A Proof of Goldbach's Conjecture
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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. We give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture, fundamental theorem of arithmetic, Euclid's proof of infinite primes

Introduction

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many “advanced mathematics tools” are used to solve them, but they are still unsolved.

I believe that prime numbers are “basic building blocks” of the natural numbers and they must follow some very simple basic rules and do not need “advanced mathematics tools” to solve them. One of the basic rules is the “fundamental theorem of arithmetic” and the “simplest tool” is Euclid’s proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors. Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof that the set of prime numbers is endless
The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that \( P_1 = 2 \), \( P_2 = 3 \), \( P_3 = 5 \) and so on. If we assume that there are just \( n \) primes, then the biggest prime will be labeled \( P_n \). Now we can form the number \( Q \) by multiplying together all these primes and adding 1, so

\[
Q = (P_1 \times P_2 \times P_3 \times P_4 \ldots \times P_n) + 1
\]

Now we can see that if we divide \( Q \) by any of our \( n \) primes there is always a remainder of 1, so \( Q \) is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either \( Q \) must be a prime or \( Q \) must be divisible by primes that are larger than \( P_n \).

Our assumption that \( P_n \) is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

**Discussions**

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

If \( N \) is an even integer:

\[
N = \frac{N}{2} + \frac{N}{2} = (\frac{N}{2}+m) + (\frac{N}{2}-m); \quad m = 0, 1, 2, 3, \ldots, M.
\]

We need to prove \([(\frac{N}{2}+m) \text{ and } (\frac{N}{2}-m)] \text{ can be primes at same time.}

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach’s conjecture is about all numbers, the pattern of prime numbers likes a “kaleidoscope” of numbers, if we divide the numbers in groups, the Goldbach’s conjecture will be much simpler.
Let $N_o$ represent any odd number, the chance of $N_o$ to be a non-prime is: \[
(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) +
(1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) +
(1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) +
(1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) +
(1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) +
(1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) +
(1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) +
(1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) +
(1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 + \ldots)\}
\text{-------Formula 1}

Let $\Sigma$ represent the sum of the infinitely terms and $\Delta=1-\Sigma$, according to Euclid's proof\(^3\) that the set of prime numbers is endless. $\Delta$ is the chance of any number to be a prime.

$\Sigma$ may be very close to 1 when $N$ is growing to $\infty$, but always less than 1. Let $\Delta=1-\Sigma$, when $N$ is growing to $\infty$, $\Delta$ may be very close to 0, but always more than 0 according to Euclid's proof\(^3\) that the set of prime numbers is endless. If $\Delta$ is 0, then there is no prime.

The chance of $N_o$ to be a prime is: $\Delta=1-[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) +
(1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) +
(1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) +
(1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) +
(1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) +
(1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) +
(1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) +
(1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) +
(1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 + \ldots)\}
\text{-------Formula 2}

Let us consider the following cases:

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 110, 120, 1120, 1130,…, then N/2 has only 0 or 5 as its last digit:
1a. Except 5, any prime must have 1, 3, 7, or 9 as its last digit. When both N and N/2 have 0 as their last digit, then N must be 20, 30, 40, 60, 80, 100, 120,…, N. For enough large number N, Let’s consider $N=O_1+O_2=(N/2+L+3)+(N/2-L-3)$, $O_1$ and $O_2$ is an odd number. $O_1>O_2$, $O_1-O_2=2L+6$, $L=0, 5, 10, 15, 20, 25, 30,…L$, $O_1-O_2=2L+6=6, 26, 46, 66, 86, 106, 126,…(2L+6)$, then $O_1$ is an odd number with 3 as its last digit, $O_2$ is an odd number with 7 as its last digit.

Also we can have $N=O_1+O_2=(N/2+L+7)+(N/2-L-7)$, $O_1$ and $O_2$ is an odd number. $O_1>O_2$, $O_1-O_2=2L+14$, $L=0, 10, 20, 30,…L$, $O_1-O_2=2L+14=14, 34, 54, 74, 94, 114, 134,…(2L+14)$, then $O_1$ is an odd number with 7 as its last digit, $O_2$ is an odd number with 3 as its last digit.

Then, we have odd number pairs as listed in table 1:

<table>
<thead>
<tr>
<th>N-7</th>
<th>N-17</th>
<th>N-37</th>
<th>N-47</th>
<th>N-67</th>
<th>N-97</th>
<th>…</th>
<th>N/2+L+3</th>
<th>N/2-L-7</th>
<th>…</th>
<th>83</th>
<th>73</th>
<th>53</th>
<th>43</th>
<th>23</th>
<th>13</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>17</td>
<td>37</td>
<td>47</td>
<td>67</td>
<td>97</td>
<td>…</td>
<td>N/2-L-3</td>
<td>N/2+L+7</td>
<td>…</td>
<td>83</td>
<td>73</td>
<td>53</td>
<td>43</td>
<td>23</td>
<td>13</td>
<td>3</td>
</tr>
</tbody>
</table>

Let $1$ represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,…; $3$ represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193,…; $7$ represents a prime with 7 as its last digit, such as 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197,…; and $9$ represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,….
Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,…; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,…; O7 represents an odd number with 7 as its last digit, such as 7, 17, 27, 37, 47, 57, 67, 77…; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,….

