On the Accelerated Expansion of the Universe and the Preponderance of Matter over Antimatter

G. G. Nyambuya

Department of Applied Physics, National University of Science & Technology, Republic of Zimbabwe

Abstract

When the Dirac equation is modified to include a universal all-pervading Cosmic Four Vector Field, \( \Lambda_\mu \), the modified Dirac equation violates all the seven discrete symmetries i.e.: C, P, T, CP, CT, PT and CPT. The violation of the C-symmetry seems to be capable of explaining why the Universe appears to be made up chiefly of matter. By applying this Field to Einstein’s original Field Equation, one is able to link this Field with Einstein’s cosmological constant term, \( \Lambda \). Since the cosmological term is invoked to explain the supposed accelerated expansion of the Universe, the inclusion of this Field into the Dirac equation may be justified on this basis of the accelerated expansion of the Universe. Thus, the preponderance of matter over antimatter may be justified on the same basis. In conclusion, the Einstein \( \Lambda \)-field (hence the \( \Lambda_\mu \)-field) which causes the acceleration of the expansion of the Universe may also be responsible for the observed asymmetry in matter and antimatter.

Keywords: Cosmological constant, dark energy, dark matter, Dirac equation, matter, antimatter, asymmetry.

“The measure of greatness in a scientific idea is the extent to which it stimulates thought and opens up new lines of research.”

– Paul Adrien Maurice Dirac (1902 – 1984)

1 Introduction

Each electronically charged elementary particle has a counterpart with the opposite electronic charge which is known as its antiparticle (antiparticles are also referred to as antimatter) and just like normal particles, antiparticles do combine, forming atoms of antimatter (Amole et al. 2012, Ahmadi et al. 2016) which some call antiatoms – albeit, unlike atoms, these do not live long. Paul Dirac’s (1928a, b) brilliant theory predicted the existence of antimatter (Dirac 1930). It [Dirac’s Theory] is one of the most successful Theories of Physics. This theory, suggested that the Laws of Nature are exactly the same for matter and antimatter; so given this symmetry, the Universe must contain matter and antimatter in equal proportions everywhere and everytime – that is, across all of spacetime. Unfortunately (or maybe fortunately – as will be argued soon), when we look into our immediate vicinity, we see that this is not the case – our terrestrial habitat seems to be dominated exclusively by matter – so the question: “Why is our measurable Universe made up chiefly of matter with no significant quantities of antimatter?”, has always been hanging in-limbo since Dirac’s (1928a, b, 1930)’s theory was set forth.

While we may wonder why the Universe is formed this way, viz matter-antimatter imbalance, we must be very thankful that the Universe is formed this way, because if it [Universe] did really have equal proportions of matter and antimatter uniformly distributed throughout all of space and time, you the reader would not be reading this because the Universe would be nothing but a hot-bath of radiation because matter and antimatter would annihilate to form radiation. Despite its great success, the search for an answer to this great cosmic mystery – why we are so lucky to have a Universe chiefly made-up of...
matter – is the main theme of the present reading and it is important to mention that our adventures to seeking an answer to this great cosmic mystery will take us to other areas of physical enquiry and new discoveries. Though it shall prove difficult, we shall try to not veer too much off the main road but keep as much as we can – to what we want to achieve here.

It is worthwhile to mention here that the first and probably current-best attempt at an answer to this question is that by the Russian Physicist, father of the hydrogen bomb and 1975 Nobel Peace Prize winner, Andrei Sakharov (1924 – 1987). The attempt by Sakharov (1967) is the widely accepted explanation as to why there exist this matter-antimatter asymmetry – we offer an asymptotically different solution! Sakharov (1967) argued, that to create an imbalance between matter and antimatter from an initial condition of balance, certain conditions must be met and these conditions have come to the called the Sakharov conditions and CP-violation is one of the conditions. CP-violation is a violation of the symmetry where the Laws of Nature are expected to act the same when we simultaneously interchange the electronic charge (C-symmetry known as charge conjugation symmetry) of a particle and invert the space coordinates P-symmetry (known as parity symmetry).

