Single-Valued Neutrosophic Graph Structures

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Abstract

A graph structure is a generalization of undirected graph which is quite useful in studying some structures, including graphs and signed graphs. In this research paper, we apply the idea of single-valued neutrosophic sets to graph structure, and explore some interesting properties of single-valued neutrosophic graph structure. We also discuss the concept of ϕ-complement of single-valued neutrosophic graph structure.

Key-words: Graph structure, Single-valued Neutrosophic graph structure, ϕ-complement.
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1 Introduction

Fuzzy set theory was introduced by Zadeh [14] to solve problems with uncertainties. At present, in modeling and controlling unsure systems in industry, society and nature, fuzzy sets and fuzzy logic are playing a vital role. In decision making, they can be used as power full mathematical tools which facilitate for approximate reasoning. They play a significant role in complex phenomena which is not easily described by classical mathematics. Atanassov [3] illustrated the extension of fuzzy sets by adding a new component, called, intuitionistic fuzzy sets. The intuitionistic fuzzy sets have essentially higher describing possibilities than fuzzy sets. The idea of intuitionistic fuzzy set is more meaningful as well as inventive due to the presence of degree of truth, degree of false and the hesitation margin. The hesitation margin of intuitionistic fuzzy set is its indeterminacy value by default. Samrandache [10] submitted the idea of neutrosophic set (NS)by combining the non-standard analysis, a tricomponent logic/set/probability theory and philosophy. "It is a branch of philosophy which studies the origin, nature and scope of neutralities as well as their interactions with different ideational spectra" [11]. A NS has three components: truth membership, indeterminacy membership and falsity membership, in which each membership value is a real standard or non-standard subset of the nonstandard unit interval $]0-, 1+[$ ([10]). To apply NSs in real-life problems more conveniently, Wang et al. [12] defined single-valued neutrosophic sets (SVNSs). A SVNS is a generalization of intuitionistic fuzzy sets [3]. In SVNS three components are not dependent and their values are contained in the standard unit interval [0, 1].
Fuzzy graphs were narrated by Rosenfeld [8] in 1975. Dinesh and Ramakrishnan [5] introduced the notion of a fuzzy graph structure and discussed some related properties. Akram and Akmal [1] introduced the concept of bipolar fuzzy graph structures. Broum et al. [4] portrayed single-valued neutrosophic graphs. Akram and Shahzadi [2] introduced the notion of neutrosophic soft graphs with applications. In this research paper, we apply the idea of single-valued neutrosophic sets to graph structure, and explore some interesting properties of single-valued neutrosophic graphs. We also discuss the concept of ϕ-complement of single-valued neutrosophic graph structure.

2 Single-Valued Neutrosophic Graph Structures

Definition 2.1. $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is called a single-valued neutrosophic graph structure (SVNSGS) of a graph stucture $\check{G} = (S, S_1, S_2..., S_n)$ if $Q =< (m, n), T(m, n), I(m, n), F(m, n) >$ is a single-valued neutrosophic (SVNS) set on S and $Q_i =< n, T_i(n), I_i(n), F_i(n) >$ is a single-valued neutrosophic set on S_i such that

$$T_i(m, n) \leq \min\{T(m), T(n)\},\ I_i(m, n) \leq \min\{I(m), I(n)\},\ F_i(m, n) \leq \max\{F(m), F(n)\},\ \forall m, n \in S.$$

Note that $T_i(m,n) = 0 = I_i(m,n) = F_i(m,n)$ for all $(m,n) \in S \times S - S_i$ and $0 \leq T_i(m,n) + I_i(m,n) + F_i(m,n) \leq 3\ \forall (m,n) \in S_i$, where S and S_i (i = 1,2,...,n) are underlying vertex and underlying i-edge sets of $\check{G}_n$ respectively.

Definition 2.2. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS of $\check{G}$. If $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ is a SVNSGS of $\check{G}$ such that

$$T'(n) \leq T(n), I'(n) \leq I(n), F'(n) \geq F(n)\ \forall n \in S,$$

$$T'_i(m,n) \leq T_i(m,n),\ I'_i(m,n) \leq I_i(m,n),\ F'_i(m,n) \geq F_i(m,n),\ \forall mn \in S_i,\ i = 1, 2, ..., n.$$

Then $\check{H}_n$ is called a SVNS subgraph structure of SVNSGS $\check{G}_n$.

Definition 2.3. A SVNSGS $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ is called a SVNS induced subgraph structure of $\check{G}_n$ by a subset R of S if

$$T'(n) = T(n), I'(n) = I(n), F'(n) = F(n)\ \forall n \in R,$$

$$T'_i(m,n) = T_i(m,n),\ I'_i(m,n) = I_i(m,n),\ F'_i(m,n) = F_i(m,n),\ \forall m, n \in R,\ i = 1, 2, ..., n.$$

Definition 2.4. A SVNSGS $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ is called a SVNS spanning subgraph structure of $\check{G}_n$ if $Q' = Q$ and

$$T'_i(m,n) \leq T_i(m,n),\ I'_i(m,n) \leq I_i(m,n),\ F'_i(m,n) \geq F_i(m,n),\ i = 1, 2, ..., n.$$

Example 2.5. Consider a GS $\check{G} = (S, S_1, S_2)$ and Q, Q_1,Q_2 be SVNS subsets of S, S_1, S_2 respectively, such that $Q = \{(n_1, .5, .2, .3), (n_2, .7, .3, .4), (n_3, .4, .3, .5), (n_4, .7, .3, .6)\}$, $Q_1 = \{(n_1n_2, .5, .2, .4), (n_2n_4, .7, .3, .6)\}$, $Q_2 = \{(n_3n_4, .4, .3, .6), (n_1n_4, .5, .2, .6)\}$.
Direct calculations show that $\check{G}_n = (Q, Q_1, Q_2)$ is a SVNSGS of $\check{G}$ as shown in Fig. 2.1.