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number with 3 as its last digit is a product of $3 \times 1$ or $7 \times 9$; $1$ is decided by $3$ and $9$ is decided by $7$, so we need to consider only $3$ and $7$.

For number $N$, there are $N/10$ odd numbers. According to Euclid's proof, primes are endless and it is easy to prove that prime with 3 as its last digit is also endless.

If a number ($N>3$) is not divisible by 3 or any prime which is smaller or equal to $N/3$, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 13, it has 1/7 chance to be divisible by 7, so on, we have terms: 1/3, 1/7x2/3, 1/13x2/3x6/7/…., For number $N$, there are $N/10$ odd number with 3 as its last digit, $N/10$ odd number with 7 as its last digit, and $N/10$ odd number with 9 as its last digit; The number (n) of primes in $N/10$ odd number with 3 as its last digit: $n = N/10 - \{N/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23) + (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…] \} \text{----------(Formula 3)}$

For infinitely terms, the number will grow slowly and will be close to 1, but never equal to 1 (if it equal to 1, we will have 0 prime) according to Euclid’s proof of endless prime numbers. Let $\sum_3$ represent the sum of the above infinitely terms. When $N$ is growing to $\infty$, and $\Delta_3=1-\sum_3 > 1-\sum$ may be close to 0, but never be 0.

The sum of first 20 terms = $[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x12/13x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) + (1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) + (1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83)$
\[\begin{align*}
+(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) \\
+(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) \\
+(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) \\
+(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + \\
(1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + \\
(1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x13/167) + \\
(1/167x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x13/167x156/157) \\
=N/10[1/3+1/10.5+1/22.75+1/32.23+1/46.33+1/77.92+1/93.07+1/104.15+1/120+1/164.61+1/171.01+1/197.14+1/233.2+1/250.2+1/262.47+1/279.80+1/317.27+1/344.97+1/398.24+1/416.11]+[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333+0.005848+0.0050773+0.004288+0.003997+0.003574+0.003152+0.002899+0.002511+0.002403+0.002331]=0.6102883
\]

For the first 20 term: \( \sum_3 = 0.6103, \Delta_3 = 1-\sum_3 = 0.3897 \)

For \( N=600 \), the smallest prime \$3 \) is 11, it decided the possible largest prime \$3 \) is 53, the smallest prime \$9 \) is 19, but \( 3 \times 3 \) is 9, so the possible largest prime \$7 \) is 47, \( 47 \times 3 \times 3 = 423 \) (the next will \( 67 \times 3 \times 3 = 603 \geq 600 \)), so we have: Prime number with \$3 \) as its last digit=600/10 – {600/10[(1/3)+ (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47)]} = 600/10-600/10{0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333=60-60x0.566654=60-34=26, it is 3 less than 29 primes (in total 60 odd number) with \$3 \) as their last digit from 1 to 600 due to the first three odd numbers 3, 13, and 23 is too small and cannot be divisible by primes 7, 11, 13, 17, or 19. When \( N \) is big enough, the calculated number will be very close to the real number. . For \( N=600 \), we have \( \Delta_3 = 1-\sum_3 = 0.566654 \), every odd number with \$3 \) as its last digit has almost 43% chance to be a prime number smaller than 600, every odd number with \$3 \) as its last digit has more than 43% chance to be a prime; for a number bigger than 600, every odd number with \$3 \) as its last digit has less than 43% chance to be a prime.

Every odd number with \$7 \) as its last digit is a product of \$3x9 or \$7x1; \$1 \) is decided by \$7 \) and \$9 \) is decided by \$3 \), so we need to consider only \$3 \) and \$7 \).
The number \( n \) of primes in \( N/10 \) odd number with 7 as its last digit is:

\[
n = \frac{N}{10} - \left\{ \frac{N}{10} \left( \frac{1}{3} + \frac{1}{7} \times \frac{2}{3} + \frac{1}{13} \times \frac{2}{3} \times \frac{6}{7} \right) + \frac{1}{17} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} + \frac{1}{23} \times \frac{2}{3} \times \frac{12}{13} \times \frac{16}{17} + \frac{1}{37} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} \times 16 \times 17 \times 22 \times 23 + \frac{1}{43} \times \frac{2}{3} \times \frac{6}{7} \times 16 \times 17 \times 36 \times 37 \times 12 \times 23 \times 42 \times 43 \right\}
\]  
--------Formula 3.

Let \( \sum_7 \) represent the sum of the above infinitely terms. \( \sum_7 = \sum_3 \), when \( N \) is growing to \( \infty \), and \( \Delta_7 = 1 - \sum_7 > 1 - \sum \) may be close to 0, but never be 0.

That mean we have almost same number of primes with 3 as their last digit as the number of primes with 7 as their last digit.

From above formula, we can know smaller number \( N \) has high percentage to be primes than bigger number \( N \).

For formula 3, we also can know: for any odd number \( (O_7) \) with 7 as its last digit, the chance of \( N - O_7 \) to be an odd number (but not prime) with 3 as its last digit is:

\[
\left( \frac{1}{3} + \frac{1}{7} \times \frac{2}{3} + \frac{1}{13} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} + \frac{1}{23} \times \frac{2}{3} \times \frac{12}{13} \times \frac{16}{17} + \frac{1}{37} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} \times 16 \times 17 \times 22 \times 23 + \frac{1}{43} \times \frac{2}{3} \times \frac{6}{7} \times 16 \times 17 \times 36 \times 37 \times 12 \times 23 \times 42 \times 43 \right) 
\]  
--------Formula 4

If this \( O_7 = \) a prime \$7, the chance of \( (N - \$7) \) to be an odd number (but not prime) with 3 as its last digit is smaller than \( (N - O_7) \). Why?