Given the need for CP-asymmetric equations in physics (as dictated by the Sakharov 1967 conditions), much to the dismay and chagrin of the physicist, the Fundamental Equations of Physics, in their bare form, do not exhibit CP-violation (or P-violation) and this – sadly and against the desiderata, has to be inserted by hand into the equations. For example, in the Standard Model of Particle Physics, the Cabibbo-Kobayashi-Maskawa matrix (Kobayashi & Maskawa 1973; Cabibbo 1963) is employed and a complex phase factor is artificially injected into this matrix to bring about CP-violation in order to explain the observed CP-violation in the Kaon system (Fanti et al. 1999) and in B-meson aswell (Abe et al. 2001) as-well. Given Sakhorov’s thesis, this CP-violation (in the B-meson and Kaon systems) is thought of as holding the key to unlocking the mystery of matter-antimatter asymmetry – albeit – some researchers (e.g. Sinha 2009) feel it [the observed CP-violation in the B-meson and Kaon system] is not enough to explain the observed matter-antimatter asymmetry.

In this reading, we make a modification to the Dirac equation whereby we add a Cosmological Four Vector Field (CFVF) to it via the canonical procedure. This modification leads to an equation that violates C-symmetry hence an equation that clearly points to the fact that the Universe can only have one form of matter; either it is filled with matter or antimatter. Further, we show that this CFVF can be linked to Einstein’s cosmological constant field and as-well to darkmatter. The possibility of the existence of such a cosmological field has sound justification if one considers the cosmological observations such as the apparent accelerated expansion of the Universe (Riess et al. 1998, Perlmutter et al. 1999) and the indication from the rotation curves of galaxies that there must exist a form of unseen matter or energy (Davis 2014, Planck Collaboration et al. 2014, Rubin & Ford 1970, Rubin et al. 1970, 1985, Zwicky 1933, 1937). This unseen matter is popularly known as the darkmatter.

2 Modified Dirac Equation

The Dirac equation is given by:

\[ i\hbar \gamma^\mu \partial_\mu - m_0 c |\psi\rangle = 0, \]  

where \( |\psi\rangle \) is the Dirac four component wavefunction and:

\[ \gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \]  

are the 4 \times 4 Dirac gamma matrices where \( I_2 \) and 0 are the 2 \times 2 identity and null matrices respectively. Throughout this reading, the Greek indices will be understood to mean \( \mu, \nu, ... = 0, 1, 2, 3 \) and lower case English alphabet indices \( i, j, k, ... = 1, 2, 3 \).

Now, we wish to modify the Dirac equation \( 2.1 \) to include an all-pervading and all-permeating cosmic four vector field \( [\Lambda_\mu = \Lambda_\mu(r, t)] \) (the dimensions of this field is per unit length, L\(^{-1}\), and \( \Lambda_\mu \in \mathbb{R} \)).
In order for us to do this: suppose we have a Dirac particle $\psi$ whose momentum is $p_\mu$. If we immerse this particle in an electromagnetic field whose four vector potential is $A_\mu$ (this vector is here assumed to have the dimensions of per unit length per unit Coulomb: $L^{-1}C^{-1}$), the particle, $\psi$, couples to this electromagnetic field and in the process acquires a momentum $q \hbar A_\mu$. In order that this electromagnetic field be taken into account, the partial derivatives are made to undergo a canonical transformation as follows:

$$\partial_\mu \mapsto \partial_\mu + i q A_\mu, \quad (2.3)$$

so that the Dirac equation is now given by:

$$[i \hbar \gamma^\mu (\partial_\mu + i q A_\mu) - m_0 c] |\psi\rangle = 0. \quad (2.4)$$

Let us imagine that the Universe is filled with an all-pervading and all-permeating cosmic four vector field $\Lambda_\mu$ and that all matter and energy couples to this field. In much that same way as we would modified the Dirac equation as we have done in (2.4), we would take into account the field $\Lambda_\mu$ by making a canonical transformation as follows:

$$\partial_\mu \mapsto \partial_\mu + \Lambda_\mu, \quad (2.5)$$

so that the Dirac equation will now be given by:

$$[i \hbar \gamma^\mu (\partial_\mu + \Lambda_\mu) - m_0 c] |\psi\rangle = 0. \quad (2.6)$$

Because of the non-zero four vector term ($\Lambda_\mu$) – which is assumed to be invariant under any form of transformation of the state of the particle; the above modified Dirac equation (2.6) violates all the seven discrete symmetries i.e.: C, P, T, CP, CT, PT and CPT. By following the same steps as those presented in §(2 & 6) of e.g. Nyambuya (2015, 2016) respectively, one can easily demonstrate the aforesaid fact – that, equation (2.6) violates all the seven discrete symmetries: C, P, T, CP, CT, PT and CPT.

