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Citation: AIP Conference Proceedings **1726**, 020045 (2016); doi: 10.1063/1.4945871
View online: http://dx.doi.org/10.1063/1.4945871
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1726?ver=pdfcov
Published by the AIP Publishing

Smarandache Curves According to Sabban Frame of Fixed Pole Curve Belonging to the Bertrand Curves Pair

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Abstract. In this paper, we investigate the Smarandache curves according to Sabban frame of fixed pole curve which drawn by the unit Darboux vector of the Bertrand partner curve. Some results have been obtained. These results were expressed as the depends Bertrand curve.
Mathematics Subject Classification (2010). 53A04.

Keywords. Bertrand curves pair, Fixed pole curve, Smarandache curves, Sabban frame, Geodesic curvature.

INTRODUCTION AND PRELIMINARIES

A regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve [1]. K. Taşköprü, M. Tosun studied special Smarandache curves according to Sabban frame on S^2 [2]. Şenyurt and Çalışkan investigated special Smarandache curves in terms of Sabban frame for fixed pole curve and spherical indicatrix and they gave some characterization of Smarandache curves [4, 6]. Let $\alpha : I \to E^3$ be a unit speed curve denote by $\{T, N, B\}$ the moving Frenet frame. For an arbitrary curve $\alpha \in E^3$, with first and second curvature, κ and τ respectively, the Frenet formulae is given by [7, 8]

$$T' = \kappa N, \quad N' = -\kappa T + \tau B, \quad B' = -\tau N \tag{1}$$

the vector W is called Darboux vector defined by

$$W = \tau T + \kappa B.$$

If we consider the normalization of the Darboux $C = \frac{1}{\|W\|}W$ we have, $\sin\varphi = \frac{\tau}{\|W\|}$ and $\cos\varphi = \frac{\kappa}{\|W\|}$

and [5]

$$C = \sin\varphi T + \cos\varphi B \tag{2}$$

where $\angle(W, B) = \varphi$.

Theorem 1 *Let $\alpha : I \to E^3$ and $\alpha^* : I \to E^3$ be the C^2-class differentiable unit speed two curves and the amounts of $\{T(s), N(s), B(s), \kappa(s), \tau(s)\}$ and $\{T^*(s), N^*(s), B^*(s), \kappa^*(s), \tau^*(s)\}$ are entirely Frenet- serret aparataus of the curves α and the Bertrand partner α^*, respectively, then*

$$T^* = \cos\theta T - \sin\theta B, \quad N^* = N, \quad B^* = \sin\theta T + \cos\theta B, \tag{3}$$

$$\kappa^* = \frac{\lambda\kappa - \sin^2\theta}{\lambda(1 - \lambda\kappa)}, \; \tau^* = \frac{\sin^2\theta}{\lambda^2\tau} \tag{4}$$

where $\angle(T, T^*) = \theta$, [8].

International Conference on Advances in Natural and Applied Sciences
AIP Conf. Proc. 1726, 020045-1–020045-4; doi: 10.1063/1.4945871
Published by AIP Publishing. 978-0-7354-1373-3/$30.00

Theorem 2 *Let (α, α^*) be a Bertrand curves pair in $\mathbb{E}^3$. We have between unit Darboux vectors [3],*

$$C^* = -C. \tag{5}$$

Theorem 3 *Let γ be a unit speed spherical curve. We denote s as the arc-length parameter of γ . Let us denote $t(s) = \gamma'(s)$, and we call $t(s)$ a unit tangent vector of γ. We now set a vector $d(s) = \gamma(s) \wedge t(s)$ along γ. This frame is called the Sabban frame of γ on S^2 (Sphere of unit radius). Then we have the following spherical Frenet formulae of γ, [2, 6]*

$$\gamma' = t, \quad t' = -\gamma + \kappa_g d, \quad d' = -\kappa_g t \tag{6}$$

where κ_g is called the geodesic curvature of γ on S^2 and

$$\kappa_g = \langle t', d \rangle. \tag{7}$$

SMARANDACHE CURVES ACCORDING TO SABBAN FRAME OF FIXED POLE CURVE BELONGING TO THE BERTRAND CURVES PAIR

In this section, we investigate Smarandache curves according to the Sabban frame of fixed pole (C^*). Let $\alpha_{C^*}(s) = C^*$ be a unit speed regular spherical curves on S^2. We denote s_{C^*} as the arc-lenght parameter of fixed pole (C^*)