Let see \$7=7 first (left side of table 1). When \$7 is 7, \( N = O_1 + O_2 \), \( O_1 - O_2 = (N-7) = 2L + 6 \). Only 56, 126, 196, ..., is divisible by 7, however, \( L=0, 10, 20, 30, 40, ..., (2L+6) = 6, 26, 46, 66, 86, 106, 126, 146, 166, 186, 206, ..., \) only 1 in 3 is shown on \( (2L+6) \) that is divisible by 7, so the term \( \frac{1}{7} \times \frac{2}{3} \), not \( \frac{1}{7} \) should be taken off from Formula 3.

The chance of \( (N-7) \) to be an odd number (not prime) with 3 as its last digit is:

\[
\left( \frac{1}{3} + \frac{1}{13} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} + \frac{1}{23} \times \frac{2}{3} \times \frac{12}{13} \times \frac{16}{17} + \frac{1}{37} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} \times 16 \times 17 \times 22 \times 23 + \frac{1}{43} \times \frac{2}{3} \times \frac{6}{7} \times 16 \times 17 \times 36 \times 37 \times 12 \times 23 \times 42 \times 43 \right) 
\]  
--------Formula 5

For the next prime \$7=17, the chance of \( (N-17) \) to be an odd number (not prime) with 3 as its last digit is:

\[
\left( \frac{1}{3} + \frac{1}{13} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} + \frac{1}{23} \times \frac{2}{3} \times \frac{12}{13} \times \frac{16}{17} + \frac{1}{37} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} \times 16 \times 17 \times 22 \times 23 + \frac{1}{43} \times \frac{2}{3} \times \frac{6}{7} \times 16 \times 17 \times 36 \times 37 \times 12 \times 23 \times 42 \times 43 \right) 
\]  
--------Formula 6

Term \( \frac{1}{17} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} \) should be taken off from Formula 3, so on.... Finally, we have:

\[
\left( \frac{1}{3} + \frac{1}{13} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} + \frac{1}{23} \times \frac{2}{3} \times \frac{12}{13} \times \frac{16}{17} + \frac{1}{37} \times \frac{2}{3} \times \frac{6}{7} \times \frac{12}{13} \times 16 \times 17 \times 22 \times 23 + \frac{1}{43} \times \frac{2}{3} \times \frac{6}{7} \times 16 \times 17 \times 36 \times 37 \times 12 \times 23 \times 42 \times 43 \right) 
\]  
--------Formula 7
For the right side of table 1, starting from $S=3$, $N = O_1+O_2$, $O_1-O_2 = (N-3)=2L+14$, The chance of $(N-3)$ to be an odd number (not prime) with 7 as its last digit is: $[(1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23) + (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43)…]----------Formula 8.

Term (1/3) should be taken off from Formula 3.

For the next prime 13, the chance of $(N-13)$ to be an odd number (not prime) with 3 as its last digit is: $[(1/3)+(1/7x2/3) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]----------Formula 9

The term (1/13x2/3x6/7) should be removed from Formula 3, so on... Finally, the remainder term in Formula 7 are cancelled.

For number $N$, let $n_7$ be the number of primes $(N-7)$ with 3 as its last digit which matches a prime ($7$) with 7 as its last digit and $n_3$ be the number of primes $(N-3)$ with 7 as its last digit which matches a prime ($3$) with 3 as its last digit $n_7 = n_3$, $n = n_7 + n_3 = 2n_3$. $n$ is the total number of primes $(N-3)$ and $(N-7)$ that have 3 or 7 as their last digital and match $7$ or $3$.

$n=n_3 + n_7 = n_3 - [(n_3-1)(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…}] + n_7 - [(n_7-1)(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…}] =2n_3 = n_3 - [(n_3-1)(\Sigma_3)] + n_7 - [(n_7-1)(\Sigma_7)] =n-n\Sigma_3 + \Sigma_3=[n(1-\Sigma_3) + \Sigma_3]=n\Delta+\Sigma = n\Delta_3 + (1-\Delta_3)=1+n\Delta_3-\Delta_3=1+(n-1)\Delta_3>1$ because $n+1$ and $\Delta_3>0$. The results show: When both $N$ and $N/2$ have 0 as their last digit, there is at least one pair primes in which one prime has 7 as its last digit and another has 3 as its last digit and their sum is $N$ (see table 1). This is an extreme situation, normally, more than 1 pair primes can be found. For $N=600$, $n=(600/2)x(1/10)=30$, $\Sigma_3=0.57$ and $\Delta_3=0.43$, $n\Delta_3+\Sigma_3=30x0.57=13.5$, in fact, 600 can be expressed as the sum of 15 pairs of primes which has 7 as one prime last digit and 3 as another prime last digit.
For enough large number \( N \), let’s consider \( N = O_1 + O_2 = (N/2 + L + 1) + (N/2 - L - 1) \), \( O_1 \) and \( O_2 \) are odd numbers. \( O_1 > O_2, O_1 - O_2 = 2L + 2, L = 0, 10, 20, 30 … L \), \( O_1 - O_2 = 2L + 2 = 2, 22, 42, 62, 82, 102, 122, … (2L + 2) \), then \( O_1 \) is an odd number with 1 as its last digit, \( O_2 \) is an odd number with 9 as its last digit.