It should be noted that the modified Dirac equation (2.6) is Lorentz invariant. In that regard, it interesting that this equation is Lorentz invariant because it – against the CPT-Theorem; violates CPT-symmetry. The CPT-Theorem (Schwinger 1951, Lüders 1954, Pauli et al. 1955) holds that the CPT-symmetry must hold for all physical phenomena, or more precisely, that: any Lorentz invariant local Quantum Field Theory (QFT) with a Hermitian Hamiltonian must uphold CPT-symmetry. The CPT-Theorem is a sanctified Symmetry of Nature which is highly regarded and considered to be an exact symmetry of Nature (see e.g. Kostelecky 1998, Greenberg 2002, Villata 2011, Stadnik et al. 2014). All the same, the CPT-Theorem is here violated.

In §(4) [equation (4.16) to be specific], we shall show that this cosmic four vector ($\Lambda_\mu$) can be linked to Einstein’s cosmological constant term and the darkmatter field. In order to account for the preponderance of matter over antimatter, a C-violating Dirac equation is sufficient. Therefore, if the Universe is indeed endowed with such an all-pervading and all-permeating cosmic field, the absence of antimatter (or the observed asymmetry in matter and antimatter) is not an undecipherable ‘mystery’, but a result of this cosmic four vector field ($\Lambda_\mu$).

3 Einstein’s Cosmological Constant and Darkmatter

Two of the greatest, if not the greatest unsolved “mysteries” in cosmology, astronomy and astrophysics are the “hot” and polemical issues of the hypothetical form of matter and energy known as darkmatter and darkenergy. In the next two sub-sections, we will give a brief discussion of these phenomenon.
3.1 Cosmological Constant

Dark energy is a hypothetical form of energy believed to permeate all of space and tending to cause the accelerated expansion of the Universe (Peebles & Ratra 2003). Assuming that the standard model of cosmology is correct, then, the current best measurements (e.g. Steinhardt & Turok 2006) indicate that dark energy contributes $\sim 68.3\%$ of the total energy in the present-day observable Universe. This dark energy is believed to come in two forms, the first of which is the cosmological constant $\Lambda$ first added by Einstein into his gravitational field equations and this cosmological constant manifests as a constant energy density filling space homogeneously (Carroll 2001) and, the second is in the form of scalar fields whose energy density can vary in time and space (Caldwell 2002, Zlatev et al. 1999, Caldwell et al. 1998, Ratra & Peebles 1988).

As is common knowledge, the cosmological constant was first introduced by Albert Einstein (1879–1955) soon after formulating his brilliantly convincing theory (Einstein 1915) – the General Theory of Relativity (GTR); and applied it to the Universe (Einstein 1917) in order to obtain a model of the Universe (i.e., the general structure of spacetime on cosmic scales) that his theory predicted. After applying his gravitational field equation: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$ (Einstein 1915), to the Universe for a model Universe; it became clear to Einstein that his beautiful theory predicted that the Universe must – as a whole be expanding. Because this prediction was not at all in tandem with the prevailing astronomical wisdom of the day, Einstein (1917), judiciously added the cosmological fudge-factor, $\Lambda g_{\mu\nu}$, into his equations so that he obtains a static model of the Universe. The resulting gravitational field equation (Einstein 1917) after the addition of the fudge-factor is:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu},$$

where: $R_{\mu\nu}$ is the Riemann curvature tensor and $(R = g^{\mu\nu} R_{\mu\nu})$ is the contracted Riemann curvature tensor and:

$$T_{\mu\nu} = \rho v_{\mu} v_{\nu} + pg_{\mu\nu},$$

is the stress and energy tensor where $\rho$ is the density of matter, $p$ is the pressure and $v_{\mu}$ the four velocity, $\kappa = 8\pi G/c^4$ is the Einstein constant of Gravitation with $G$ being Newton’s universal constant of gravitation and $c$ is the speed of light in vacuum. Apart from the astronomical wisdom of the day, Einstein 1917 was motivated by Ernst Waldfried Josef Wenzel Mach (1838–1916)’s ideas of the origins of inertia. The cosmological field fulfilled Mach’s Principle (Mach 1893), a principle that had inspired Einstein to search for the GTR. Einstein thought that the GTR will have this naturally embedded in it. Mach’s Principle forbids the existence of a truly empty space and at the same time supra that the inertia of an object is due to the induction effect(s) of the totality of all-matter in the Universe.

After America’s celebrated astronomer – Edwin Powell Hubble (1889–1953), brought forth evidence that the Universe is in-fact expanding (Hubble 1929), Einstein – melodramatically – dropped the cosmological fudge-factor – famously calling its introduction (by him) into his (otherwise beautiful) original equations as “...the greatest blunder of my life...” (Gamov 1970 p.44). However, modern theoretical physics has taken up the idea of Einstein’s cosmological constant after the unprecedented landmarking measurements by Riess et al. 1998 & Perlmutter et al. 1999 which revealed that the Universe is undergoing an accelerated expansion. In order to explain this supposed accelerated expansion from within the provinces of Einstein’s GTR, one needs the cosmological constant term. On cosmological scales, this term (cosmological constant) acts as a cosmic repulsive gravitational force capable to explaining the supposed accelerated expansion.

3.2 Darkmatter

In the case of dark matter, this hypothetical form of matter is thought to pervade and permeate the entire Universe – for, upon a closer and meticulous inspection, when the tangential orbital speeds ($v_{\phi}$) of
stars and star systems (that lie along the galactic disk and are found) orbiting about the massive central bulge of spiral galaxies are measured, most surprisingly, it is found that the speed of these stars and star systems, instead of falling off in a Keplerian fashion \( v_\phi \propto r^{-\frac{1}{2}} \) as one would expect from Newton’s theory of gravitation, these speeds are roughly constant across the entire galactic disk. Assuming that Newtonian gravitation is correct description of the gravitational phenomenon up to the scale of galactic systems, then, the only explanation to the constancy of the tangential orbital speeds is that there must exist some non-luminous material in the interstices of the intervening spaces between the edge of the galactic bulge and the far end of the galactic disk. This hypothetical non-luminous material is what has come to be known as dark matter. If dark matter really exists, then, it neither emits nor absorbs light or any other electromagnetic radiation at any significant level.

The supposed presence of this hypothetical dark matter first come to notice in 1932 – 1933. While studying stellar motions in the local galactic neighbourhood, the renowned Dutch astronomer – after whom the Ort cloud is named; Jan Oort (1900 – 1992) in 1932, noted some discrepancies between the mass of large astronomical objects determined from their gravitational effects, and the mass calculated from the “luminous matter” they contained. Despite the inadequate evidence (cf. Kuijken & Gilmore 1989), Jan Oort postulated the existence of unseen matter. In the subsequent year in 1933, independently of Jan Oort, the eccentric Swiss astrophysicist, Fritz Zwicky (1898 – 1974), then working at the California Institute of Technology in the United States of America, examined the Coma galaxy cluster and applied it – the virial theorem, in-which event he reached the same conclusion as that of Oort – that; galaxies must contain more matter than is inferred from the light they emit – he referred to this unseen matter as “Dunkle Materie” which is German for ‘Dark Matter’ (Zwicky 1933, 1937). More convincing evidence of this in spiral galaxies was availed by Rubin & Ford (1970), Rubin et al. (1970, 1985). Today (see e.g. Davis 2014, Planck Collaboration et al. 2014, Angus et al. 2013), the idea of dark matter is commonplace (it is not disputed, but universally accepted) and is subject of serious research – theoretically, observationally and experimentally. We will show shortly that the proposed CFVF, \( \Lambda_{\mu} \), can be used to investigate this dark matter phenomenon. We introduce here this CFVF, \( \Lambda_{\mu} \), and link it to Einstein’s cosmological constant.

4 Modification of Einstein’s Original Field Equation

The original Einstein Field Equation is Einstein’s Field Equation (3.1) with a vanishing cosmological term (i.e., \( \Lambda = 0 \)). What we shall do here is to demonstrate that if one where to include the canonical cosmic momentum in the form of the cosmic four vector \( \Lambda_{\mu} \) into the original Einstein Field Equation, they will be able to account not only for Einstein’s cosmological constant \( \Lambda \), but the dark matter field as well. To do this, we start off from the Riemann curvature tensor. Written in full, the Riemann curvature tensor is given by:

\[
P_{\mu\lambda\nu} = \Gamma_{\mu\nu,\lambda} - \Gamma_{\mu\lambda,\nu} + \Gamma_{\sigma\lambda} \Gamma_{\mu\nu} - \Gamma_{\sigma\nu} \Gamma_{\mu\lambda},
\]

(4.1)

If we are to apply the canonical transformation (2.5) to this curvature tensor (i.e., \( \partial_\mu \mapsto \partial_\mu + \Lambda_\alpha \)), it follows that the affine \( \Gamma_{\mu\nu}^\alpha \) will have to undergo a transformation as follows:

\[
\Gamma_{\mu\nu}^\alpha \mapsto \bar{\Gamma}_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha + \gamma_{\mu\nu}^\alpha,
\]

(4.2)

where:

\[
\gamma_{\mu\nu}^\alpha = \frac{1}{2} \left( \delta_{\mu}^\alpha \Lambda_\nu + \delta_{\nu}^\alpha \Lambda_\mu - g_{\mu\nu} \Lambda^\alpha \right),
\]

(4.3)

is an affine connection arising due to the four vector field \( \Lambda_\mu \). This affine connection \( \gamma_{\mu\nu}^\alpha \) is a tensor and is identical in form and structure to Weyl (1918)’s affine connection. However, the meaning of \( \gamma_{\mu\nu}^\alpha \) as a connection is very different from that of Weyl’s theory, because Weyl’s theory, the vector \( \Lambda_\mu \) is the
electromagnetic vector potential of the particle in-question while in the present is not, but is something completely different. The resulting spacetime is still very much Riemannian whereas Weyl’s theory, the resulting spacetime is a pseudo-Riemann space. S, our spacetime is a kind of new Weyl-space.

Now, with the introduction of this connection, γ^α_µν, where-in (Γ^α_µν ↦→ ¯Γ^α_µν), the Riemann tensor will transform as:

\[
R^α_μλν ↦→ \bar{R}^α_μλν = \bar{Γ}^α_μλν,λ - \bar{Γ}^α_μλ,ν + \bar{Γ}^σ_α_σλ,ν_µ - \bar{Γ}^σ_α_σν,λ_µ. \tag{4.4}
\]

This new Riemann tensor can be written as a sum of three curvature tensors, i.e.:

\[
\bar{R}^α_μλν = R^α_μλν - κD^α_μλν + \mathcal{R}^α_μλν \tag{4.5}
\]

where:

\[
-κD^α_μλν = γ^α_μλ,ν - γ^α_μλ,σ_σν + γ^σ_α_σλ,σ_νλ - γ^σ_α_σν,σ_λμ, \tag{4.6}
\]

is the curvature due to the four vector field Λ_µ and:

\[
\mathcal{R}^α_μλν = Γ^σ_α_σλ,σ_νμ + γ^σ_α_σλ,σ_νμ - Γ^σ_α_σμ,σ_λν - γ^σ_α_σμ,σ_λν_σμ. \tag{4.7}
\]

is the curvature tensor arising from the interference of the particle and the Λ_µ-field. We shall set as a precondition that for all conditions of existence, we must have:

\[
\mathcal{R}^α_μλν = 0, \tag{4.8}
\]

so that the resultant curvature tensor \( \bar{R}^α_μλν \) is such that:

\[
\bar{R}^α_μλν = R^α_μλν - κD^α_μλν. \tag{4.9}
\]

Further, let us set \( \mathcal{R}_{μλ} \) and \( \mathcal{R} \) such that:

\[
\mathcal{R}_{μλ} = \mathcal{R}^α_μλ = 0, \tag{4.10}
\]

\[
\mathcal{R} = g^{μν}\mathcal{R}_{μν} = 0. \tag{4.11}
\]

From all this, it follows that the new Einstein Field Equation “without Einstein’s cosmological term” will now be given by:

\[
\bar{R}_{μν} - \frac{1}{2}\bar{R}g_{μν} = κT_{μν}, \tag{4.12}
\]

where \( \bar{R}_{μν} = \bar{R}^α_μαν = \bar{R}^α_μαν \) and \( \bar{R} = g^{μν}\bar{R}_{μν} \). Furthermore, let us set \( \mathcal{D}_{μν} \) and \( \mathcal{D} \) such that:

\[
\mathcal{D}_{μν} = \mathcal{D}^α_μλν = \mathcal{D}_{μν} ≠ 0, \tag{4.13}
\]

\[
-κD = g^{μν}\Lambda_{μν} = 2Λ ≠ 0 ⇒ \Lambda = -\frac{1}{2}κD, \tag{4.14}
\]

where \( \mathcal{D}_{μν} \) is the darkmatter field and \( \Lambda \) (with dimensions of per unit square length) is a cosmological constant that we shall identify with Einstein’s cosmological term. The above equation (\( \Lambda = -κD/2 \)) links the CFVF, \( \Lambda_µ \) (with dimensions of per unit length), and Einstein’s cosmological constant term, \( \Lambda \). From all this, it follows that the Einstein Field Equation now with a cosmological term and the additional dark matter field will be given by:

\[
R_{μν} - \frac{1}{2}RG_{μν} = κT_{μν} + κD_{μν} + \Lambda g_{μν}. \tag{4.15}
\]
In-order for the conservation of mass-energy, we must have the fields $D_{\mu\nu}$ and $\Lambda g_{\mu\nu}$ constrained such that:

$$ \partial^\mu (\kappa D_{\mu\nu} + \Lambda g_{\mu\nu}) = 0. \quad (4.16) $$

We thus have shown here that the cosmic four vector field $\Lambda_{\mu}$ can be used to justify not only the inclusion of Einstein’s cosmological term, but a darkmatter term as-well.

5 Discussion

We first conceived of the idea of the CFVF in the unpublished manuscript – Nyambuya (2008). There-in the reading Nyambuya (2008), we where unable to connect this CFVF to anything that is accepted by the majority of scientists such as Einstein’s cosmological constant. The CFVF of Nyambuya (2008) stood in there as a hypothesis far removed from anything associated with the present world. Our thrust in this reading (Nyambuya 2008) was to find a $C$-violating Dirac equation in-order to explain the preponderance of matter over antimatter. Because of this failure to connect the CFVF to something that is thought to be physical and existing, we did not feel confident of this modification. However, in the present reading, we have been able to connect this CFVF with Einstein’s cosmological constant which is assumed to be the cause of the observed acceleration of the Universe. Apart from this, we have also been able to link this vector with darkmatter via the darkmatter stress, energy and momentum tensor $D_{\mu\nu}$. What this all means is that a platform for the falsification of the idea has here been established. Clearly, if the CFVF really exists, then one will be able to “hit two birds with one stone” as this would simultaneously explain darkmatter and darkenergy.

6 Conclusion

Assuming the acceptability of the ideas presented here-in, we hereby put the following forward as our conclusion:

1. Einstein’s $\Lambda$ cosmological constant (hence the $\Lambda_{\mu}$-field), which at present is assumed to be the most likely cause of the acceleration of the the expansion of the Universe, this same field can – under the present proposal, be held responsible for the observed asymmetry in matter and antimatter as this constant can be linked – via equation (4.16); to the CFVF that leads to a symmetry violating Dirac equation.

2. The revered, seemingly and highly regarded $CPT$-Theorem that holds that all Lorentz invariant theories must uphold $CPT$-symmetry, this symmetry is violated by the presence of the CFVF, hence, by – Einstein’s $\Lambda$ cosmological constant [i.e., if the connection between $\Lambda_{\mu}$ and $\Lambda$ as given in equation (4.16), is accepted/or exists in reality].

References


Nyambuya, G. G., On the Accelerated Expansion of the Universe and the Preponderance of Matter over Antimatter


