

Definition 5. [10]Let $U$ be an initial set and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$, and let $A \rightarrow E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$.

Definition 6.[34]A complex neutrosophic set $A$ defined on a universe of discourse $X$, is characterized by a truth membership function $T_{A}(x)$, an indeterminacy membership function $I_{A}(x)$, and a falsity membership function $F_{A}(x)$ that assigns a complex-valued grade for each of these membership function in $A$ for any $x \in X$. The values of $T_{A}(x), I_{A}(x)$ and $F_{A}(x)$ and their sum may assume any values within a unit circle in the complex plane, and is of the form $T_{A}(x)=\rho_{A}(x) \cdot e^{j \mu_{A}(x)}, I_{A}(x)=q_{A}(x) \cdot e^{j \nu_{A}(x)}, \quad$ and $F_{A}(x)=r_{A}(x) \cdot e^{j \omega_{A}(x)}$. All the amplitude and phase terms are real-valued and $p_{A}(x), q_{A}(x), r_{A}(x) \in[0,1]$, whereas $\mu_{A}(x), v_{A}(x), \omega_{A}(x) \in[0,2 \pi]$, such that the following condition is satisfied:

$$
\left.{ }^{-} 0 \leq p_{A}(x)+q_{A}(x)+r_{A}(x) \leq 3^{+} . \text {. } 5\right)
$$

A complex neutrosophic set $A$ can thus be represented in set form as:
$A=\left\{\left(x, T_{A}(x)=a_{T}, I_{A}(x)=a_{I}, F_{A}(x)=a_{F}\right): x \in X\right\},$,
where $T_{A}: X \rightarrow\left\{a_{T}: a_{T} \in C,\left|a_{T}\right| \leq 1\right\}$,
$I_{A}: X \rightarrow\left\{a_{I}: a_{I} \in C,\left|a_{I}\right| \leq 1\right\}, F_{A}: X \rightarrow\left\{a_{F}: a_{F} \in C,\left|a_{F}\right| \leq 1\right\}$, and

$$
\begin{equation*}
\left|T_{A}(x)+I_{A}(x)+F_{A}(x)\right| \leq 3 . \tag{6}
\end{equation*}
$$

Definition 7. [34]Let $A=\left\{\left\langle x, T_{A}(x), I_{A}(x), F_{A}(x)\right\rangle: x \in X\right\}$ be a complexneutrosophic set over $X$. Then the complement of $A$, denoted by $A^{c}$, is defined as:
$A^{c}=\left\{\left\langle x, T_{A}^{c}(x), I_{A}^{c}(x), F_{A}^{c}(x)\right\rangle: x \in X\right\}$,
where
$T_{A}^{c}(x)=r_{A}(x) e^{j \mu_{A}^{c}(x)}$, such that $\mu_{A}^{c}(x)=\mu_{A}(x)$ or $2 \pi-\mu_{A}(x)$
or $\mu_{A}(x)+\pi$;
$I_{A}^{c}(x)=1-q_{A}(x) e^{j v_{A}^{c}(x)}$, such that $v_{A}^{c}(x)=v_{A}(x)$ or $2 \pi-v_{A}(x)$ or $v_{A}(x)+\pi$;
$F_{A}^{c}(x)=p_{A}(x) e^{j \omega_{A}^{c}(x)}$, such that $\omega_{A}^{c}(x)=\omega_{A}(x)$ or $2 \pi-\omega_{A}(x)$ or $\omega_{A}(x)+\pi$.