Also we can have \( N = O_1 + O_2 = (N/2 + L + 9) + (N/2 - L - 9) \), \( O_1 \) and \( O_2 \) are odd numbers. \( O_1 > O_2, O_1 - O_2 = 2L + 18, L = 0, 10, 20, 30 … L \), \( O_1 - O_2 = 2L + 18 = 18, 38, 58, 78, 98, 118, 138, … (2L + 18) \), then \( O_1 \) is an odd number with 9 as its last digit, \( O_2 \) is an odd number with 1 as its last digit.

These odd number pairs are listed in Table 2:

Table 2. The odd number pairs in \( N = O_1 + O_2 = (N/2 + L + 1) + (N/2 - L - 1) \) and \( N = O_1 + O_2 = (N/2 + L + 9) + (N/2 - L - 9) \)

| N-9 | N-19 | N-29 | N-39 | N-59 | N-79 | N-89 | … | N/2+L+1 | N/2-L-9 | … | 101 | 71 | 61 | 41 | 31 | 11 |
|-----|------|------|------|------|------|------|----|--------|--------|----|-----|----|----|----|----|
| 9(3x3) | 19 | 29 | 39 | 59 | 79 | 89 | … | N/2-L-1 | N/2+L+9 | … | N-101 | N-71 | N-61 | N-41 | N-31 | N-11 |

Every odd number \((O1)\) with 1 as its last digit is a product of \(1\times1, 3\times7, \) and \(9\times9\). The first \(9\) is \(19\), but the odd number \(9 = 3\times3\), so \(3\) is the smallest prime for \(9\) and the number \(n\) of primes in \(N/10\) odd number with 1 as its last digit: \(n = N/10 - \{1/3 + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) + (1/23x2/3x10/11x12/13x18/19x22/23) + (1/29x2/3x10/11x12/13x18/19x22/23x28/29) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41) + …\}\n
\[ \text{Formula 10} \]

The sum of first 20 terms = \((1/3 + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) + (1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + \)
\[(1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31)\] + \[(1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)\] + \[(1/53x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43)\] + \[(1/59x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53)\] + \[(1/61x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59)\] + \[(1/71x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61)\] + \[(1/73x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71)\] + \[(1/79x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73)\] + \[(1/83x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79)\] + \[(1/89x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83)\] + \[(1/101x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89)\] + \[(1/103x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101)\] + \[(1/109x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101x102/103)\] = 0.333333 + 0.046620 + 0.0029444 + 0.0046586 + 0.01748 + 0.01568 + 0.017047 + 0.008517 + 0.007506 + 0.006484 + 0.006032 + 0.005783 + 0.00529 + 0.004953 + 0.004564 + 0.003976 + 0.003612 = 0.63254

Every odd number (O9) with 9 as its last digit is a product term of $1x9$, $3x3$ or $7x7$, for $3x3$ and $7x7$, every odd number with 1 as its last digit will cost 2 $3$ or $7$, so the total terms will be same as O3 or O7. We need to consider only $1$, $3$, and $7$ and the number (n) of primes in N/10 odd number with 9 as its last digit: n = N/10 - \((N/10)((1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)\) \to \text{Formula 11}

The sum of first 20 terms = \((1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13)\) + \((1/21/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47) +
Let see $9=9(3x3)$ first (left side of table 2). When $9$ is $9$, $N = O_1+O_2$. $O_1-O_2 = (N-9)-2L+18$. Only, 18, 48, 78, 108, 138, 168, 198, 228, 258, 288, ..., is divisible by 3 or 9, however, $L=0, 10, 20, 30, 40,...$ and $(2L+18)=18, 38, 58, 78, 98, 118, 138, 158, 178, 198, 218, 238, 258, 278, 298, ...$, only 1 in 3 is shown on $(2L+18)$ that is divisible by 3, so the term $(1/3)$ should be taken off from Formula 3.

The chance of $(N-9)$ to be an odd number (not prime) with 1 as its last digit is: $[(1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x18/19) + (1/23x2/3x10/11x18/19x18/19x22/23) + (1/31x2/3x10/11x18/19x18/19x22/23x28/29) + (1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41)+...]

For the next prime $9=19$, the chance of $(N-19)$ to be an odd number (not prime) with 3 as its last digit is: $[(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/23x2/3x10/11x18/19x18/19x22/23) + (1/31x2/3x10/11x18/19x18/19x22/23x28/29) + (1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41)+...]

Term $(1/19x2/3x10/11x18/19)$ should be taken off from Formula 3, so on.... Finally, we have:
When \( \$1 \) is 11, \( N = O_1 + O_2 \), \( O_1 - O_2 = (N-9)-9 = 2L + 2 \) (right side of table 2), Only, 22, 132, 242, ..., is divisible by 11, however, \( L=0, 10, 20, 30, 40 \ldots \) and \( (2L+2) = 2, 22, 42, 62, 82, 102, 122, 142, 162, 182, 202, 222, 242, ..., \) only \( 1/11 \) numbers are divisible by 11.

