



## **A STUDY ON NEUTROSOPHIC GRAPHS**

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### **ABSTRACT**

In this paper, we are study about some basic definitions related to Graphs and Neutrosophic graphs. Some properties for the neutrosophic graphs associated with the Neutrosophic bigraphs. By applying some neutrosophic cognitive map and techniques in Neutrosophic models.

**Keywords:** Graphs, Neutrosophic graphs, Neutrosophic bigraphs and Neutrosophic

**MSC CODE: 05C99**

### **1. INTRODUCTION**

Graph theory has several interesting applications in system analysis, operations research, and economics. Euler (1707-1782) became the father of graph theory. In 1847 Kirchoff developed the theory of trees, in order to solve the system of simultaneous linear equation, which give the current in each branch and each circuit of an electric network. In 1857 Cayley discovered the important class of graphs called trees by considering the changes of variable in the differential calculus. Jordan in 1869 independently discovered trees as a purely mathematical discipline and Sylvester 1882 wrote that Jordan did so without having any suspicion of its bearing on modern chemical doctrine. Lewin the psychologist proposed in 1936 that the life span of an individual be represented by a planar map. Thus finally in the 21<sup>st</sup> century the graph theory has been fully exploited by fuzzy theory.

### 1.1. Neutrosophic Graphs:

#### Definition 1.1.1:

A **graph**  $G$  is an order triple  $G (V (G), E (G), \psi_G )$  consisting of a non empty set  $V(G)$  of vertices, a set  $E(G)$  disjoint from  $V(G)$ , of edges and an incidence function  $\psi_G$  that associates each edge of  $G$  an unorder pair of vertices of  $G$ .

If  $e$  is an edge and  $u$  and  $v$  are vertices  $\psi_G (e) =uv$ , then  $e$  is said to join  $u$  and  $v$ , the vertices  $u$  and  $v$  are called the ends of  $e$ .

#### Example 1.1.1:

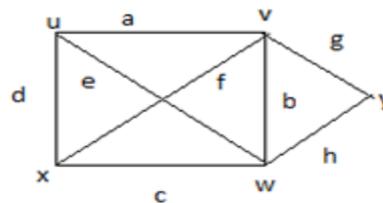


figure 1.1.1. Graph

#### Definition 1.1.2:

A **Neutrosophic graph** is a graph in which at least one edge is an indeterminacy denoted by dotted lines.

#### Example 1.1.2:

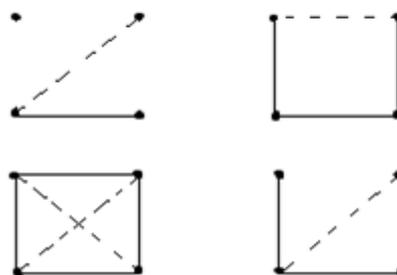


figure 1.1.2. neutrosophic graphs

**Definition 1.1.3:**

A *neutrosophic directed graph* is a directed graph which has at least one edge to be an indeterminacy. A *neutrosophic oriented graph* is a neutrosophic directed graph having no symmetric pair of directed indeterminacy lines. A *neutrosophic subgraph* H of a neutrosophic graph G is a subgraph H which is itself a neutrosophic graph.

**Theorem 1.1.1:**

Let G be a neutrosophic graph. All subgraphs of G are not neutrosophic subgraphs of G.

**Proof:** Consider the neutrosophic graph given in figure 1.1.3.

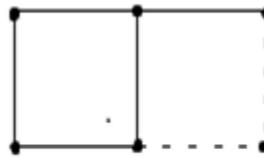


figure 1.1.3.

This has a subgraph given by figure 1.1.4.

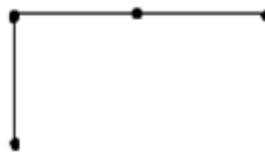


figure 1.1.4

Which is not a neutrosophic. hence proved.

**Theorem 2.2.2:**

Let G be a neutrosophic graph. In general the removal of a point from G need not be a neutrosophic subgraph.

**Proof:**

Consider the graph G given in figure2.2.4.

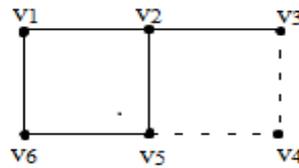


figure2.2.4.

$G \setminus v_4$  is only a subgraph of G but is not a neutrosophic subgraph of G. Thus it is interesting to note that this is a main feature by which a graph differs from a neutrosophic graph.

### 2.1. Neutrosophic bigraphs

**Definition 2.1.1:**

$G = G_1 \cup G_2$  is said to be a **bigraph** if  $G_1$  and  $G_2$  are two graphs such that  $G_1$  is not a subgraph of  $G_2$  or  $G_2$  is not a subgraph of  $G_1$ , i.e., they have either distinct vertices or edges.