## III. COMPLEX NEUTROSOPHIC SOFT SETS

In this section, we introduce the complex neutrosophic soft set (CNSS) model which is a hybrid of the CNS and soft set models. The formal definition of this model as well as some concepts related to this modelare as given below:

Definition 8. Let $U$ be universal set, $E$ be the set of parameters under consideration, $A \subseteq E$, and $\psi_{A}$ be a CNS over $U$ for all $x \in U$. Then a complex neutrosophic soft set $\chi_{A}$ over $U$ is defined as a mapping $\chi_{A}: E \rightarrow C N(U)$, where $C N(U)$ denotes the set of complex neutrosophic sets of $U$, and $\psi_{A}(x)=\varnothing$ if $x \notin A$. Here $\psi_{A}(x)$ is called a complex neutrosophic approximate function of $\chi_{A}$ and the values of $\psi_{A}(x)$ is called thex-elements of the CNSS for all $x \in U$. Thus, $\chi_{A}$ can be represented by the set of ordered pairs of the following form:

$$
\chi_{A}=\left\{\left(x, \psi_{A}(x)\right): x \in E, \psi_{A}(x) \in C N(U)\right\},
$$

where $\psi_{A}(x)=\left(p_{A}(x) e^{j \mu_{A}(x)}, q_{A}(x) e^{j \nu_{A}(x)}, r_{A}(x) e^{j \omega_{A}(x)}\right)$.
Example 1. LetUbe a set of developing countries in the Southeast Asian (SEA) region, that are under consideration, $E$ be a set of parameters that describe a country's economic indicators, and $A=\left\{e_{1}, e_{2}, e_{3}, e_{4}\right\} \subseteq E$, where these sets are as defined below:
$U=\left\{h_{1}=\right.$ Republic of Philippines, $h_{2}=$ Vietnam,
$h_{3}=$ Myanmar, $h_{4}=$ Indonesia $\}$,
and
$E=\left\{e_{1}=\right.$ inflation rate, $e_{2}=$ population growth, $\mathrm{e}_{3}=$ GDP growth rate,$e_{4}=$ unemployment rate, $\mathrm{e}_{5}=$ volume of exports $\}$.

The CNS $\psi_{A}\left(e_{1}\right)$ and $\psi_{A}\left(e_{3}\right)$ are defined as:

$$
\psi_{A}\left(e_{1}\right)
$$

$$
=\left\{\begin{array}{l}
\frac{\left(0.6 \mathrm{e}^{j 0.8 \pi}, 0.3 e^{j \frac{3 \pi}{4}}, 0.5 e^{j 0.3 \pi}\right)}{h_{1}}, \frac{\left(0.7 \mathrm{e}^{j 0 \pi}, 0.2 e^{j 0.9 \pi}, 0.1 e^{j \frac{2 \pi}{3}}\right)}{h_{2}}, \\
\frac{\left(0.9 \mathrm{e}^{j 0.1 \pi}, 0.4 e^{j \pi}, 0.7 e^{j 0.7 \pi}\right)}{h_{3}}, \frac{\left(0.3 \mathrm{e}^{j 0.4 \pi}, 0.2 e^{j 0.6 \pi}, 0.7 e^{j 0.5 \pi}\right)}{h_{4}}
\end{array}\right\},
$$

$$
\psi_{A}\left(e_{2}\right)
$$

$$
=\left\{\begin{array}{l}
\frac{\left(0.2 \mathrm{e}^{j 0.2 \pi}, 0.5 e^{j \frac{3 \pi}{4}}, 0.6 e^{j 0.3 \pi}\right)}{h_{1}}, \frac{\left(0.4 \mathrm{e}^{j 0.3 \pi}, 0.4 e^{j 0.4 \pi}, 0.2 e^{j \frac{j \pi}{3}}\right)}{h_{2}}, \\
\frac{\left(0.3 \mathrm{e}^{j 0.1 \pi}, 0.2 e^{j 0.1 \pi}, 0.3 e^{j 0.7 \pi}\right)}{h_{3}}, \frac{\left(0.1 \mathrm{e}^{j 0.4 \pi}, 0.5 e^{j 1.2 \pi}, 0.3 e^{j 0.1 \pi}\right)}{h_{4}}
\end{array}\right\},
$$