The chance of \( (N-11) \) to be an odd number (not prime) with 9 as its last digit is: \([1/3] + (1/13x2/3x10/11) + (1/19x2/3x10/11) + (1/23x2/3x10/11x18/19x18/19x22/23x28/29) + (1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41)+\ldots\]

The term \((1/11x2/3)\) is cancelled out.

The next \( \$1 \) is 31, so \((1/31x2/3x10/11x18/19x18/19x22/23x28/29)\) should be cancelled, so on…

\[\[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)\ldots]\]

For number \( N \), let \( n_9 \) be the number of primes \((N-9)\) with 1 as its last digit which matches a prime \((9)\) with 9 as its last digit and \( n_1 \) be the number of primes \((N-1)\) with 9 as its last digit which matches a prime \((1)\) with 1 as its last digit \( n = n_9 + n_1, n_0 \sim n_1 \) \( n \) is the total number of primes \((N-9)\) and \((N-1)\) that have 1 or 9 as their last digit and match \$1 or \$9:

\[n = n_1 + n_9 = n_1 - \{ (n_1 - 1)\} [((1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x18/19) + (1/23x2/3x10/11x18/19x18/19x22/23) + (1/31x2/3x10/11x18/19x18/19x22/23x28/29) + (1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41)+\ldots] + [(1/11x2/3) + (1/13x2/3x10/11x18/19x18/19x22/23) + (1/31x2/3x10/11x18/19x18/19x22/23x28/29) + (1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41)+\ldots]\] + \(\{n_9-1\}[(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)+\ldots] + [(1/3) + (1/7x2/3) + (1/11x2/3x6/7x10/11)+\ldots]
(1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17) + (1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43)…\} = n_1-[(n_1-1)(\sum_1)] + n_9-[(n_9-1)(\sum_9)]

By comparison between \( \sum \) and \( \sum_1 \) term by term, when \( n \) and \( n_1 \geq 5 \):

\[
(n-1)\sum_9 = (n-1)[((1/3) + (1/7x2/3) + (1/11x2/3x6/7)) + (1/13x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31)] + (1/41x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43)…\] > (n-1)[\{(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31)] + (1/41x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43)…\] > (n-1)[\{(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31)] + (1/41x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43)…\] and

\[
(n-1)\sum = (n-1)[((1/3) + (1/7x2/3) + (1/11x2/3x6/7)) + (1/13x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31)] + (1/41x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43)…\] > (n-1)[\{(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/31x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31)] + (1/41x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x10x11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43)…\]
\[
+ \left( \frac{1}{43}x2/3x10/11x18/19x18/19x22/23x28/29x30/31x40/41/1 \right) + \ldots \} = (n-1) \left\{ \frac{1}{3} + \left( \frac{1}{11}x2/3 \right) + \left( \frac{1}{13}x2/3x10/11x18/19x22/23 \right) + \left( \frac{1}{31}x2/3x10/11x18/19x18/19x22/23x28/29 \right) + (1/41x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x18/19x18/19x22/23x28/29x30/31) + \ldots \} \]

\[ n_1 \left( (n_1 - 1)(\sum) \right) + n_9 \left( (n_9 - 1)(\sum) \right) \leq \left( n_1 - 1 \right) \sum = n - n_1 \sum \leq \sum = n + 1 \Delta - 1 + (n - 1) \Delta > 1 \text{ because } n > 1 \text{ and } \Delta > 0 \text{, The results show: When both } N \text{ and } N/2 \text{ have 0 as their last digit, there is at least one pair primes in which one prime has 9 as its last digit and another has 1 as its last digit and their sum is } N \text{ (see table 2). This is an extreme situation, normally, more than 1 pair primes can be found.} \]

For \( N = 600 \) (see table 3), 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit and 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime with 9 as its last digit.
|   | 7   | 387 | 397 | 407 | 417 | 427 | 437 | 447 | 457 | 467 | 477 | 487 | 497 | 507 | 517 | 527 | 537 | 547 | 557 | 567 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|   |     | 3x3x4 | prime | 11x37 | 3x139 | 7x61 | 23x19 | 3x149 | prime | 3x3x5 | 7x71 | 3x13x13 | 11x47 | 3x179 | prime | 3x3x3x3x7 | |
| 213 | 203 | 3x29 | 193 | prime | 183 | 3x61 | 173 | Prime | 163 | prime | 153 | 3x3x17 | 143 | 11x13 | 133 | 7x19 | 123 | 3x41 | 113 | 3x3x13 | 103 | prime | 93 | 3x31 | prime | 73 | 3x3x7 | 63 | prime | 53 | prime | 43 | prime | 33 | 3x11 | |
|   |     | 3 | Prime | 13 | Prime | 23 | Prime | 3 | Prime | 597 | 3x199 | 587 | Prime | 577 | prime |
| 589 | 579 | 569 | prime | 559 | 13x43 | 549 | 3x3x61 | 539 | 7x7x11 | 529 | 23x23 | 519 | 3x178 | 509 | Prime | 499 | Prime | 489 | 3x163 | 479 | Prime | 469 | 7x67 | 459 | 3x3x3x17 | 449 | Prime | 439 | Prime | 429 | 3x11x13 | 419 | Prime | 409 | Prime |
| 11 | 21 | Prime | 3x7 | 31 | Prime | 41 | Prime | 51 | 3x17 | 61 | Prime | 71 | Prime | 81 | 3x3x3 | 91 | 7x13 | 101 | Prime | 111 | 3x37 | 121 | 11x11 | 131 | Prime | 141 | 3x47 | 151 | Prime | 161 | 7x23 | 171 | 3x3x19 | 181 | Prime | 191 | Prime |
1b. When both N has 0 as its last digit, and N/2 has 5 as its last digit.