$$\alpha_{C^*}(s) = C^*(s). \tag{8}$$

Differentiating (8), we have

$$T_{C^*} = \cos\varphi^* T^* - \sin\varphi^* B^*$$

and

$$C^* \wedge T_{C^*} = N^*.$$

From the equation

$$C^* = \sin\varphi^* T^* + \cos\varphi^* B^*, \quad T_{C^*} = \cos\varphi^* T^* - \sin\varphi^* B^*, \quad C^* \wedge T_{C^*} = N^* \tag{9}$$

is called the Sabban frame of fixed pole curve (C^*). From the (6)

$$\kappa_g = \langle T_{C^*}', C^* \wedge T_{C^*} \rangle \Longrightarrow \kappa_g = \frac{\|W^*\|}{\varphi^{*\prime}}.$$

Then from the (4) we have the following spherical Frenet formulae of (C^*):

$$C^{*\prime} = T_{C^*}, \quad T_{C^*}' + \frac{\|W^*\|}{\varphi^{*\prime}} C^* \wedge T_{C^*}, \quad (C^* \wedge T_{C^*})' = -\frac{\|W^*\|}{\varphi^{*\prime}} T_{C^*}. \tag{10}$$

i.) $C^* T_{C^*}$-Smarandache Curves

Let S^2 be a unit sphere in E^3 and suppose that the unit speed regular Bertrand partner curve $\alpha_{C^*}(s) = C^*(s)$ lying fully on S^2. In this case, $C^* T_{C^*}$ - Smarandache curve can be defined by

$$\beta_1(s) = \frac{1}{\sqrt{2}}(C^* + T_{C^*}). \tag{11}$$

Substituting the equation (9) into equation (11), we reach

$$\beta_1(s) = \frac{1}{\sqrt{2}}\Big((\sin\varphi^* + \cos\varphi^*)T^* + (\cos\varphi^* - \sin\varphi^*)B^*\Big). \tag{12}$$

Differentiating (11), we can write the tangent vector of β_1-Smarandache curve according to Bertrand partner curve

$$T_{\beta_1} = \frac{(\varphi^{*\prime} - \sin\varphi^*)}{\sqrt{2\varphi^{*\prime 2} + \|W^*\|^2}} T^* + \frac{\|W^*\|}{\sqrt{2\varphi^{*\prime 2} + \|W^*\|^2}} N^* - \frac{(\varphi^{*\prime} + \sin\varphi^*)}{\sqrt{2\varphi^{*\prime 2} + \|W^*\|^2}} B^*. \tag{13}$$

Differentiating (13), we get

$$T'_{\beta_1} = \frac{\varphi^{*\prime 4}\sqrt{2}(\chi_1 \sin\varphi^* + \chi_2 \cos\varphi^*)}{\left(\|W^*\|^2 + \varphi^{*\prime 2}\right)^2} T^* + \frac{\chi_3 \varphi^{*\prime 4}\sqrt{2}}{\left(\|W^*\|^2 + \varphi^{*\prime 2}\right)^2} N^* + \frac{\varphi^{*\prime 4}\sqrt{2}(\chi_1 \cos\varphi^* - \chi_2 \sin\varphi^*)}{\left(\|W^*\|^2 + \varphi^{*\prime 2}\right)^2} B^* \tag{14}$$

where

$$\begin{aligned} \chi_1 &= -2 - (\frac{\|W^*\|}{\varphi^{*\prime}})^2 + (\frac{\|W^*\|}{\varphi^{*\prime}})'(\frac{\|W^*\|}{\varphi^{*\prime}}),\ \chi_2 = -2 - 3(\frac{\|W^*\|}{\varphi^{*\prime}})^2 - (\frac{\|W^*\|}{\varphi^{*\prime}})^4 - (\frac{\|W^*\|}{\varphi^{*\prime}})'(\frac{\|W^*\|}{\varphi^{*\prime}}) \\ \chi_3 &= 2(\frac{\|W^*\|}{\varphi^{*\prime}}) + (\frac{\|W^*\|}{\varphi^{*\prime}})^3 + (\frac{\|W^*\|}{\varphi^{*\prime}})'. \end{aligned} \tag{15}$$