$$
\psi_{A}\left(e_{3}\right)
$$

$$
=\left\{\begin{array}{l}
\frac{\left(0.4 \mathrm{e}^{j 0.4 \pi}, 0.1 e^{j \frac{\pi}{4}}, 0.2 e^{j 0.1 \pi}\right)}{h_{1}}, \frac{\left(0.3 \mathrm{e}^{j 0.2 \pi}, 0.3 e^{j 0.4 \pi}, 0.2 e^{j \frac{\pi}{3}}\right)}{h_{2}}, \\
\frac{\left(0.2 \mathrm{e}^{j 0.1 \pi}, 0.4 e^{j 0.5 \pi}, 0.5 e^{j 0.2 \pi}\right)}{h_{3}}, \frac{\left(0.5 \mathrm{e}^{j 0.2 \pi}, 0.4 e^{j 2 \pi}, 0.6 e^{j 0.1 \pi}\right)}{h_{4}}
\end{array}\right\}
$$

and

$$
\begin{aligned}
& \psi_{A}\left(e_{4}\right) \\
& =\left\{\begin{array}{l}
\frac{\left(0.3 \mathrm{e}^{j 0.2 \pi}, 0.5 e^{j \frac{\pi}{4}}, 0.5 e^{j 0.1 \pi}\right)}{h_{1}}, \frac{\left(0.1 \mathrm{e}^{j 0 \pi}, 0.6 e^{j 0.4 \pi}, 0.4 e^{j \frac{\pi}{3}}\right)}{h_{2}}, \\
\frac{\left(0.1 \mathrm{e}^{j 0.1 \pi}, 0.2 e^{j 0.2 \pi}, 0.4 e^{j 0.2 \pi}\right)}{h_{3}}, \frac{\left(0.2 \mathrm{e}^{j 0.2 \pi}, 0.5 e^{j 0.3 \pi}, 0.3 e^{j 0.1 \pi}\right)}{h_{4}}
\end{array}\right\} .
\end{aligned}
$$

Then the complex neutrosophic soft set $\chi_{A}$ can be written as a collection of CNSs in the form $\chi_{A}=\left\{\psi_{A}\left(e_{1}\right), \psi_{A}\left(e_{2}\right), \psi_{A}\left(e_{3}\right), \psi_{A}\left(e_{4}\right)\right\}$.

Defintion 9.Let $\chi_{A}$ and $\chi_{B}$ be two CNSSs over a universe $U$. Then we have the following:
(i) $\chi_{A}$ is said to be an empty CNSS, denoted by $\chi_{A_{8}}$, if $\psi_{A}(x)=\varnothing$, for all $x \in U$;
(ii) $\chi_{A}$ is said to be an absolute CNSS, denoted by $\chi_{A_{v}}$, if $\psi_{A}(x)=U$, for all $x \in U$;
(iii) $\chi_{A}$ is said to be a CNS-subset of $\chi_{B}$, denoted by $\chi_{A} \subseteq \chi_{B}$, if for all $x \in U, \psi_{A}(\mathrm{e}) \subseteq \psi_{B}(\mathrm{e})$, that is the following conditions are satisfied:
$\rho_{A}(e) \leq \rho_{B}(e), q_{A}(e) \leq q_{B}(e), r_{A}(e) \leq r_{B}(e)$,
and $\mu_{A}(e) \leq \mu_{B}(e), v_{A}(e) \leq v_{B}(e), \omega_{A}(e) \leq \omega_{B}(e)$.
(iv) $\chi_{A}$ is said to be equal to $\chi_{B}$, denoted by $\chi_{A}=\chi_{B}$,
iffor all $x \in U$, the following conditions are satisfied:

$$
\begin{aligned}
& \rho_{A}(e)=\rho_{B}(e), q_{A}(e)=q_{B}(e), r_{A}(e)=r_{B}(e) \\
& \text { and } \mu_{A}(e)=\mu_{B}(e), v_{A}(e)=v_{B}(e), \omega_{A}(e)=\omega_{B}(e)
\end{aligned}
$$

Proposition 1.Let $\chi_{A} \in C N(U)$. Then the following hold:
(i) $\left(\chi_{A}^{c}\right)^{c}=\chi_{A}$;
(ii) $\chi_{A_{b}}^{c}=\chi_{A_{v}}$.