Table 3. The odd number pairs in N=O₁+O₂=(N/2+L-2)+(N/2-L+2) and N=O₁+O₂=(N/2+L+2)+(N/2-L-2)
In same way as 1a, we can prove there is at least one pair primes in which one prime has 3 as its last digit and another has 7 as its last digit and their sum is N (see table 3).

Table 4. The odd number pairs in \( N=O_1+O_2=(N/2+L-4)+(N/2-L+4) \) and \( N=O_1+O_2=(N/2+L-6)+(N/2-L+6) \)

| N-9 | N-19 | N-29 | N-59 | N-79 | N-89 | N/2+1-4 | N/2-L+6 | ... | 101 | 71 | 61 | 41 | 31 | 11 |
|-----|------|------|------|------|------|---------|---------|-----|-----|----|----|----|----|----|----|
| 9   | 3x3  | 19   | 29   | 59   | 79   | 89      | N/2-L+4 | ... | N-101| N-71| N-61| N-41| N-31| N-11|

In the same way as 1b, it is easy to approve there is at least one pair primes in which one prime has 9 as its last digit and another has 1 as its last digit and their sum is N (see table 4).

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 112, 122, 1122, 1132,…, then N/2 has only 6 or 1 as its last digit.

2a. When any even integer (N) has 2 as its last digit, such as 12, 32, 52, 112, 1132,…, then N/2 has 6 as its last digit:

Table 5. The odd number pairs in \( N=O_1+O_2=(N/2+L-3)+(N/2-L+3) \) and \( N=O_1+O_2=(N/2+L+3)+(N/2-L-3) \)
Table 6. The odd number pairs in $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$ and $N=O_1+O_2=(N/2+L-5)+(N/2-L+5)$

<table>
<thead>
<tr>
<th>N-9</th>
<th>N-19</th>
<th>N-29</th>
<th>N-39</th>
<th>N-49</th>
<th>N-59</th>
<th>N-79</th>
<th>N-89</th>
<th>N/2+L-3</th>
<th>N/2-L-3</th>
<th>...</th>
<th>83</th>
<th>73</th>
<th>53</th>
<th>43</th>
<th>23</th>
<th>13</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9(3x3)</td>
<td>19</td>
<td>29</td>
<td>59</td>
<td>79</td>
<td>89</td>
<td>...</td>
<td>N/2-L+3</td>
<td>N/2+L+3</td>
<td>...</td>
<td>N-83</td>
<td>N-73</td>
<td>N-53</td>
<td>N-43</td>
<td>N-23</td>
<td>N-13</td>
<td>N-3</td>
<td></td>
</tr>
</tbody>
</table>

2b. When any even integer (N) has 2 as its last digit, such as 22, 42, 62, 82, 102, 1122, ..., then N/2 has 1 as its last digit:

Table 7. The odd number pairs in $N=O_1+O_2=(N/2+L+2)+(N/2-L+8)$ and $N=O_1+O_2=(N/2-L+2)+(N/2+L+8)$

| N-9 | N-19 | N-29 | N-39 | N-49 | N-59 | N-69 | N-79 | N/2+L+2 | N/2-L+2 | ... | 83 | 73 | 63 | 53 | 43 | 33 | 23 | 13 | 3 |
|-----|------|------|------|------|------|------|------|---------|---------|-----|-----|-----|----|----|----|----|---|---|---|---|---|---|
| 11 | 31 | 41 | 61 | 71 | 101 | ... | N/2-L+5 | N/2+L-5 | ... | N-101 | N-71 | N-61 | N-41 | N-31 | N-11 |
Table 8. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L-0)$ and $N=O_1+O_2=(N/2+L-0)+(N/2-L+0)$

<table>
<thead>
<tr>
<th>N-9</th>
<th>N-19</th>
<th>N-29</th>
<th>N-39</th>
<th>N-49</th>
<th>N-59</th>
<th>N-69</th>
<th>N-79</th>
<th>...</th>
<th>N/2-L+8</th>
<th>N/2+L+8</th>
<th>...</th>
<th>N-83</th>
<th>N-73</th>
<th>N-63</th>
<th>N-53</th>
<th>N-43</th>
<th>N-33</th>
<th>N-23</th>
<th>N-13</th>
<th>N-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>19</td>
<td>29</td>
<td>39</td>
<td>49</td>
<td>59</td>
<td>69</td>
<td>79</td>
<td>...</td>
<td>N/2-L+8</td>
<td>N/2+L+8</td>
<td>...</td>
<td>N-83</td>
<td>N-73</td>
<td>N-63</td>
<td>N-53</td>
<td>N-43</td>
<td>N-33</td>
<td>N-23</td>
<td>N-13</td>
<td>N-3</td>
</tr>
</tbody>
</table>

Table 9. The odd number pairs in $N=O_1+O_2=(N/2+L+1)+(N/2-L+7)$ and $N=O_1+O_2=(N/2-L+1)+(N/2+L+7)$