Considering the equations (12) and (13), it easily seen that

$$(C^* \wedge T_{C^*})_{\beta_1} = \frac{\|W^*\|(\cos\varphi^* + \sin\varphi^*)}{\sqrt{2\|W^*\|^2 + 4\varphi^{*\prime 2}}} T^* - \frac{\varphi^{*\prime}}{\sqrt{2\|W^*\|^2 + 4\varphi^{*\prime 2}}} N^* + \frac{\|W^*\|(\cos\varphi^* + \sin\varphi^*)}{\sqrt{2\|W^*\|^2 + 4\varphi^{*\prime 2}}} B^*. \tag{16}$$

Substituting the (3) and (4) into equation (12), (13), (14) and (16), Sabban aparataus of the β_1-Smarandache curve according to Bertrand curve

$$\beta_1(s) = \frac{1}{\sqrt{2}}\Big((-\sin\varphi - \cos\varphi)T + (-\cos\varphi + \sin\varphi)B\Big),\quad T_{\beta_1} = \frac{\varphi'(\sin\varphi - \cos\varphi)}{\sqrt{2\varphi'^2 + \|W\|^2}} T - \frac{\|W\|}{\sqrt{2\varphi'^2 + \|W\|^2}} N + \frac{\varphi'(\cos\varphi + \sin\varphi)}{\sqrt{2\varphi'^2 + \|W\|^2}} B,$$

$$(C^* \wedge T_{C^*})_{\beta_1} = \frac{\|W\|(\cos\varphi - \sin\varphi)}{\sqrt{2\|W\|^2 + 4\varphi'^2}} T - \frac{\varphi'}{\sqrt{2\|W\|^2 + 4\varphi'^2}} N - \frac{\|W\|(\cos\varphi + \sin\varphi)}{\sqrt{2\|W\|^2 + 4\varphi'^2}} B,$$

$$T'_{\beta_1} = \frac{\varphi'^4\sqrt{2}(-\overline{\chi}_1 \sin\varphi - \overline{\chi}_2 \cos\varphi)}{(\|W\|^2 + 2\varphi'^2)^2} T - \frac{\overline{\chi}_3 \varphi'^4 \sqrt{2}}{(\|W\|^2 + 2\varphi'^2)^2} N + \frac{\varphi'^4\sqrt{2}(-\overline{\chi}_1 \cos\varphi + \overline{\chi}_2 \sin\varphi)}{(\|W\|^2 + 2\varphi'^2)^2} B,$$

where

$$\begin{aligned} \overline{\chi}_1 &= -2 - (\frac{\|W\|}{\varphi'})^2 + (\frac{\|W\|}{\varphi'})'(\frac{\|W\|}{\varphi'}),\ \overline{\chi}_2 = -2 - 3(\frac{\|W\|}{\varphi'})^2 - (\frac{\|W\|}{\varphi'})^4 - (\frac{\|W\|}{\varphi'})'(\frac{\|W\|}{\varphi'}) \\ \overline{\chi}_3 &= 2(\frac{\|W\|}{\varphi'}) + (\frac{\|W\|}{\varphi'})^3 + 2(\frac{\|W\|}{\varphi'})'. \end{aligned} \tag{17}$$

Geodesic curvatures of the $\beta_1(s_{\beta_1})$-Smarandache curve according to Bertrand partner and Bertrand curves, recpectively,

$$\kappa_g^{\beta_1} = \frac{1}{(2 + (\frac{\|W^*\|}{\varphi^{*\prime}})^2)^{\frac{5}{2}}}\Big(\frac{\|W^*\|}{\varphi^{*\prime}}\chi_1 - \frac{\|W^*\|}{\varphi^{*\prime}}\chi_2 + 2\chi_3\Big),\quad \kappa_g^{\beta_1} = \frac{1}{(2 + (\frac{\|W\|}{\varphi'})^2)^{\frac{5}{2}}}\Big(\frac{\|W\|}{\varphi'}\overline{\chi}_1 - \frac{\|W\|}{\varphi'}\overline{\chi}_2 + 2\overline{\chi}_3\Big).$$

ii.) $T_{C^*}(C^* \wedge T_{C^*})$-Smarandache Curves

$T_{C^*}(C^* \wedge T_{C^*})$-Smarandache curve can be defined by

$$\beta_2(s) = \frac{1}{\sqrt{2}}(T_{C^*} + C^* \wedge T_{C^*}). \tag{18}$$

Solving the above equation by substitution of T_{C^*} and $C^* \wedge T_{C^*}$ from (9), (3) and (4), we reach β_2-Smarandache curve according to Bertrand partner and Bertrand curves, respectively,

$$\beta_2(s) = \frac{1}{\sqrt{2}}(\cos\varphi^* T^* + N^* - \sin\varphi^* B^*),\quad \beta_2(s) = \frac{1}{\sqrt{2}}(-\cos\varphi T + N + \sin\varphi B). \tag{19}$$