Proof. The proofs are straightforward from Definition 9.

## IV. OPERATIONS ON COMPLEX NEUTROSOPHIC SOFT SETS

In this section we define the basic set theoretic operations on CNSSs, namely the complement, union and intersection.

Let $\chi_{A}$ and $\chi_{B}$ be two CNSSs over a universe $U$.

Definition 10.The complement of $\chi_{A}$, denoted by $\chi_{A}^{c}$, is a CNSS defined by $\chi_{A}^{c}=\left\{\left(x, \psi_{A}^{c}(x)\right): x \in U\right\}$, where $\psi_{A}^{c}(x)$ is the complex neutrosophic complement of $\psi_{A}$.

Example 2. Consider Example 1. The complement of $\chi_{A}$ is given by $\chi_{A}^{c}=\left\{\psi_{A}^{c}\left(e_{1}\right), \psi_{A}^{c}\left(e_{2}\right), \psi_{A}^{c}\left(e_{3}\right), \psi_{A}^{c}\left(e_{4}\right)\right\}$. For the sake of brevity, we only give the complement for $\psi\left(e_{1}\right)$ below:

$$
\begin{aligned}
& \psi_{A}^{c}\left(e_{1}\right) \\
& =\left\{\begin{array}{l}
\frac{\left(0.5 \mathrm{e}^{j 1.2 \pi}, 0.7 e^{j \frac{5 \pi}{4}}, 0.6 e^{j 1.7 \pi}\right)}{h_{1}}, \frac{\left(0.1 \mathrm{e}^{j 2 \pi}, 0.8 e^{j 1.1 \pi}, 0.7 e^{j \frac{4 \pi}{3}}\right)}{h_{2}} \\
\frac{\left(0.7 \mathrm{e}^{j 1.9 \pi}, 0.6 e^{j \pi}, 0.9 e^{j 1.3 \pi}\right)}{h_{3}}, \frac{\left(0.7 \mathrm{e}^{j 1.6 \pi}, 0.8 e^{j 1.4 \pi}, 0.3 e^{j 1.5 \pi}\right)}{h_{4}}
\end{array}\right\} .
\end{aligned}
$$

Definition11.The union of $\chi_{A}$ and $\chi_{B}$, denoted by $\chi_{A} \tilde{\cup} \chi_{B}$, is defined as:

$$
\chi_{C}=\chi_{A} \tilde{\cup} \chi_{B}=\left(x, \psi_{A}(x) \tilde{\cup} \psi_{B}(x)\right)
$$

where $C=A \cup B, x \in U$, and

$$
\psi_{A}(x) \tilde{\cup} \psi_{B}(x)=\left\{\begin{array}{l}
\left(p_{A}(x) \vee p_{B}(x)\right) e^{j\left(\mu_{A}(x) \cup \mu_{B}(x)\right)} \\
\left(q_{A}(x) \wedge q_{B}(x)\right) e^{j\left(v_{A}(x) \cup v_{B}(x)\right)} \\
\left(r_{A}(x) \wedge r_{B}(x)\right) e^{j\left(\omega_{A}(x) \cup \omega_{B}(x)\right)}
\end{array}\right\}
$$