<table>
<thead>
<tr>
<th>N-9</th>
<th>N-19</th>
<th>N-29</th>
<th>N-39</th>
<th>N-49</th>
<th>N-59</th>
<th>N-69</th>
<th>N-79</th>
<th>...</th>
<th>N/2-L+7</th>
<th>N/2+L+7</th>
<th>...</th>
<th>N-83</th>
<th>N-73</th>
<th>N-63</th>
<th>N-53</th>
<th>N-43</th>
<th>N-33</th>
<th>N-23</th>
<th>N-13</th>
<th>N-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>19</td>
<td>29</td>
<td>39</td>
<td>49</td>
<td>59</td>
<td>69</td>
<td>79</td>
<td>...</td>
<td>N/2-L+7</td>
<td>N/2+L+7</td>
<td>...</td>
<td>N-83</td>
<td>N-73</td>
<td>N-63</td>
<td>N-53</td>
<td>N-43</td>
<td>N-33</td>
<td>N-23</td>
<td>N-13</td>
<td>N-3</td>
</tr>
</tbody>
</table>

Table 10. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L-1)$ and $N=O_1+O_2=(N/2+L-1)+(N/2-L-1)$

3a. When any even integer ($N$) has 4 as its last digit, such as 24, 44, 64, 84, 104, 1124,..., then $N/2$ has 2 as its last digit:
3b. When any even integer (N) has 4 as its last digit, such as 14, 34, 54, 74, 94, 1114,…, then N/2 has 7 as its last digit:

Table 11. The odd number pairs in \(N = O_1 + O_2 = (N/2 + L + 6) + (N/2 - L + 2)\) and \(N = O_1 + O_2 = (N/2 - L + 6) + (N/2 + L + 2)\)

| N-9 | N-19 | N-29 | N-39 | N-49 | N-59 | N-69 | N-79 | … | N/2+L+6 | N/2-L+6 | … | 83 | 73 | 63 | 53 | 43 | 33 | 23 | 13 | 3 |
|-----|-----|-----|-----|-----|-----|-----|-----|---|-------|-------|---|---|---|---|---|---|---|---|---|---|---|

Table 12. The odd number pairs in \(N = O_1 + O_2 = (N/2 + L + 4) + (N/2 - L + 4)\) and \(N = O_1 + O_2 = (N/2 + L + 4) + (N/2 - L + 4)\)

<table>
<thead>
<tr>
<th>N-11</th>
<th>N-21</th>
<th>N-31</th>
<th>N-41</th>
<th>N-51</th>
<th>N-61</th>
<th>N-71</th>
<th>N-81</th>
<th>…</th>
<th>N/2+L+4</th>
<th>N/2-L+4</th>
<th>…</th>
<th>91</th>
<th>81</th>
<th>71</th>
<th>61</th>
<th>51</th>
<th>41</th>
<th>31</th>
<th>21</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>21</td>
<td>31</td>
<td>41</td>
<td>51</td>
<td>61</td>
<td>71</td>
<td>81</td>
<td>…</td>
<td>N/2-L+4</td>
<td>N/2+L+4</td>
<td>…</td>
<td>N-91</td>
<td>N-81</td>
<td>N-71</td>
<td>N-61</td>
<td>N-51</td>
<td>N-41</td>
<td>N-31</td>
<td>N-21</td>
<td>N-11</td>
</tr>
</tbody>
</table>
4a. When any even integer (N) has 6 as its last digit, such as 26, 46, 66, 86, 106, 1126, ..., then N/2 has 3 as its last digit:

Table 13. The odd number pairs in $N=O_1+O_2=(N/2+L+0)+(N/2-L+6)$ and $N=O_1+O_2=(N/2-L+0)+(N/2+L+6)$

| N-9 | N-19 | N-29 | N-39 | N-49 | N-59 | N-69 | N-79 | ... | N/2+L+0 | N/2-L+0 | ... | 83 | 73 | 63 | 53 | 43 | 33 | 23 | 13 | 3 |
|-----|------|------|------|------|------|------|------|-----|--------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 9   | 19   | 29   | 39   | 49   | 59   | 69   | 79   | ... | N/2-L+6 | N/2+L+6 | ... | N-83 | N-73 | N-63 | N-43 | N-33 | N-23 | N-13 | N-3 |

4b. When any even integer (N) has 6 as its last digit, such as 16, 36, 56, 76, 96, 1116, ..., then N/2 has 8 as its last digit:

Table 14. The odd number pairs in $N=O_1+O_2=(N/2+L-2)+(N/2-L+8)$ and $N=O_1+O_2=(N/2+L+8)+(N/2-L-2)$

| N-11 | N-21 | N-31 | N-41 | N-51 | N-61 | N-71 | N-81 | ... | N/2+L-2 | N/2-L-2 | ... | 91 | 81 | 71 | 61 | 51 | 41 | 31 | 21 | 11 |
|------|------|------|------|------|------|------|------|-----|--------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 11   | 21   | 31   | 41   | 51   | 61   | 71   | 81   | ... | N/2-L+8 | N/2+L+8 | ... | N-91 | N-81 | N-71 | N-61 | N-41 | N-31 | N-21 | N-11 |
Table 16. The odd number pairs in $N=O_1+O_2=(N/2+L+3)+(N/2-L+3)$ and $N=O_1+O_2=(N/2+L+3)+(N/2-L+3)$