Geodesic curvature of the $\beta_2(s_{\beta_2})$-Smarandache curve according to Bertrand curve

$$\kappa_g^{\beta_2} = \frac{1}{\left(1+2(\frac{\|W\|}{\varphi'})^2\right)^{\frac{5}{2}}}\left(2\frac{\|W\|}{\varphi'}\overline{\delta}_1 - \overline{\delta}_2 + \overline{\delta}_3\right)$$

where

$$\overline{\delta}_1 = (\frac{\|W\|}{\varphi'})+(\frac{\|W\|}{\varphi'})^3+2(\frac{\|W\|}{\varphi'})'(\frac{\|W\|}{\varphi'}),\ \overline{\delta}_2 = -1-3(\frac{\|W\|}{\varphi'})^2-2(\frac{\|W\|}{\varphi'})^4-(\frac{\|W\|}{\varphi'})',\ \overline{\delta}_3 = -(\frac{\|W\|}{\varphi'})^2-2(\frac{\|W\|}{\varphi'})^4+(\frac{\|W\|}{\varphi'})'.$$

iii.) $C^*T_{C^*}(C^* \wedge T_{C^*})$-Smarandache Curves

$C^*T_{C^*}(C^* \wedge T_{C^*})$-Smarandache curve can be defined by

$$\beta_3(s) = \frac{1}{\sqrt{3}}(C^* + T_{C^*} + C^* \wedge T_{C^*}). \tag{20}$$

Solving the above equation by substitution of C^*, T_{C^*} and $C^* \wedge T_{C^*}$ from (9), (3) and (4), we reach β_3-Smarandache curve according to Bertrand partner and Bertrand curves, respectively,

$$\beta_3(s) = \frac{1}{\sqrt{3}}((\sin\varphi^* + \cos\varphi^*)T^* + N^* + (\cos\varphi^* - \sin\varphi^*)B^*),\ \beta_3(s) = \frac{1}{\sqrt{3}}((-\sin\varphi - \cos\varphi)T + N + (\sin\varphi - \cos\varphi)B).$$

Geodesic curvature of the $\beta_3(s_{\beta_3})$-Smarandache curve according to Bertrand curve,

$$\kappa_g^{\beta_3} = \frac{\varphi'^5(2\|W\| - \varphi')\overline{\rho}_1 - \varphi'^5(\|W\| + \varphi')\overline{\rho}_2 + \varphi'^5(2\varphi' - \|W\|)\overline{\rho}_3}{4\sqrt{2}\left(\varphi'^2 - \varphi'\|W\| + \|W\|^2\right)^{\frac{5}{2}}}$$

where

$$\begin{aligned}
\overline{\rho}_1 &= -2 + 4(\frac{\|W\|}{\varphi'}) + 4(\frac{\|W\|}{\varphi'}) - (\frac{\|W\|}{\varphi'})^2 + 2(\frac{\|W\|}{\varphi'})^3 + (\frac{\|W\|}{\varphi'})'(2\frac{\|W\|}{\varphi'} - 1),\\
\overline{\rho}_2 &= -2 + 2(\frac{\|W\|}{\varphi'}) - 4(\frac{\|W\|}{\varphi'})^2 + (\frac{\|W\|}{\varphi'})^3 - 2(\frac{\|W\|}{\varphi'})^4 - (\frac{\|W\|}{\varphi'})'(1 + \frac{\|W\|}{\varphi'}),\\
\overline{\rho}_3 &= 2(\frac{\|W\|}{\varphi'}) - 4(\frac{\|W\|}{\varphi'})^2 + 4(\frac{\|W\|}{\varphi'})^3 - 2(\frac{\|W\|}{\varphi'})^4 + (\frac{\|W\|}{\varphi'})'(2 - \frac{\|W\|}{\varphi'}).
\end{aligned}$$

ACKNOWLEDGEMENT

This work was supported by BAP (The Scientific Research Projects Coordination Unit), Ordu University.

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