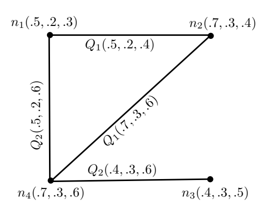

Figure 2.1: A single-valued neutrosophic graph structure

Example 2.6. SVNSGS $\check{K}_n = (Q', Q_{11}, Q_{12})$ is a SVNS subgraph structure of $\check{G}_n$ as shown in Fig. 2.2.

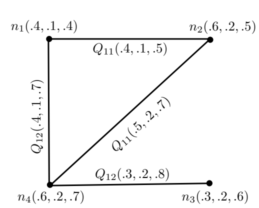

Figure 2.2: A SVNS subgraph structure $\check{K}_n$

Definition 2.7. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS of $\check{G}$. Then $mn \in S_i$ is called a SVNS Q_i-edge or simply Q_i-edge, if $T_i(m,n) > 0$ or $I_i(m,n) > 0$ or $F_i(m,n) > 0$ or all three conditions hold. Consequently, support of Q_i; i=1,2,...,n is:

$$supp(Q_i) = \{mn \in Q_i : T_i(m,n) > 0\} \cup \{mn \in Q_i : I_i(m,n) > 0\} \cup \{mn \in Q_i : F_i(m,n) > 0\}.$$

Definition 2.8. Q_i-path in a SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a sequence of distinct vertices $n_1, n_2, ..., n_m$ (except choice that $n_m = n_1$) in S, such that $n_{j-i}n_j$ is a SVNS Q_i-edge $\forall j = 1, 2, ..., m$.

Definition 2.9. A SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is called Q_i-strong for some $i \in \{1, 2, ..., n\}$ if $T_i(m, n) = \min\{T(m), T(n)\}$, $I_i(m, n) = \min\{I(m), I(n)\}$, $F_i(m, n) = \max\{F(m), F(n)\}, \forall mn \in supp(Q_i)$. SVNSGS $\check{G}_n$ is called strong if it is Q_i-strong for all $i \in \{1, 2, ..., n\}$.

Example 2.10. Consider SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Fig. 2.3. Then $\check{G}_n$ is a strong SVNSGS since it is both Q_1- and $Q_2 - strong$.

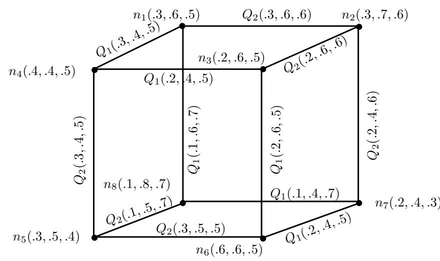

Figure 2.3: A strong SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$

Definition 2.11. A SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is called complete or $Q_1Q_2...Q_n$-complete, if $\check{G}_n$ is a strong SVNSGS, $supp(Q_i) \neq \phi$ for all i=1,2,...,n and for every pair of vertices $m, n \in S$, mn is an $Q_i - edge$ for some i.

Example 2.12. Let $\check{G}_n = (Q, Q_1, Q_2)$ be a SVNSGS of graph structure $\check{G} = (S, S_1, S_2)$ such that $S = \{n_1, n_2, n_3\}$, $S_1 = \{n_1n_2\}$, $S_2 = \{n_2n_3, n_1n_3\}$ as shown in Fig. 2.4. by simple calculations, it can be seen that $\check{G}_n$ is a strong SVNSGS.

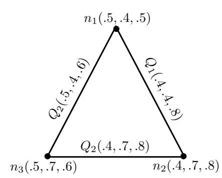

Figure 2.4: A complte SVNSGS

Moreover, $supp(Q_1) \neq \phi$, $supp(Q_2) \neq \phi$ and each pair of vertices in S, is either a Q_1-edge or an Q_2-edge. So $\check{G}_n$ is a complete, i.e., Q_1Q_2-complete SVNSGS.

Definition 2.13. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS. Then truth strength, indeterminacy strength and falsity strength of a Q_i-path P_{Q_i}=$n_1, n_2, ..., n_m$ are denoted by $T.P_{Q_i}$, $I.P_{Q_i}$ and $F.P_{Q_i}$ respectively and defined as

$$T.P_{Q_i}= \bigwedge_{j=2}^{m} [T^P_{Q_i}(n_{j-1}n_j)],\ I.P_{Q_i}= \bigwedge_{j=2}^{m} [I^P_{Q_i}(n_{j-1}n_j)],\ F.P_{Q_i}= \bigvee_{j=2}^{m} [F^P_{Q_i}(n_{j-1}n_j)].$$

Example 2.14. Consider a SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Fig. 2.4. We found that $P_{Q_2} = n_2, n_1, n_3$ is a Q_2-path. So $T.P_{Q_2}$= 0.4, $I.P_{Q_2}$= 0.4 and $F.P_{Q_2}$= 0.8.