where $\vee$ and $\wedge$ denote the max and min operators respectively, whereas the phase terms of the truth, indeterminacy and falsity
functions lie in the interval $(0,2 \pi]$, and can be calculated using any one of the following operators:
(i) Sum:
$\mu_{A \cup B}(x)=\mu_{A}(x)+\mu_{B}(x), v_{A \cup B}(x)=v_{A}(x)+v_{B}(x)$, and $\omega_{A \cup B}(x)=\omega_{A}(x)+\omega_{B}(x)$.
(ii) Max:
$\mu_{A \cup B}(x)=\max \left(\mu_{A}(x), \mu_{B}(x)\right), v_{A \cup B}(x)=\max \left(v_{A}(x), v_{B}(x)\right)$, and $\omega_{A \cup B}(x)=\max \left(\omega_{A}(x), \omega_{B}(x)\right)$.
(iii) Min:
$\mu_{A \cup B}(x)=\min \left(\mu_{A}(x), \mu_{B}(x)\right), v_{A \cup B}(x)=\min \left(v_{A}(x), v_{B}(x)\right)$, and $\omega_{A \cup B}(x)=\min \left(\omega_{A}(x), \omega_{B}(x)\right)$.
(iv) "The game of winner, neutral, and loser":
$\mu_{A \cup B}(x)=\left\{\begin{array}{lll}\mu_{A}(x) & \text { if } & p_{A}>p_{B} \\ \mu_{B}(x) & \text { if } & p_{B}>p_{A}\end{array}\right.$,
$v_{A \cup B}(x)=\left\{\begin{array}{lll}v_{A}(x) & \text { if } & q_{A}<q_{B} \\ v_{B}(x) & \text { if } & q_{B}<q_{A}\end{array}\right.$, and
$\omega_{A \cup B}(x)=\left\{\begin{array}{lll}\omega_{A}(x) & \text { if } & r_{A}<r_{B} \\ \omega_{B}(x) & \text { if } & r_{B}<r_{A}\end{array}\right.$.
The game of winner, neutral, and loser is an adaptation of the similar operator defined in [23].

Definition 12.Theintersection of $\chi_{A}$ and $\chi_{B}$, denoted by $\chi_{D}=\chi_{A} \tilde{\sim} \chi_{B}$, is defined as:

$$
\chi_{D}=\chi_{A} \tilde{\cap} \chi_{B}=\left(x, \psi_{A}(x) \tilde{\cap} \psi_{B}(x)\right)
$$

where $D=A \cap B, x \in U$, and

$$
\psi_{A}(x) \tilde{\sim} \psi_{B}(x)=\left\{\begin{array}{l}
\left(p_{A}(x) \wedge p_{B}(x)\right) e^{j\left(\mu_{A}(x) \cap \mu_{B}(x)\right)} \\
\left(q_{A}(x) \vee q_{B}(x)\right) e^{j\left(v_{A}(x) \cap \nu_{B}(x)\right)} \\
\left(r_{A}(x) \vee r_{B}(x)\right) e^{j\left(\omega_{A}(x) \cap \omega_{B}(x)\right)}
\end{array}\right\}
$$

Here $\vee$ and $\wedge$ once again denote the max and min operators respectively, whereas the phase terms of the truth, indeterminacy and falsity functions also lie in the interval $(0,2 \pi]$, and can be calculated using any one of theoperators given in Definition 11.

## V. APPLICATION OF THE CNSS MODEL IN A DECISION MAKING PROBLEM

In Example 1, we presented an example related to the economic indicators of four countries. In this section, we use the same information to determine which one of the four countries that are studied has the strongest economic indicators. To achieve this, a modified algorithm and an accompanying score function is presented in Definition 13 and 14. This algorithm and score function are an adaptation of the corresponding concepts
introduced in [35], which was then made compatible with the structure of the CNSS model. The steps involved in the decision making process, in the context of this example, until a final decision is reached, is as given below.

Definition 13. A comparison matrix is a matrix whose rows consists of the elements of the universal set $U, u_{1}, u_{2}, \ldots, u_{m}$, whereas the columns consists of the corresponding parameters $e_{1}, e_{2}, \ldots, e_{n}$ that are being considered in the problem. The entries of this matrix are $c_{i j}$, where $c_{i j}=\left(\alpha_{a m p}+\beta_{\text {amp }}-\gamma_{a m p}\right)+\left(\alpha_{\text {phase }}+\beta_{\text {phase }}-\gamma_{\text {phase }}\right)$, where the components of this formula are as defined below for all $b_{k} \in U$, such that $b_{i} \neq b_{k}$ :
$\alpha_{\text {amp }}=$ the number of times the value of the amplitude term of

$$
T_{b_{i}}\left(e_{j}\right) \geq T_{b_{k}}\left(e_{j}\right)
$$

$\beta_{\text {amp }}=$ the number of times the value of the amplitude term of

$$
I_{b_{i}}\left(e_{j}\right) \geq I_{b_{k}}\left(e_{j}\right)
$$

$\gamma_{a m p}=$ the number of times the value of the amplitude term of

$$
F_{b_{i}}\left(e_{j}\right) \geq F_{b_{k}}\left(e_{j}\right),
$$

and
$\alpha_{\text {phase }}=$ the number of times the value of the phase term of

$$
T_{b_{i}}\left(e_{j}\right) \geq T_{b_{k}}\left(e_{j}\right)
$$

$\beta_{\text {phase }}=$ the number of times the value of the phase term of

$$
I_{b_{i}}\left(e_{j}\right) \geq I_{b_{k}}\left(e_{j}\right)
$$

$\gamma_{\text {phase }}=$ the number of times the value of the phase term of

$$
F_{b_{i}}\left(e_{j}\right) \geq F_{b_{k}}\left(e_{j}\right) .
$$

Definition 14. The score of an element $u_{i}$ can be calculated by the score function $S_{i}$ which is defined as $S_{i}=\sum_{j} c_{i j}$.
Next, we apply the algorithm and score function in a decision making problem. The steps are as given below:

## Step 1: Define a CNSS

Construct a CNSS for the problem that is being studied, which includes the elements $u_{i}(i=1,2, \ldots, m)$, and the set of parameters $e_{j}(j=1,2, \ldots, n)$, that are being considered.
In the context of this example, the universal set $U$, set of parameters $A$, and the CNSS $\chi_{A}$ that were defined in Example 1 will be used.

Step 2: Construct and compute the comparison matrix
A comparison matrix is constructed, and the values of $c_{i j}$ for each element $u_{i}$ and the corresponding parameter $e_{j}$ is calculated using the formula given in Definition 13. For this example, the comparison matrix is given in Table 1.

Table 1.Comparison matrix for $\chi_{A}$

| $U$ | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ |
| ---: | ---: | ---: | ---: | ---: |
| $h_{1}$ | 6 | 3 | 3 | 5 |
| $h_{2}$ | 3 | 5 | 1 | 2 |
| $h_{3}$ | 4 | -3 | 1 | -1 |
| $h_{4}$ | -1 | 7 | 5 | 8 |

## Step 3: Calculate the score function

Compute the scores $S_{i}$ for each element $u_{i}(i=1,2, \ldots, m)$ using the score function given in Definition 14. The score values obtained for this example are given in Table 2.

Table 2. Score function for $\chi_{A}$

| $U$ | $S_{i}$ |
| :---: | :---: |
| $h_{1}$ | 17 |
| $h_{2}$ | 11 |
| $h_{3}$ | 1 |
| $h_{4}$ | 19 |

Step 4: Conclusion and discussion
The values of the score function are compared and the element with the maximum score will be chosen as the optimal alternative. In the event that there are more than one elementwith the maximum score, any of the elements may be chosen as the optimal alternative.
In the context of this example, $\max _{u_{i} \in U}\left\{S_{i}\right\}=h_{4}$. As such, it can be concluded that country $h_{4}$ i.e. the Indonesia is the country with
the strongest economic indicators, followed closely by the Republic of Phillipines and Vietnam among the four developing countries in the SEA region that were considered, whereas Myanmar is identified as the country with the weakest and slowest growing economy, among the four countries.

## VI. CONCLUSION

In this paper, we introduced the complex neutrosophic soft set model which is a hybrid between the complex neutrosophic set and soft set models. This model has a more generalized framework than the fuzzy soft set, neutrosophic set, complex fuzzy set models and their respective generalizations. The basic set theoretic operations were defined. The CNSS model was then applied in a decision making problem involving to demonstrate its utility in representing the uncertainty and indeterminacy that exists when dealing with uncertain and subjective data.

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