**Definition 2.1.2:**

A neutrosophic graph  $G_N = G_1 \cup G_2$  is said to be a **neutrosophic bigraph** if both  $G_1$  and  $G_2$  are neutrosophic graphs that the set of vertices of  $G_1$  and  $G_2$  are different at least by one coordinate i.e.  $V(G_1) \not\subseteq V(G_2)$  or  $V(G_2) \not\subseteq V(G_1)$  i.e.  $V(G_1) \cap V(G_2) = \emptyset$  is also possible but is not a condition i.e. the vertex set of  $G_1$  is not a proper subset of the vertex set of  $G_2$  or vice versa or at least one edge is different in the graphs  $G_1$  and  $G_2$ . ‘or’ is not used in the mutually exclusive sense.

**Example 2.1.1:**

Let  $G = G_1 \cup G_2$  be the neutrosophic bigraph given by the following figure:

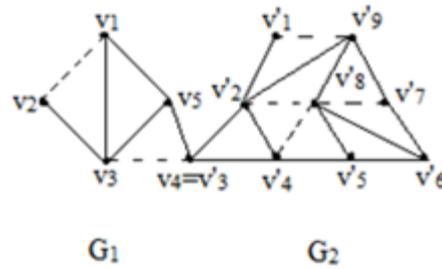


figure 2.1.1.

$$V(G_1) = \{ v_1, v_2, v_3, v_4, v_5 \}$$

$$V(G_2) = \{ v'_1, v'_2, v'_3, v'_4, v'_5, v'_6, v'_7, v'_8, v'_9 \}$$

dotted edges are the neutrosophic edges. Thus  $G_N = G_1 \cup G_2$  is a neutrosophic bigraph.

**Definition 2.1.3:**

A *neutrosophic weak bigraph*  $G = G_1 \cup G_2$  is a bigraph in which at least one of  $G_1$  or  $G_2$  is a neutrosophic graph and the other need not be a neutrosophic graph.

**Theorem 2.1.2:**

All neutrosophic bigraph are neutrosophic weak bigraph but a neutrosophic weak bigraph in general is not a neutrosophic bigraph.

**Proof:**

By the very definition we see all neutrosophic bigraphs are weak neutrosophic bigraphs. To show a weak neutrosophic bigraph in general is not a neutrosophic bigraph. Consider the weak neutrosophic bigraph  $G = G_1 \cup G_2$  given by the following figure 4.1.2 given by the following example.

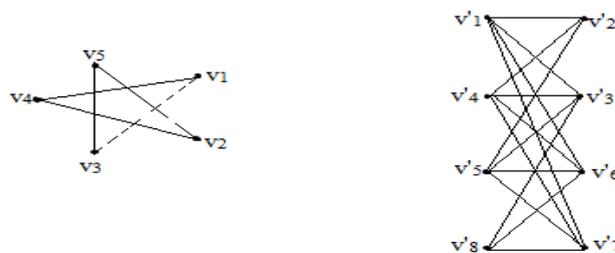


figure 2.1.2

$$G=G_1 \cup G_2$$

Clearly  $G_1$  is a neutrosophic graph but  $G_2$  is not a neutrosophic graph. So  $G=G_1 \cup G_2$  is not a neutrosophic bigraph but only a weak neutrosophic bigraph.

**Definition 2.1.4:**

Let  $G=G_1 \cup G_2$  be a neutrosophic bigraph  $G$  is said to *neutrosophically glued neutrosophic bigraph* if the bigraph is edge glued and at least one edge is a neutrosophic edge i.e. atleast one edge is joined by dotted lines.

**Theorem 2.1.2:**

Let  $G=G_1 \cup G_2$  be a weak neutrosophic graph which is not a neutrosophic bigraph. Then  $G=G_1 \cup G_2$  cannot be a neutrosophically glued neutrosophic bigraph.

**Proof:**

Let  $G=G_1 \cup G_2$  is a weak neutrosophic bigraph which is not a neutrosophic bigraph; i.e. without loss in generality we assume  $G_1$  is a neutrosophic graph and  $G_2$  is not a neutrosophic graph.  $G=G_1 \cup G_2$  is a weak neutrosophic bigraph only.

Suppose  $G= G_1 \cup G_2$  is neutrosophic bigraph then both the graphs  $G_1$  and  $G_2$  becomes neutrosophic as both  $G_1$  and  $G_2$  have only one neutrosophic edge in common, which is a very contradiction to our assumption that  $G$  is only a weak neutrosophic

**Definition 2.1.5:**

Let  $G=G_1 \cup G_2$  be a neutrosophic bigraph.  $G$  is said to be a *neutrosophic subbigraph connected* if the graph  $G_1$  and  $G_2$  have neutrosophic subbigraph in common.

**Theorem 2.1.3:**

Let  $G=G_1 \cup G_2$  be a weak neutrosophic bigraph which is not a neutrosophic bigraph  $G$  cannot be neutrosophic subbigraph connected.

**Proof:**

Given  $G= G_1 \cup G_2$  is a weak neutrosophic bigraph which is not a neutrosophic bigraph. That is only one of  $G_1$  or  $G_2$  is a neutrosophic graph. So by Theorem the weak

neutrosophic bigraph cannot be even neutrosophic edge connected so if, this is to be neutrosophic subbigraph connected  $G_1$  and  $G_2$  must have a neutrosophic subgraph. Which is not possible as only one of  $G_1$  or  $G_2$  is a neutrosophic graph. Hence the claim.

We know to every neutrosophic graph  $G$  there is a neutrosophic matrix associated with it. Likewise with every neutrosophic bigraph, we have neutrosophic bimatrix associated with it. Further with every weak neutrosophic bigraph we have a weak neutrosophic bimatrix associated with it.

### **3.1. Neutrosophic Cognitive map applied in Neutrosophic model**

#### **Definition 3.1.1:**

A *neutrosophic cognitive map (NCM)* is a neutrosophic directed graph with concepts like policies, events etc. as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts.

#### **Example 3.1.1:**

Here Analysis of strategic planning simulation based on NCMs knowledge and differential game is given. We use the map of FCM but after discussing with an expert converts it into an NCM by adjoining the edges which are indeterminate, and this is mainly carried out for easy comparison.

Now according to this expert, competitiveness and market demand is an indeterminate. Also sales price and economic condition is an indeterminate. Also according to him the productivity and market share is an indeterminate whether a relation exists directly cannot be said but he is not able to state that there is no relation between these concepts so he says let it be an indeterminate. Also according to him quality control and market share is an indeterminate. Thus on the whole the market share is an FCM with a lot of indeterminacy so is best fit with an NCM model. Thus obtain the initial version of NCM matrix and refined version of NCM matrix, also give the corresponding comment. Study the factor of indeterminacy and prove the result is nearer to truth for finding solutions to the market share problem. Compare FCM and NCM in the case of market share problem.

**Definition 3.1.2:**

An NCM is imbalanced if we can find two paths between the same two nodes that create causal relations of different sign. In the opposite case the NCM is balanced. The term ‘balanced’ *neutrosophic digraph* is used in the following sense that is in a imbalanced NCM we cannot determine the sign or the presence of indeterminacy of the total effect of a concept to another.

Now an similar lines based on the idea that as the length of the path increases, the indirect causal relations become weakened the total effect should have the sign of the shortest path between two nodes.

**Example 3.1.2:**

Illustration of neutrosophic cognitive state maps of users web behavior is described. Searching for information in general is complex, with lot of indeterminacies and it is an uncertain process for it depends on the search engine; number of key words, sensitivity of search, seriousness of search etc. Hence we can see several of the factors will remain as indeterminate for the  $C_1, C_2, \dots, C_7$  and we can remodel using NCM.

The NCM modeling of the users web behavior is given by the following neutrosophic digraph and the corresponding  $N(E)$  built using an expert opinion is given by the following neutrosophic matrix:

$$N(E) = \begin{pmatrix} 0 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & 0 & -1 & I & 1 & 1 \\ -1 & -1 & -1 & 0 & 0 & I & I \\ 1 & 1 & 1 & 1 & 0 & 1 & -1 \\ 1 & 1 & I & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 & I & 0 \end{pmatrix}$$

Several results and conclusions can be derived for each of the state vectors.

## **2. CONCLUSION:**

In this paper, fundamental features of graphs and its properties, we have discussed and definitions, examples and theorems of Neutrosophic graphs also discussed. The concepts of Neutrosophic bigraphs also have been discussed. Further these are implemented in neutrosophic models and the neutrosophic cognitive map is also implemented in Neutrosophic Models.

## **BIBLIOGRAPHY:**

- 1) **Balakrishnan .R., and K.Ranganathan,** A textbook of Graph Theory, Springer,1999.
- 2) **Bondy ,J.A., and Murthy, U.S.R.,** Graph Theory and application, The Mac Milan Press 1976.
- 3) **Feischner, H.,** Eulerian Graphs and related topics, Annals of Disc. Math, 45, North Holland, 1990.
- 4) **Fiorini, S., and Wilson, R.J.,** Edge colorings of graphs, in Research Notes in Mathematics, 16, pittman, London, 1971.
- 5) **Gibbons, A.,** Algorithmic Graph Theory, Cambridge. Univ. Press, 1985.