Definition 2.15. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS. Then

Q_i-truth strength of connectedness between m and n is defined by $T^{\infty}_{Q_i}(mn) = \bigvee_{j\geq 1} \{T^j_{Q_i}(mn)\}$, such that $T^j_{Q_i}(mn)$ = $(T^{j-1}_{Q_i} \circ T^1_{Q_i})(mn)$ for $j \geq 2$ and $T^2_{Q_i}(mn) = (T^1_{Q_i} \circ T^1_{Q_i})(mn) = \bigvee_{z}(T^1_{Q_i}(mz) \wedge T^1_{Q_i})(zn)$.

Q_i-indeterminacy strength of connectedness between m and n is defined by $I^{\infty}_{Q_i}(mn) = \bigvee_{j\geq 1} \{I^j_{Q_i}(mn)\}$, such that $I^j_{Q_i}(mn) = (I^{j-1}_{Q_i} \circ I^1_{Q_i})(mn)$ for $j \geq 2$ and $I^2_{Q_i}(mn) = (I^1_{Q_i} \circ I^1_{Q_i})(mn) = \bigvee_{z}(I^1_{Q_i}(mz) \wedge I^1_{Q_i})(zn)$.

Q_i-Falsity strength of connectedness between m and n is defined by $F^{\infty}_{Q_i}(mn) = \bigwedge_{j\geq 1}\{F^{j}_{Q_i}(mn)\}$, such that $F^{j}_{Q_i}(mn) = (F^{j-1}_{Q_i} \circ F^{1}_{Q_i})(mn)$ for $j \geq 2$ and $F^{2}_{Q_i}(mn) = (F^{1}_{Q_i} \circ F^{1}_{Q_i})(mn) = \bigwedge_{z}(F^{1}_{Q_i}(mz) \vee F^{1}_{Q_i})(zn)$.

Definition 2.16. A SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a Q_i-cycle if $(supp(Q), supp(Q_1), supp(Q_2), ..., supp(Q_n))$ is a $Q_i - cycle$.

Definition 2.17. A SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a SVNS fuzzy Q_i-cycle (for some i) if $\check{G}_n$ is a Q_i-cycle, no unique Q_i-edge mn is in $\check{G}_n$ such that
$T_{Q_i}(mn) = \min\{T_{Q_i}(rs) : rs \in S_i = supp(Q_i)\}$ *or* $I_{Q_i}(mn) = \min\{I_{Q_i}(rs) : rs \in S_i = supp(Q_i)\}$ *or*
$F_{Q_i}(mn) = \max\{F_{Q_i}(rs) : rs \in S_i = supp(Q_i)\}$.

Example 2.18. Consider a SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Fig. 2.3. Then $\check{G}_n$ is a Q_1-cycle and SVNS fuzzy $Q_1 - cycle$, since $(supp(Q), supp(Q_1), supp(Q_2))$ is a Q_1-cycle and there is no unique Q_1-edge satisfying above condition.

Definition 2.19. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS and p be a vertex in $\check{G}_n$. Let $(Q', Q'_1, Q'_2, ..., Q'_n)$ be a SVNSGS induced by $S \setminus \{p\}$ such that $\forall v \neq p, w \neq p$
$T_{Q'}(p) = 0 = I_{Q'}(p) = F_{Q'}(p)$, $T_{Q'_i}(pv) = 0 = I_{Q'_i}(pv) = F_{Q'_i}(pv)$ $\forall$ edges $pv \in \check{G}_n$. $T_{Q'}(v) = T_Q(v)$, $I_{Q'}(v) = I_Q(v)$, $F_{Q'}(v) = F_Q(v)$, $\forall v \neq p$. $T_{Q'_i}(vw) = T_{Q_i}(vw)$, $I_{Q'_i}(vw) = I_{Q_i}(vw)$, $F_{Q'_i}(vw) = F_{Q_i}(vw)$,
Then p is SVNS fuzzy Q_i-cut vertex for any i, if
$T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$, for some $v, w \in S \setminus \{p\}$.
Note that p is a $Q_i - T$ SVNS fuzzy cut vertex if $T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $Q_i - I$ SVNS fuzzy cut vertex if $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $Q_i - F$ SVNS fuzzy cut vertex if $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$.

Example 2.20. Consider the SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Fig.2.5 and $\check{G}'_n = (Q', Q'_1, Q'_2)$ be SVNS subgraph structure of SVNSGS $\check{G}_n$ found by deleting vertex n_2. Deleted vertex n_2 is a SVNS fuzzy Q_1-I cut vertex since $I^{\infty}_{Q_1}(n_2n_5) = 0.4 > 0.3 = I^{\infty}_{Q'_1}(n_2n_5)$, $I^{\infty}_{Q_1}(n_3n_4) = 0.7 = I^{\infty}_{Q'_1}(n_3n_4)$,
$I^{\infty}_{Q_1}(n_3n_5) = 0.4 > 0.3 = I^{\infty}_{Q'_1}(n_3n_5)$.

Definition 2.21. Suppose $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS and mn be $Q_i - edge$.
Let $(Q', Q'_1, Q'_2, ..., Q'_n)$ be a SVNS fuzzy spanning subgraph structure of $\check{G}_n$, such that $\forall$ edges $mn \neq rs$, $T_{Q'_i}(mn) = 0 = I_{Q'_i}(mn) = F_{Q'_i}(mn)$, $T_{Q'_i}(rs) = T_{Q_i}(rs)$, $I_{Q'_i}(rs) = I_{Q_i}(rs)$, $F_{Q'_i}(rs) = F_{Q_i}(rs)$.
Then mn is a SVNS fuzzy Q_i-bridge if
$T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$, for some $v, w \in S$.
Note that mn is a $Q_i - T$ SVNS fuzzy bridge if $T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $Q_i - I$ SVNS fuzzy bridge if $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $Q_i - F$ SVNS fuzzy bridge if $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$.

Example 2.22. Consider the SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Fig. 2.5 and $\check{G}'_n = (Q', Q'_1, Q'_2)$ be SVNS spanning subgraph structure of SVNSGS $\check{G}_n$ found by deleting Q_1-edge (n_2n_5).

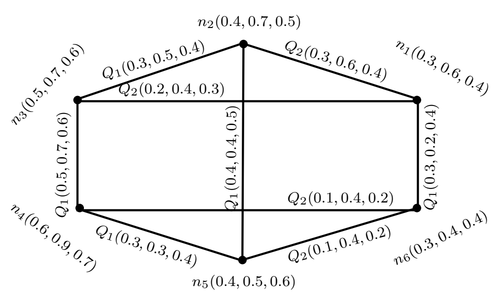

Figure 2.5: A SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$

Edge (n_2n_5) is a SVNS fuzzy Q_1-bridge. Since $T^{\infty}_{Q_1}(n_2n_5) = 0.4 > 0.3 = T^{\infty}_{Q'_1}(n_2n_5)$,
$I^{\infty}_{Q_1}(n_2n_5) = 0.4 > 0.3 = I^{\infty}_{Q'_1}(n_2n_5)$ and $F^{\infty}_{Q_1}(n_2n_5) = 0.5 > 0 = F^{\infty}_{Q'_1}(n_2n_5)$.

Definition 2.23. A SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a Q_i-tree if $(supp(Q), supp(Q_1), supp(Q_2), ..., supp(Q_n))$ is a $Q_i - tree$. In other words, $\check{G}_n$ is a Q_i-tree if a subgraph of $\check{G}_n$ induced by $supp(Q_i)$ generates a tree.

Definition 2.24. A SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a SVNS fuzzy Q_i-tree if $\check{G}_n$ has a SVNS fuzzy spanning subgraph structure $\check{H}_n = (Q'', Q''_1, Q''_2, ..., Q''_n)$ such that $\forall Q_i$-edges mn not in $\check{H}_n$,
$\check{H}_n$ is a Q''_i-tree, $T_{Q_i}(mn) < T^{\infty}_{Q''_i}(mn)$, $I_{Q_i}(mn) < I^{\infty}_{Q''_i}(mn)$, $F_{Q_i}(mn) < F^{\infty}_{Q''_i}(mn)$.
Inparticular, $\check{G}_n$ is a SVNS fuzzy Q_i-T tree if $T_{Q_i}(mn) < T^{\infty}_{Q''_i}(mn)$, a SVNS fuzzy Q_i-I tree if $I_{Q_i}(mn) < I^{\infty}_{Q''_i}(mn)$ and a SVNS fuzzy Q_i-F tree if $F_{Q_i}(mn) > F^{\infty}_{Q''_i}(mn)$.

Example 2.25. Consider the SVNSGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Fig.2.6, which is a Q_2-tree. It is not a Q_1-tree but a SVNS fuzzy Q_1-tree since it has a single-vlaued neutrosophic fuzzy spanning subgraph (Q', Q'_1, Q'_2) as a Q'_1-tree, which is obtained by deleting Q_1-edge n_2n_5 from $\check{G}_n$. Moreover,
$T_{Q_1}(n_2n_5) = 0.2 < 0.3 = T^{\infty}_{Q'_1}(n_2n_5)$, $I_{Q_1}(n_2n_5) = 0.1 < 0.3 = I^{\infty}_{Q_1}{}'(n_2n_5$ and
$F_{Q_1}(n_2n_5) = 0.6 > 0.5 = F^{\infty}_{Q_1}{}'(n_2n_5)$.

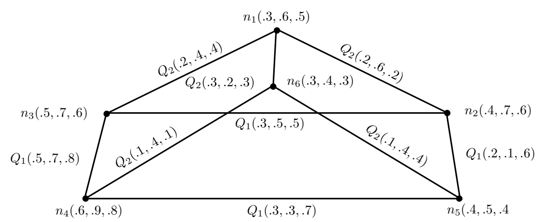

Figure 2.6: A single-valued neutrosophic fuzzy Q_1-tree

Definition 2.26. A SVNSGS $\check{G}_{s1} = (Q_1, Q_{11}, Q_{12}, ..., Q_{1n})$ of graph structure $\check{G}_1 = (S_1, S_{11}, S_{12}, ..., S_{1n})$ is isomorphic to SVNSGS $\check{G}_{s2} = (Q_2, Q_{21}, Q_{22}, ..., Q_{2n})$ of graph structure $\check{G}_2 = (S_2, S_{21}, Q_{22}, ..., S_{2n})$ if we have (f, ϕ) where $f : S_1 \to S_2$ is a bijection and ϕ is a permutation on set $\{1, 2, ..., n\}$ and following relations are satisfied;

$$T_{Q_1}(m) = T_{Q_2}(f(m)),\ I_{Q_1}(m) = I_{Q_2}(f(m)),\ F_{Q_1}(m) = F_{Q_2}(f(m)),\ \forall m \in S_1 \text{ and}$$
$$T_{Q_{1i}}(mn) = T_{Q_{2\phi(i)}}(f(m)f(n)),\ I_{Q_{1i}}(mn) = I_{Q_{2\phi(i)}}(f(m)f(n)\ F_{Q_{1i}}(mn) = F_{Q_{2\phi(i)}}(f(m)f(n)),$$

$\forall mn \in S_{1i}$, i=1,2,...,n.

Example 2.27. Let $\check{G}_{n1} = (Q, Q_1, Q_2)$ and $\check{G}_{n2} = (Q', Q'_1, Q'_2)$ be two SVNSGSs as shown in Fig. 2.7.

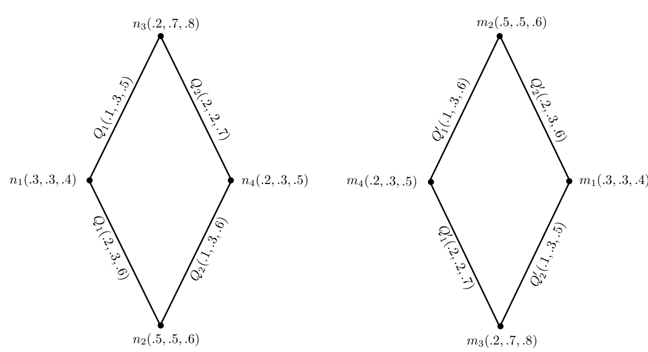

Figure 2.7: Isomorphic SVNS graph structures

$\check{G}_{n1}$ is isomorphic $\check{G}_{n2}$ under (f, ϕ) where $f : S \to S'$ is a bijection and ϕ is a permutation on set $\{1, 2\}$ defined as $\phi(1) = 2$, $\phi(2) = 1$ and following relations are satisfied;
$T_Q(n_i) = T_{Q'}(f(n_i))$, $I_Q(n_i) = I_{Q'}(f(n_i))$, $F_Q(n_i) = F_{Q'}(f(n_i))$, $\forall n_i \in S$ and $T_{Q_i}(n_in_j) = T_{Q'_{\phi(i)}}(f(n_i)f(n_j))$, $I_{Q_i}(n_in_j) = I_{Q'_{\phi(i)}}(f(n_i)f(n_j))$, $F_{Q_i}(n_in_j) = F_{Q'_{\phi(i)}}(f(n_i)f(n_j))$, $\forall n_in_j \in S_i$, i=1,2.

Definition 2.28. A SVNSGS $\check{G}_{s1} = (Q_1, Q_{11}, Q_{12}, ..., Q_{1n})$ of graph structure $\check{G}_1 = (S_1, S_{11}, S_{12}, ..., S_{1n})$ is identical to SVNSGS $\check{G}_{s2} = (Q_2, Q_{21}, Q_{22}, ..., Q_{2n})$ of graph structure $\check{G}_2 = (S_2, S_{21}, Q_{22}, ..., S_{2n})$ if $f : S_1 \to S_2$ is a bijection and following relations are satisfied;
$T_{Q_1}(m) = T_{Q_2}(f(m))$, $I_{Q_1}(m) = I_{Q_2}(f(m))$, $F_{Q_1}(m) = F_{Q_2}(f(m))$, $\forall m \in S_1$ and $T_{Q_{1i}}(mn) = T_{Q_{2i}}(f(m)f(n))$, $I_{Q_{1i}}(mn) = I_{Q_{2i}}(f(m)f(n))$, $F_{Q_{1i}}(mn) = F_{Q_{2(i)}}(f(m)f(n))$, $\forall mn \in S_{1i}$, i=1,2,...,n.

Example 2.29. Let Let $\check{G}_{n1} = (Q, Q_1, Q_2)$ and $\check{G}_{n2} = (Q', Q'_1, Q'_2)$ be two SVNSGSs of GSs $\check{G}_1 = (S, S_1, S_2)$, $\check{G}_2 = (S', S'_1, S'_2)$ respectively as shown in Fig. 2.8 and Fig. 2.9.

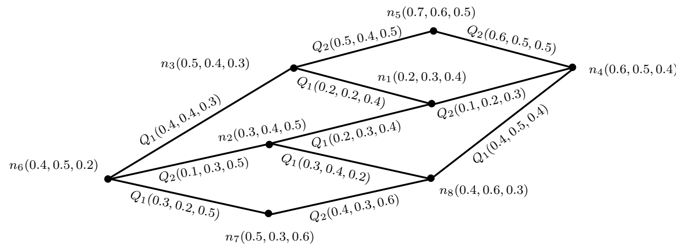

Figure 2.8: A SVNSGS $\check{G}_{n1}$

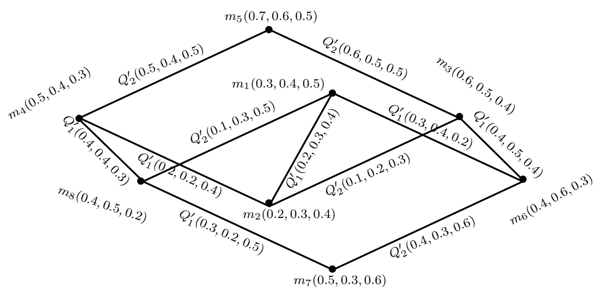

Figure 2.9: A SVNSGS $\check{G}_{n2}$

SVNGS $\check{G}_{n1}$ is identical with $\check{G}_{n2}$ under $f : S \to S'$ defined as ;
$f(n_1) = m_2$, $f(n_2) = m_1$, $f(n_3) = m_4$, $f(n_4) = m_3$, $f(n_5) = m_5$, $f(n_6) = m_8$, $f(n_7) = m_7$, $f(n_8) = m_6$, $T_Q(n_i) = T_{Q'}(f(n_i))$, $I_Q(n_i) = I_{Q'}(f(n_i))$, $F_Q(n_i) = F_{Q'}(f(n_i))$, $\forall n_i \in S$ and $T_{Q_i}(n_in_j) = T_{Q'_i}(f(n_i)f(n_j))$, $I_{Q_i}(n_in_j) = I_{Q'_i}(f(n_i)f(n_j))$, $F_{Q_i}(n_in_j) = F_{Q'_i}(f(n_i)f(n_j))$, $\forall n_in_j \in S_i$, i=1,2.

Definition 2.30. Suppose $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS and ϕ be a permutation on $\{Q_1, Q_2, ..., Q_n\}$ and on $\{1, 2, ..., n\}$ that is $\phi(Q_i) = Q_j$ iff $\phi(i) = j$ $\forall i$. If $mn \in Q_i$ for any i and
$T_{Q_i^\phi}(mn) = T_Q(m) \wedge T_Q(n) - \bigvee_{j \neq i} T_{\phi(Q_j)}(mn)$, $I_{Q_i^\phi}(mn) = I_Q(m) \wedge I_Q(n) - \bigvee_{j \neq i} I_{\phi(Q_j)}(mn)$,
$F_{Q_i^\phi}(mn) = F_Q(m) \vee F_Q(n) - \bigwedge_{j \neq i} T_{\phi(Q_j)}(mn)$, $i = 1, 2, ..., n$, then $mn \in Q_k^\phi$, where k is selected such that;
$T_{Q_k^\phi}(mn) \geq T_{Q_i^\phi}(mn)$, $I_{Q_k^\phi}(mn) \geq I_{Q_i^\phi}(mn)$, $F_{Q_k^\phi}(mn) \geq F_{Q_i^\phi}(mn)$ $\forall i$.
And SVNSGS $(Q, Q_1^\phi, Q_2^\phi, ..., Q_n^\phi)$ is named as ϕ-complement of SVNSGS $\check{G}_n$ and symbolized as $\check{G}_n^{\phi c}$.

Example 2.31. Let $\check{G}_n = (Q, Q_1, Q_2, Q_3)$ be a SVNSGS shown in Fig. 2.10, $\phi(1) = 2, \phi(2) = 3, \phi(3) = 1$. As a result of simple calculations, $n_1n_3 \in Q_3^\phi$, $n_2n_3 \in Q_1^\phi$, $n_1n_2 \in Q_2^\phi$ So, $\check{G}_n^{\phi c}$ =$(Q, Q_1^\phi, Q_2^\phi, Q_3^\phi)$ is ϕ-complement of SVNSGS $\check{G}_n$ as showm in Fig. 2.10.

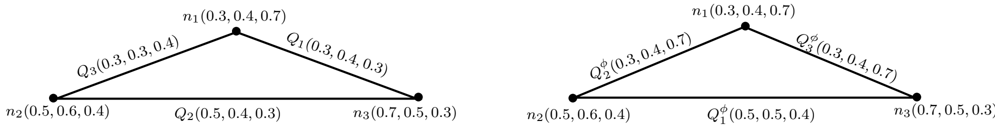

Figure 2.10: SVNSGSs $\check{G}_n$, $\check{G}_n^{\phi c}$

Proposition 2.32. *A ϕ-complement of a SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is always a strong SVNSGS. Moreover, if $\phi(i) = k$, where $i, k \in \{1, 2, ..., n\}$, then all Q_k-edges in SVNSGS $(Q, Q_1, Q_2, ..., Q_n)$ become Q_i^{ϕ}-edges in $(Q, Q_1^{\phi}, Q_2^{\phi}, ..., Q_n^{\phi})$.*

Proof. According to the definition of ϕ-complement,
$T_{Q_i^{\phi}}(mn) = T_Q(m) \wedge T_Q(n) - \bigvee_{j \neq i} T_{\phi(Q_j)}(mn), I_{Q_i^{\phi}}(mn) = I_Q(m) \wedge I_Q(n) - \bigvee_{j \neq i} I_{\phi(Q_j)}(mn),$
$F_{Q_i^{\phi}}(mn) = F_Q(m) \vee F_Q(n) - \bigwedge_{j \neq i} F_{\phi(Q_j)}(mn)$, for $i \in \{1, 2, ..., n\}$.
For expression of truthness in ϕ-complement requirements are shown as:
Since $T_Q(m) \wedge T_Q(n) \geq 0$, $\bigvee_{j \neq i} T_{\phi(Q_j)}(mn) \geq 0$ and $T_{Q_i}(mn) \leq T_Q(m) \wedge T_Q(n)$ $\forall Q_i$.
$\Rightarrow \bigvee_{j \neq i} T_{\phi(Q_j)}(mn) \leq T_Q(m) \wedge T_Q(n) \Rightarrow T_Q(m) \wedge T_Q(n) - \bigvee_{j \neq i} T_{\phi(Q_j)}(mn) \geq 0.$
Therefore, $T_{Q_i^{\phi}}(mn) \geq 0$ $\forall i$. Moreover, $T_{Q_i^{\phi}}(mn)$ achieves its maximum value when $\bigvee_{j \neq i} T_{\phi(Q_j)}(mn)$ is zero. It is obvious that when $\phi(Q_i) = Q_k$ and mn is a Q_k-edge then $\bigvee_{j \neq i} T_{\phi(Q_j)}(mn)$ gets zero value. So,
$T_{Q_i^{\phi}}(mn) = T_Q(m) \wedge T_Q(n),\ for\ (mn) \in Q_k,\ \phi(Q_i) = Q_k$. Similarly,
$I_{Q_i^{\phi}}(mn) = I_Q(m) \wedge I_Q(n),\ for\ (mn) \in Q_k,\ \phi(Q_i) = Q_k$.
In the similar way for expression of falsity in ϕ-complement requirements are shown as:
Since $F_Q(m) \vee F_Q(n) \geq 0$, $\bigwedge_{j \neq i} F_{\phi(Q_j)}(mn) \geq 0$ and $F_{Q_i}(mn) \leq F_Q(m) \vee F_Q(n)$ $\forall Q_i$.
$\Rightarrow \bigwedge_{j \neq i} F_{\phi(Q_j)}(mn) \leq F_Q(m) \vee F_Q(n) \Rightarrow F_Q(m) \vee F_Q(n) - \bigwedge_{j \neq i} F_{\phi(Q_j)}(mn) \geq 0.$
Therefore, $F_{Q_i^{\phi}}(mn)$ is non-negative for all i. Moreover, $F_{Q_i^{\phi}}(mn)$ attains its maximum value when $\bigwedge_{j \neq i} F_{\phi(Q_j)}(mn)$ becomes zero. It is clear that when $\phi(Q_i) = Q_k$ and mn is a Q_k-edge then $\bigwedge_{j \neq i} F_{\phi(Q_j)}(mn)$ gets zero value. So, $F_{Q_i^{\phi}}(mn) = F_Q(m) \vee F_Q(n),\ for\ (mn) \in Q_k,\ \phi(Q_i) = Q_k$. From these expressions of truthness, indeteminacy and falsity required results are achieved. □

Definition 2.33. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS and ϕ a permutation on $\{1, 2, ..., n\}$. Then

(i) If $\check{G}_n$ is isomorphic to $\check{G}_n^{\phi c}$, then $\check{G}_n$ is said to be self-complementary.

(ii) If $\check{G}_n$ is identical to $\check{G}_n^{\phi c}$, then $\check{G}_n$ is said to be strong self-complementary.

Definition 2.34. Suppose $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNSGS. Then

(i) If $\check{G}_n$ is isomorphic to $\check{G}_n^{\phi c}$, $\forall$ permutations ϕ on {1,2,...,n}, then $\check{G}_n$ is totally self-complementary.

(ii) If $\check{G}_n$ is identical to $\check{G}_n^{\phi c}$, $\forall$ permutations ϕ on {1,2,...,n}, then $\check{G}_n$ is totally self-complementary.

Example 2.35. All strong SVNSGSs are self-complementary or totally self-complementary SVNSGSs.

Example 2.36. SVNSGS $\check{G}_n = (Q, Q_1, Q_2, Q_3)$ in Fig.2.11 is totally strong self-complementary SVNSGS.

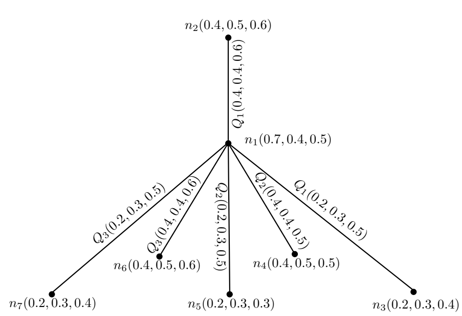

Figure 2.11: A totally strong self-complementary SVNSGS

Theorem 2.37. *A SVNSGS is totally self-complementary if and only if it is strong SVNSGS.*

Proof. Consider a strong SVNSGS $\check{G}_n$ and a permutation ϕ on {1,2,..., n}. By proposition 2.32, ϕ-complement of a SVNSGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is always a strong SVNSGS.
Moreover, if $\phi(i) = k$, where $i, k \in \{1, 2, ..., n\}$, then all Q_k-edges in SVNSGS $(Q, Q_1, Q_2, ..., Q_n)$ become Q_i^ϕ-edges in $(Q, Q_1^\phi, Q_2^\phi, ..., Q_n^\phi)$.
This leads $T_{Q_k}(mn) = T_Q(m) \wedge T_Q(n) = T_{Q_i^\phi}(mn)$, $I_{Q_k}(mn) = I_Q(m) \wedge I_Q(n) = I_{Q_i^\phi}(mn)$, $F_{Q_k}(mn) = F_Q(m) \vee F_Q(n) = F_{Q_i^\phi}(mn)$. Hence under the mapping(identity mapping) $f : S \to S$, $\check{G}_n$ and $\check{G}_n^\phi$ are isomorphic such that,

$$T_Q(m) = T_Q(f(m)),\ I_Q(m) = I_Q(f(m)),\ F_Q(m) = F_Q(f(m)) \text{ and } T_{Q_k}(mn) = T_{Q_i^\phi}(f(m)f(n)) = T_{Q_i^\phi}(mn),$$
$$I_{Q_k}(mn) = I_{Q_i^\phi}(f(m)f(n)) = I_{Q_i^\phi}(mn)\ ,\ F_{Q_k}(mn) = F_{Q_i^\phi}(f(m)f(n)) = F_{Q_i^\phi}(mn)\ ,$$

$\forall mn \in S_k$, for $\phi^{-1}(k) = i$; i,k = 1,2,...,n. All this is satisfied for every permutation ϕ on {1,2,..., n}. Hence $\check{G}_n$ is totally self-complementary SVNSGS. Conversely, let for every permutation ϕ on {1,2,..., n}, $\check{G}_n$ and $\check{G}_n^\phi$ are isomorphic. Then according to the definition of isomorphism of SVNSGSs and ϕ-complement of SVNSGS,

$$T_{Q_k}(mn) = T_{Q_i^\phi}(f(m)f(n)) = T_Q(f(m)) \wedge T_Q(f(n)) = T_Q(m) \wedge T_Q(n),$$

$$I_{Q_k}(mn) = I_{Q_i^\phi}(f(m)f(n)) = I_Q(f(m)) \wedge I_Q(f(n)) = T_Q(m) \wedge I_Q(n),$$

$$F_{Q_k}(mn) = F_{Q_i^\phi}(f(m)f(n)) = F_Q(f(m)) \vee T_Q(f(n)) = F_Q(m) \wedge T_Q(n),$$

$\forall mn \in S_k$, k = 1,2,...,n. Hence $\check{G}_n$ is strong SVNSGS. ◻

Remark 2.38. Every SVNSGS which is self-complementary is definitely totally self-complementary.

Theorem 2.39. *If $\check{G} = (S, S_1, S_2, ..., S_n)$ is a strong and totally self-complementary GS and $Q = (T_Q, I_Q, F_Q)$ is a SVNS subset of S where T_Q, I_Q, F_Q are constant valued functions then a strong SVNSGS of $\check{G}$ with SVNS vertex set Q is always a strong totally self-complementary SVNSGS.*

Proof. Consider three constants p, q, $r \in [0, 1]$, such that $T_Q(m) = p, I_Q(m) = q, F_Q(m) = r\ \forall m \in S$ Since $\check{G}$ is totally self-complementary strong GS, so there is a bijection $f : S \to S$ for any permutation ϕ^{-1} on {1,2,...,n}, such that for any S_k-edge (mn), (f(m)f(n)) [a S_i-edge in $\check{G}$] is a S_k-edge in $\check{G}^{\phi^{-1}c}$. Hence for every Q_k-edge (mn), (f(m)f(n)) [a Q_i-edge in $\check{G}_n$] is a Q_k^ϕ-edge in ${\check{G}_n}^{\phi^{-1}c}$.
Moreover $\check{G}_n$ is strong SVNSGS, so

$$T_Q(m) = p = T_Q(f(m)),\ I_Q(m) = q = I_Q(f(m)),\ F_Q(m) = r = F_Q(f(m))\ \forall m \in S \text{ and}$$
$$T_{Q_k}(mn) = T_Q(m) \wedge T_Q(n) = T_Q(f(m)) \wedge T_Q(f(n)) = T_{Q_i^\phi}(f(m)f(n)),$$
$$I_{Q_k}(mn) = I_Q(m) \wedge I_Q(n) = I_Q(f(m)) \wedge I_Q(f(n)) = I_{Q_i^\phi}(f(m)f(n)),$$
$$F_{Q_k}(mn) = F_Q(m) \vee I_Q(n) = F_Q(f(m)) \vee F_Q(f(n)) = F_{Q_i^\phi}(f(m)f(n)),$$

$\forall mn \in S_i, i = 1, 2, ..., n$. This shows $\check{G}_n$ is seif-complementary strong SVNSGS. Every permutation ϕ, ϕ^{-1} on {1,2,...,n} satisfies above expressions, thus $\check{G}_n$ is strong totally self-complementary SVNSGS. Hence required result is obtained. ◻

Remark 2.40. Converse of theorem 2.39 may not be true, as a SVNSGS shown in Fig. 2.39 is strong totally self-complementary, it is strong and its underlying GS is a strong totally self-complementary but T_Q, I_Q, F_Q are not constant functions.

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