<table>
<thead>
<tr>
<th>N-9</th>
<th>N-19</th>
<th>N-29</th>
<th>N-39</th>
<th>N-49</th>
<th>N-59</th>
<th>N-69</th>
<th>N-79</th>
<th>...</th>
<th>N/2+L-5</th>
<th>N/2-L+5</th>
<th>...</th>
<th>83</th>
<th>73</th>
<th>63</th>
<th>53</th>
<th>43</th>
<th>33</th>
<th>23</th>
<th>13</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>19</td>
<td>29</td>
<td>39</td>
<td>49</td>
<td>59</td>
<td>69</td>
<td>79</td>
<td>...</td>
<td>N/2-L+1</td>
<td>N/2+L-1</td>
<td>...</td>
<td>N-83</td>
<td>N-73</td>
<td>N-63</td>
<td>N-53</td>
<td>N-43</td>
<td>N-33</td>
<td>N-23</td>
<td>N-13</td>
<td>N-3</td>
</tr>
</tbody>
</table>

Table 16 is same to table 14, there is at least one pair primes in which both primes have 1 as its last digit and their sum is $N$.

5a. When any even integer $(N)$ has 8 as its last digit, such as 28, 48, 68, 88, 108, 1128,..., then $N/2$ has 4 as its last digit:

Table 17. The odd number pairs in $N=O_1+O_2=(N/2+L-1)+(N/2-L+5)$ and $N=O_1+O_2=(N/2-L-1)+(N/2+L+5)$

<table>
<thead>
<tr>
<th>N-9</th>
<th>N-19</th>
<th>N-29</th>
<th>N-39</th>
<th>N-49</th>
<th>N-59</th>
<th>N-69</th>
<th>N-79</th>
<th>...</th>
<th>N/2+L-1</th>
<th>N/2-L-1</th>
<th>...</th>
<th>83</th>
<th>73</th>
<th>63</th>
<th>53</th>
<th>43</th>
<th>33</th>
<th>23</th>
<th>13</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>19</td>
<td>29</td>
<td>39</td>
<td>49</td>
<td>59</td>
<td>69</td>
<td>79</td>
<td>...</td>
<td>N/2-L+5</td>
<td>N/2+L-5</td>
<td>...</td>
<td>N-83</td>
<td>N-73</td>
<td>N-63</td>
<td>N-53</td>
<td>N-43</td>
<td>N-33</td>
<td>N-23</td>
<td>N-13</td>
<td>N-3</td>
</tr>
</tbody>
</table>
Table 18. The odd number pairs in $N = O_1 + O_2 = (N/2 + L - 3) + (N/2 - L + 7)$ and $N = O_1 + O_2 = (N/2 + L + 7) + (N/2 - L - 3)$

| N-11 | N-21 | N-31 | N-41 | N-51 | N-61 | N-71 | N-81 | N/2+L-3 | N/2-L-3 | ... | 91  | 81  | 71  | 61  | 51  | 41  | 31  | 21  | 11  |
|------|------|------|------|------|------|------|------|---------|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 11   | 21   | 31   | 41   | 51   | 61   | 71   | 81   | ...     | N/2+L+7 | N/2+L+7 | ... | N-91| N-81| N-71| N-61| N-51| N-41| N-31| N-21| N-11|

5b. When any even integer (N) has 8 as its last digit, such as 18, 38, 58, 78, 98, 1118, ..., then N/2 has 9 as its last digit:

Table 19. The odd number pairs in $N = O_1 + O_2 = (N/2 + L - 6) + (N/2 - L + 0)$ and $N = O_1 + O_2 = (N/2 - L - 6) + (N/2 + L + 0)$

| N-9  | N-19 | N-29 | N-39 | N-49 | N-59 | N-69 | N-79 | N/2+L-6 | N/2-L-6 | ... | 83  | 73  | 63  | 53  | 43  | 33  | 23  | 13  | 3   |
|------|------|------|------|------|------|------|------|---------|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 9    | 19   | 29   | 39   | 49   | 59   | 69   | 79   | ...     | N/2+L+0 | N/2+L+0 | ... | N-83| N-73| N-63| N-53| N-43| N-33| N-23| N-13| N-3 |

Table 20. The odd number pairs in $N = O_1 + O_2 = (N/2 + L + 2) + (N/2 - L + 2)$ and $N = O_1 + O_2 = (N/2 + L + 2) + (N/2 - L + 2)$

| N-11 | N-21 | N-31 | N-41 | N-51 | N-61 | N-71 | N-81 | N/2+L+2 | N/2-L+2 | ... | 91  | 81  | 71  | 61  | 51  | 41  | 31  | 21  | 11  |
|------|------|------|------|------|------|------|------|---------|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 11   | 21   | 31   | 41   | 51   | 61   | 71   | 81   | ...     | N/2+L+2 | N/2+L+2 | ... | N-91| N-81| N-71| N-61| N-51| N-41| N-31| N-21| N-11|
From 2a to 5 b, it is easy to approve there is at least one pair primes in which one prime has 9 as its last digit and another has 3 as its last digit and their sum is N or both primes have 1 as its last digit and their sum is N because a prime has higher chance to meet another prime than an odd (not prime) number to meet another prime when their sum is N from formula 1 and 2.

For any even number, Goldbach's conjecture is true.

References: