δ-equalities of Neutrosophic Sets

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***Abstract*— Fuzzy sets and intuitionistic fuzzy sets can't handle imprecise, indeterminate, inconsistent, and incomplete information. Neutrosophic sets play an important role to overcome this difficulty. A neutrosophic set has a truth membership function, indeterminate membership function, and a falsehood membership functions that can handle all types of ambiguous information. New type of union and intersection has been proposed in this paper. In this paper, δ -equalities of neutrosophic sets have been introduced. Further, some basic properties of δ -equalities have been discussed. Moreover, these δ -equalities have been applied to set theoretic operations of neutrosophic set such as union, intersection, complement, product, probabilistic sum, bold sum, bold intersection, bounded difference, symmetrical difference, and convex linear sum of min and max. These δ -equalities of neutrosophic sets have been further extended to neutrosophic relations and neutrosophic norms respectively. In this paper, δ -equalities also applied in the composition of neutrosophic relations, Cartesian product and neutrosophic triangular norms. The applications and utilizations of δ -equalities have been presented in this paper. In this regards, δ -equalities have been successfully applied in Fault Tree Analysis and Neutrosophic Reliability (generalization of Profust Reliability).**

Keywords*— Fuzzy set, intuitionistic fuzzy set, neutrosophic set, neutrosophic relation, δ -equalities.

I. Introduction

Fuzzy sets were introduced by Zadeh [31] in 1965. This novel mathematical framework is used to handle uncertainty in several areas of our real life. The characterization of a fuzzy set is made by a membership function μ which has the range $[0,1]$. The applications and utilization of fuzzy sets have been extensively found in different aspects from the last few decades such as control [31], pattern recognition[13], and computer vision[32] etc. This theory also become an important area for the researchers in medical diagnosis [32], engineering [13], social sciences [32] etc. A huge amount of literature on fuzzy set theory can be found in [13,22,32]. In fuzzy set, the membership function μ is a single value between in the unit interval [0,1]. Therefore it is not always true that the non-membership function ν of an element is equal to $1-\mu$, because there is some kind of hesitation. Thus in 1986, Atanassov [1] introduced intuitionistic fuzzy sets to explain this situation by incorporating the hesitation degree called hesitation margin. The hesitation margin is defined as $1-\mu-\nu$. Thus an intuitionistic fuzzy set has a membership function μ and non-membership function ν which has range $[0,1]$ with an extra condition that $0 \le \mu+\nu \le 1$. In this way, the intuitionistic fuzzy set theory became the generalization of fuzzy set theory. As an application point of view, the intuitionistic fuzzy set theory have been successfully applied in medical diagnosis [21], pattern recognition [22], social sciences [6] and decision making [11] etc.

Fuzzy sets and intuitionistic fuzzy sets can't handle imprecise, indeterminate, inconsistent, and incomplete information. Therefore, Smarandache [20] in 1998, introduced neutrosophic logic and set inspired from Neutrosophy, a branch of philosophy that deals with the origin, nature, and scope of neutralities and their interactions with different ideational spectra. A neutrosophic set has a truth membership function T, an indeterminate membership function I and a falsehood membership function F. The indeterminacy degree I plays a very important role of mediocrity. Thus, neutrosophic set theory generalizes the concept of classical set theory [20], fuzzy sets theory [31], intuitionistic fuzzy sets theory [1], interval valued fuzzy set theory [22], paraconsistent theory [20], dialetheist theory [20], paradoxist theory [20], and tautological theory [20]. Neutrosophic set is a powerful tool to handle the indeterminate and inconsistent information that exists in our real world. The researchers have been successfully applied neutrosophic set theory in several areas. In this regard, Wang et al. [26] introduced single valued neutrosophic sets in order to use them in scientific and engineering that gives some additional possibility to represent

978-1-5090-0626-7/16/$31.00

uncertain, incomplete, imprecise, and inconsistent data. Hanafy et.al discussed the correlation coefficient of neutrosophic set [8,9]. Ye [27] conducted study on the correlation coefficient of single valued neutrosophic sets. Broumi and Smaradache studied the correlation coefficient of interval neutrosophic set in [2]. Salama et al. [18] discussed neutrosophic topological spaces. Some more literature on neutrosophic set can be seen in [5, 12, 14, 17, 19, 25, 28, 29, 30]. Neutrosophic set have been applied successfully in decision making theory [5, 27-30], data base [25], medical diagnosis [30], pattern recognition [7,15] and so on.

The notion of proximity measure was introduced by Pappis [16] to show that values of precise membership has no practical significance. He believed that the maxmin compositional rule of inference is preserved with approximately equal fuzzy sets. Hong and Hwang [10] proposed another important generalization of the work of Pappis [16] which is mainly based that the maxmin compositional rule of inference is preserved with respect to 'approximately equal fuzzy sets' and 'approximately equal' fuzzy relation respectively. But, Cai [3, 4] felt that both the Pappis [16] and Hong and Hwang [10] tactics were limited to fixed ε. Therefore, Cai [3, 4] took a different methodology to introduced δ-equalities of fuzzy sets. Cai [3, 4] proposed that if two fuzzy sets are equal to a degree of δ, then they are said to be δ-equal. The approach of δ-equalities have significances in the fuzzy statistics as well as fuzzy reasoning. Virant [23] applied δ-equalities of fuzzy sets in synthesis of real-time fuzzy systems while Cai [3,4] used them for assessing the robustness of fuzzy reasoning. Cai [3,4] also explain several reliability examples of δ-equalities.

This paper extends the theory of δ-equalities to neutrosophic sets. Basically we followed the philosophy of Cai [3, 4] to studied δ-equalities of neutrosophic sets. The organization of the rest of the paper is followed. In section 2, some basic and fundamental concepts of neutrosophic sets were presented. New type of union and intersection has been introduced. δ-equalities on neutrosophic sets were introduced in section 3. Moreover, these δ-equalities have been applied to set theoretic operations of neutrosophic set such as union, intersection, complement, product, probabilistic sum, bold sum, bold intersection, bounded difference, symmetrical difference, and convex linear sum of min and max. In section 4, these δ-equalities of neutrosophic sets have been further extended to neutrosophic relations and neutrosophic norms respectively. The applications and utilizations of δ-equalities have been presented in section 5. In this regards, δ-equalities have been successfully applied in Fault Tree Analysis and Neutrosophic Reliability (generalization of Profust Reliability). Conclusion is given in section 6.

We now review some basic concepts of neutrosophic sets and other related notion which will be used in this paper.

II. Literature Review

In this section, some basic concepts of neutrosophic sets and other related notions have been presented. These notions and definitions have been taken from [3], [4], [20], and [26].

Definition 2.1 [20]. Neutrosophic Set

Let U be a space of points and let $u \in U$. A neutrosophic set S in U is characterized by a truth membership function T_S, an indeterminacy membership function I_S, and a falsity membership function F_S. $T_S(u)$, $I_S(u)$ and $F_S(u)$ are real standard or non-standard subsets of $]0^-,1^+[$, that is $T_S, \mathrm{I}_S, \mathrm{F}_S : X \to]0^-,1^+[$. The neutrosophic set can be represented as

$$S = \{(u, T_S(u), I_S(u), F_S(u)) : u \in U\}$$

There is no restriction on the sum of $T_S(u), I_S(u)$ and $F_S(u)$, so $0^- \le T_S(u) + I_S(u) + F_S(u) \le 3^+$. From philosophical point view, the neutrosophic set takes the value from real standard or non-standard subsets of $]0^-,1^+[$. Thus it is necessary to take the interval $[0,1]$ instead of $]0^-,1^+[$ for technical applications. It is difficult to apply $]0^-,1^+[$ in the real life applications such as engineering and scientific problems. We now give some set theoretic operations of neutrosophic sets.

Definition 2.2 [26]. Complement of Neutrosophic Set

The complement of a neutrosophic set S is denoted by S^c and is defined by

$$T_{S^c}(u) = 1 - T_S(u),\ I_{S^c}(u) = 1 - I_S(u),$$
$$F_{S^c}(u) = 1 - F_S(u) \text{ for all } u \in U.$$

Definition 2.3 [26]. Union of Neutrosophic Sets

Let A and B be two neutrosophic sets in a universe of discourse X. Then the union of A and B is denoted by $A \cup B$, which is defined by

$$A \cup B = \{(x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)) : x \in X\}$$

for all $x \in X$, and $\vee$ denote the max-operator and $\wedge$ denote the min-operator respectively.

Definition 2.4 [26]. Intersection of Neutrosophic Sets

Let A and B be two neutrosophic sets in a universe of discourse X. Then the intersection of A and B is denoted as $A \cap B$, which is defined by

$$A \cap B = \{(x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)) : x \in X\}$$

Lemma 2.1 [3,4]: Let $\delta_1 * \delta_2 = \max(0, \delta_1 + \delta_2 - 1)$, where $0 \le \delta_1, \delta_2 \le 1$. Then

1. $0 * \delta_1 = 0$; for all $\delta_1 \in [0,1]$,
2. $1 * \delta_1 = \delta_1$; for all $\delta_1 \in [0,1]$,
3. $0 \le \delta_1 * \delta_2 \le 1$; for all $\delta_1, \delta_2 \in [0,1]$,

4. $\delta_1 \le \delta_1' \Rightarrow \delta_1 * \delta_2 \le \delta_1' * \delta_2$; for all $\delta_1, \delta_1', \delta_2 \in [0,1]$,
5. $\delta_1 * \delta_2 = \delta_2 * \delta_1$; for all $\delta_1, \delta_2 \in [0,1]$,
6. $(\delta_1 * \delta_2) * \delta_3 = \delta_1 * (\delta_2 * \delta_3)$; for all $\delta_1, \delta_2, \delta_3 \in [0,1]$.

III. δ-Equalities of Neutrosophic Sets

In this section, new type of union and intersection of neutrosophic set introduced which will be used in this paper. Further, δ-equalities of neutrosophic sets introduced and discussed some of their properties. Moreover, these δ-equalities utilized in set theoretic operations of neutrosophic set such as union, intersection, complement, product, probabilistic sum, bold sum, bold intersection, bounded difference, symmetrical difference, and convex linear sum of min and max.

Definition 3.1: Let A and B be two complex neutrosophic sets in a universe of discourse U. Then the union of A and B is denoted by $A \cup B$, which is defined by

$$A \cup B = \left\{\left(u, T_A(u) \vee T_B(u), I_A(u) \vee I_B(u), F_A(u) \vee F_B(u)\right) : u \in U\right\}$$

for all $u \in U$, and $\vee$ denote the max-operator.

Definition 3.2: Let A and B be two complex neutrosophic sets in a universe of discourse U. Then the intersection of A and B is denoted as $A \cap B$, which is defined by

$$A \cap B = \left\{\left(u, T_A(u) \wedge T_B(u), I_A(u) \wedge I_B(u), F_A(u) \wedge F_B(u)\right) : u \in U\right\}$$

for all $u \in U$, and $\wedge$ denote the min-operator.

Definition 3.3: Let U be a universe of discourse. Let A and B be two neutrosophic sets on U, and $T_A(u)$, $I_A(u)$, $F_A(u)$ and $T_B(u)$, $I_B(u)$, $F_B(u)$, their truth membership functions, indeterminate membership functions and falsehood membership functions respectively. Then A and B are said to be δ-equal if and only if

$\sup_{u\in U}\left|T_A(u) - T_B(u)\right| \le 1-\delta$, $\sup_{u\in U}\left|I_A(u) - I_B(u)\right| \le 1-\delta$,

$\sup_{u\in U}\left|F_A(u) - F_B(u)\right| \le 1-\delta$, for all $u \in U$ and $0 \le \delta \le 1$.

We denote it as $A = (\delta)B$. From the definition it is clear that $1-\delta$ is the maximum difference or proximity measure between A and B and δ is the degree of equality between them. It is customary to be noted that δ-equality of neutrosophic sets construct the class of neutrosophic relations.

Proposition 3.1: For two neutrosophic sets A and B, defined on U. The following assertions hold.

1. $A = (0)B$,
2. $A = (1)B$ if and only if $A = B$,
3. $A = (\delta)B$ if and only if $B = (\delta)A$,
4. $A = (\delta_1)B$ and if $\delta_1 \ge \delta_2$, then $A = (\delta_2)B$,
5. If $A = (\delta_\alpha)B$ for all $\alpha \in J$, where J is an index set, then $A = \left(\sup_{\alpha\in J}\delta_\alpha\right)B$,
6. For all A, B, there exist a unique δ such that $A = (\delta)B$ and if $A = (\delta')B$, then $\delta' \le \delta$.

Proposition 3.2: If $A = (\delta_1)B$ and $B = (\delta_2)C$, then $A = (\delta)C$ where $\delta = \delta_1 * \delta_2$.

Proposition 3.3: Let $A_1 = (\delta_1)B_1$ and $A_2 = (\delta_2)B_2$. Then

$$A_1 \cup A_2 = \left(\min(\delta_1, \delta_2)\right) B_1 \cup B_2 .$$

Proposition 3.4: Let $A_\alpha = (\delta_\alpha)B_\alpha$, for all $\alpha \in J$, where J is an index set. Let $\bigcup_{\alpha\in J} A_\alpha$ represents the union of $\{A_\alpha : \alpha \in J\}$ and $\bigcup_{\alpha\in J} B_\alpha$ represents the union of $\{B_\alpha : \alpha \in J\}$, and

$T_{\bigcup_{\alpha\in J} A_\alpha}(u) = \sup_{\alpha\in J} T_{A_\alpha}(u)$, $I_{\bigcup_{\alpha\in J} A_\alpha}(u) = \inf_{\alpha\in J} I_{A_\alpha}(u)$, $F_{\bigcup_{\alpha\in J} A_\alpha}(u) = \inf_{\alpha\in J} F_{A_\alpha}(u)$ and $T_{\bigcup_{\alpha\in J} B_\alpha}(u) = \sup_{\alpha\in J} T_{B_\alpha}(u)$, $I_{\bigcup_{\alpha\in J} B_\alpha}(u) = \inf_{\alpha\in J} I_{B_\alpha}(u)$, $F_{\bigcup_{\alpha\in J} B_\alpha}(u) = \inf_{\alpha\in J} F_{B_\alpha}(u)$ their truth membership functions, indeterminacy membership functions and falsity membership functions, respectively. Then

$$\bigcup_{\alpha\in J} A_\alpha = \left(\inf_{\alpha\in J}\delta_\alpha\right) \bigcup_{\alpha\in J} B_\alpha .$$

Proposition 3.5: Let A^c be the complement of A and B^c be the complement of B. Further let $A = (\delta)B$. Then

$$A^c = (\delta)B^c .$$

Proposition 3.6: Let $A_\alpha = (\delta_\alpha)B_\alpha$, for all $\alpha \in J$, where J is an index set. Let $\bigcap_{\alpha\in J} A_\alpha$ represents the intersection of $\{A_\alpha : \alpha \in J\}$ and $\bigcap_{\alpha\in J} B_\alpha$ represents the intersection of $\{B_\alpha : \alpha \in J\}$, and $T_{\bigcap_{\alpha\in J} A_\alpha}(u) = \inf_{\alpha\in J} T_{A_\alpha}(u)$, $I_{\bigcap_{\alpha\in J} A_\alpha}(u) = \sup_{\alpha\in J} I_{A_\alpha}(u)$, $F_{\bigcap_{\alpha\in J} A_\alpha}(u) = \sup_{\alpha\in J} F_{A_\alpha}(u)$ and $T_{\bigcap_{\alpha\in J} B_\alpha}(u) = \inf_{\alpha\in J} T_{B_\alpha}(u)$, $I_{\bigcap_{\alpha\in J} B_\alpha}(u) = \sup_{\alpha\in J} I_{B_\alpha}(u)$, $F_{\bigcap_{\alpha\in J} B_\alpha}(u) = \sup_{\alpha\in J} F_{B_\alpha}(u)$ their truth membership functions, indeterminacy membership functions and falsity membership functions, respectively. Then

$$\bigcap_{\alpha\in J} A_\alpha = \left(\inf_{\alpha\in J}\delta_\alpha\right) \bigcap_{\alpha\in J} B_\alpha .$$

Corollary 3.1: Let $A_{\alpha\beta} = (\delta_{\alpha\beta})B_{\alpha\beta}$; $\alpha \in J_1$ and $\beta \in J_2$ where J_1 and J_2 are index sets. Then

$$\bigcup_{\alpha\in J_1}\bigcap_{\beta\in J_2} A_{\alpha\beta} = \left(\inf_{\alpha\in J_1}\inf_{\beta\in J_2}\delta_\alpha\right) \bigcup_{\alpha\in J_1}\bigcap_{\beta\in J_2} B_{\alpha\beta}, \text{ and}$$

$$\bigcap_{\alpha\in J_1}\bigcup_{\beta\in J_2} A_{\alpha\beta} = \left(\inf_{\alpha\in J_1}\inf_{\beta\in J_2}\delta_\alpha\right) \bigcap_{\alpha\in J_1}\bigcup_{\beta\in J_2} B_{\alpha\beta} .$$

Corollary 3.2: Let $A_k=(\delta_k)B_k$, where $k=1,2,3,\cdots$ and let $\limsup_{n\to\infty} A_n=\bigcap_{n=1}^{\infty}\bigcup_{k=n}^{\infty}A_k$, $\liminf_{n\to\infty} A_n=\bigcup_{n=1}^{\infty}\bigcap_{k=n}^{\infty}A_k$;

$$\limsup_{n\to\infty}\mathrm{B}_n=\bigcap_{n=1}^{\infty}\bigcup_{k=n}^{\infty}B_k,\ \liminf_{n\to\infty} B_n=\bigcup_{n=1}^{\infty}\bigcap_{k=n}^{\infty}B_k.$$

Then

$$\limsup_{n\to\infty} A_n=\left(\inf_{n\geq 1}\delta_n\right)\limsup_{n\to\infty} B_n,$$

$$\liminf_{n\to\infty} A_n=\left(\inf_{n\geq 1}\delta_n\right)\liminf_{n\to\infty} B_n.$$

Proposition 3.7: Let $A_1=(\delta_1)B_1$, $A_2=(\delta_2)B_2$. Let A_1A_2 represent the product of A_1, A_2 ,and B_1B_2 represent the product of B_1,B_2. Let $T_{A_1A_2}(u)=T_{A_1}(u)T_{A_2}(u)$, $I_{A_1A_2}(u)=I_{A_1}(u)I_{A_2}(u)$, $F_{A_1A_2}(u)=F_{A_1}(u)F_{A_2}(u)$ and $T_{B_1B_2}(u)=T_{B_1}(u)T_{B_2}(u)$, $I_{B_1B_2}(u)=I_{B_1}(u)I_{B_2}(u)$, $F_{B_1B_2}(u)=F_{B_1}(u)F_{B_2}(u)$ their truth membership functions, indeterminacy membership functions and falsity membership functions, respectively. Then

$$A_1A_2=(\delta_1*\delta_2)B_1B_2.$$

Proposition 3.8: Let $A_j=(\delta_j)B_j$, where $j=1,2,3,\cdots,n$. Then

$$A_1\cdots A_n=(\delta_1*\cdots\delta_n)B_1\cdots B_n.$$

Proposition 3.9: Let $A_1=(\delta_1)B_1$ and $A_2=(\delta_2)B_2$. Let $A_1\hat{+}A_2$ represent the probabilistic sum of A_1 and A_2, and $B_1\hat{+}B_2$ represent the probablisitc sum of B_1 and B_2 respectively. Let $T_{A_1\hat{+}A_2}(u)$, $I_{A_1\hat{+}A_2}(u)$, $F_{A_1\hat{+}A_2}(u)$ and $T_{B_1\hat{+}B_2}(u)$, $I_{B_1\hat{+}B_2}(u)$, $F_{B_1\hat{+}B_2}(u)$ be their truth membership functions, indeterminacy membership functions and falsity membership functions respectively, where

$$T_{A_1\hat{+}A_2}(u)=T_{A_1}(u)+T_{A_2}(u)-T_{A_1}(u)T_{A_2}(u),$$

$$I_{A_1\hat{+}A_2}(u)=I_{A_1}(u)+I_{A_2}(u)-I_{A_1}(u)I_{A_2}(u),$$

$$F_{A_1\hat{+}A_2}(u)=F_{A_1}(u)+F_{A_2}(u)-F_{A_1}(u)F_{A_2}(u),\text{ and}$$

$$T_{B_1\hat{+}B_2}(u)=T_{B_1}(u)+T_{B_2}(u)-T_{B_1}(u)T_{B_2}(u),$$

$$I_{B_1\hat{+}B_2}(u)=I_{B_1}(u)+I_{B_2}(u)-I_{B_1}(u)I_{B_2}(u),$$

$$F_{B_1\hat{+}B_2}(u)=F_{B_1}(u)+F_{B_2}(u)-F_{B_1}(u)F_{B_2}(u).\text{ Then}$$

$$A_1\hat{+}A_2=(\delta_1*\delta_2)B_1\hat{+}B_2.$$

Corollary 3.3: Suppose $A_j=(\delta_j)B_j$, where $j=1,2,3,\cdots,n$. Then

$$A_1\hat{+}\cdots\hat{+}A_n=(\delta_1*\cdots*\delta_n)B_1\hat{+}\cdots\hat{+}B_n.$$

Proposition 3.10: Suppose $A_1=(\delta_1)B_1$ and $A_2=(\delta_2)B_2$ and let $A_1\dot{\bigcup}A_2$ represent the bold sum of A_1 and A_2, and $B_1\dot{\bigcup}B_2$ represent the bold sum of B_1 and B_2 respectively. Let $T_{A_1\dot{\cup}A_2}(u)$, $I_{A_1\dot{\cup}A_2}(u)$, $F_{A_1\dot{\cup}A_2}(u)$ and $T_{B_1\dot{\cup}B_2}(u)$, $I_{B_1\dot{\cup}B_2}(u)$, $F_{B_1\dot{\cup}B_2}(u)$ their truth membership functions, indeterminacy functions and falsity membership functions respectively, where

$$T_{A_1\dot{\cup}A_2}(u)=\min\left(1,T_{A_1}(u)+T_{A_2}(u)\right),$$

$$I_{A_1\dot{\cup}A_2}(u)=\min\left(1,\mathrm{I}_{A_1}(u)+I_{A_2}(u)\right),$$

$$F_{A_1\dot{\cup}A_2}(u)=\min\left(1,\mathrm{F}_{A_1}(u)+F_{A_2}(u)\right)\text{ and}$$

$$T_{B_1\dot{\cup}B_2}(u)=\min\left(1,T_{B_1}(u)+T_{B_2}(u)\right),$$

$$I_{B_1\dot{\cup}B_2}(u)=\min\left(1,I_{B_1}(u)+I_{B_2}(u)\right),$$

$$F_{B_1\dot{\cup}B_2}(u)=\min\left(1,\mathrm{F}_{B_1}(u)+F_{B_2}(u)\right).\text{ Then}$$

$$A_1\dot{\bigcup}A_2=(\delta_1*\delta_2)B_1\dot{\bigcup}B_2.$$

Proposition 3.11: Suppose $A_1=(\delta_1)B_1$ and $A_2=(\delta_2)B_2$ and let $A_1\underset{\bullet}{\bigcap}A_2$ represent the bold intersection of A_1 and A_2, and $B_1\underset{\bullet}{\bigcap}B_2$ represent the bold intersection of B_1 and B_2 respectively. Let $T_{A_1\underset{\bullet}{\cap}A_2}(u)$, $I_{A_1\underset{\bullet}{\cap}A_2}(u)$, $F_{A_1\underset{\bullet}{\cap}A_2}(u)$ and $T_{B_1\underset{\bullet}{\cap}B_2}(u)$, $I_{B_1\underset{\bullet}{\cap}B_2}(u)$, $F_{B_1\underset{\bullet}{\cap}B_2}(u)$ their truth membership functions, indeterminacy functions and falsity membership functions respectively, where

$$T_{A_1\underset{\bullet}{\cap}A_2}(u)=\max\left(0,T_{A_1}(u)+T_{A_2}(u)-1\right),$$

$$I_{A_1\underset{\bullet}{\cap}A_2}(u)=\max\left(0,\mathrm{I}_{A_1}(u)+I_{A_2}(u)-1\right),$$

$$F_{A_1\underset{\bullet}{\cap}A_2}(u)=\max\left(0,\mathrm{F}_{A_1}(u)+F_{A_2}(u)-1\right)\text{ and}$$

$$T_{B_1\underset{\bullet}{\cap}B_2}(u)=\max\left(0,T_{B_1}(u)+T_{B_2}(u)-1\right),$$

$$I_{B_1\underset{\bullet}{\cap}B_2}(u)=\max\left(0,\mathrm{I}_{B_1}(u)+I_{B_2}(u)-1\right),$$

$$F_{B_1\underset{\bullet}{\cap}B_2}(u)=\max\left(0,\mathrm{F}_{B_1}(u)+F_{B_2}(u)-1\right).\text{ Then}$$

$$A_1\underset{\bullet}{\bigcap}A_2=(\delta_1*\delta_2)B_1\underset{\bullet}{\bigcap}B_2.$$

Proposition 3.12: Suppose $A_1=(\delta_1)B_1$ and $A_2=(\delta_2)B_2$ and let $A_1|-|A_2$ represent the bounded difference of A_1 and A_2, and $B_1|-|B_2$ represent the bounded difference of B_1 and B_2 respectively. Let $T_{A_1|-|A_2}(u)$, $I_{A_1|-|A_2}(u)$, $F_{A_1|-|A_2}(u)$

and $T_{B_1|-|B_2}(u)$, $I_{B_1|-|B_2}(u)$, $F_{B_1|-|B_2}(u)$ their truth membership functions, indeterminacy functions and falsity membership functions respectively, where

$$T_{A_1|-|A_2}(u)=\max\left(0,T_{A_1}(u)-T_{A_2}(u)\right),$$

$$I_{A_1|-|A_2}(u)=\max\left(0,\mathrm{I}_{A_1}(u)-I_{A_2}(u)\right),$$

$$F_{A_1|-|A_2}(u)=\max\left(0,\mathrm{F}_{A_1}(u)-F_{A_2}(u)\right) \text{ and}$$

$$T_{B_1|-|B_2}(u)=\max\left(0,T_{B_1}(u)-T_{B_2}(u)\right),$$

$$I_{B_1|-|B_2}(u)=\max\left(0,\mathrm{I}_{B_1}(u)-I_{B_2}(u)\right),$$

$$F_{B_1|-|B_2}(u)=\max\left(0,\mathrm{F}_{B_1}(u)-F_{B_2}(u)\right). \text{ Then}$$

$$A_1|-|A_2=(\delta_1*\delta_2)B_1|-|B_2.$$

Proposition 3.13: Suppose $A_1=(\delta_1)B_1$ and $A_2=(\delta_2)B_2$ and let $A_1\nabla A_2$ represent the symmetrical difference of A_1 and A_2, and $B_1\nabla B_2$ represent the symmetrical difference of B_1 and B_2 respectively. Let $T_{A_1\nabla A_2}(u)$, $I_{A_1\nabla A_2}(u)$, $F_{A_1\nabla A_2}(u)$ and $T_{B_1\nabla B_2}(u)$, $I_{B_1\nabla B_2}(u)$, $F_{B_1\nabla B_2}(u)$ their truth membership functions, indeterminacy functions and falsity membership functions respectively, where

$T_{A_1\nabla A_2}(u)=\left|T_{A_1}(x)-T_{A_2}(x)\right|$, $I_{A_1\nabla A_2}(u)=\left|I_{A_1}(x)-I_{A_2}(x)\right|$,

$F_{A_1\nabla A_2}(u)=\left|F_{A_1}(x)-F_{A_2}(x)\right|$ an $T_{B_1\nabla B_2}(u)=\left|T_{B_1}(x)-T_{B_2}(x)\right|$,

$I_{B_1\nabla B_2}(u)=\left|I_{B_1}(x)-I_{B_2}(x)\right|, F_{B_1\nabla B_2}(u)=\left|F_{B_1}(x)-F_{B_2}(x)\right|.$

Then $A_1\nabla A_2=(\delta_1*\delta_2)B_1\nabla B_2$.

Proposition 3.14: Suppose $A_1=(\delta_1)B_1$ and $A_2=(\delta_2)B_2$ and let $A_1\|_\lambda A_2$ represent the convex linear sum of min and max of A_1 and A_2, and $B_1\|_\lambda B_2$ represent the convex linear sum of B_1 and B_2 respectively. Let $T_{A_1\|_\lambda A_2}(u)$, $I_{A_1\|_\lambda A_2}(u)$, $F_{A_1\|_\lambda A_2}(u)$ and $T_{B_1\|_\lambda B_2}(u)$, $I_{B_1\|_\lambda B_2}(u)$, $F_{B_1\|_\lambda B_2}(u)$ their truth membership functions, indeterminacy functions and falsity membership functions respectively, where

$$T_{A_1\|_\lambda A_2}(u)=\lambda\min\left(T_{A_1}(u),T_{A_2}(u)\right)+(1-\lambda)\max\left(T_{A_1}(u),T_{A_2}(u)\right),$$

$$I_{A_1\|_\lambda A_2}(u)=\lambda\min\left(I_{A_1}(u),\mathrm{I}_{A_2}(u)\right)+(1-\lambda)\max\left(I_{A_1}(u),I_{A_2}(u)\right),$$

$$F_{A_1\|_\lambda A_2}(u)=\lambda\min\left(F_{A_1}(u),F_{A_2}(u)\right)+(1-\lambda)\max\left(F_{A_1}(u),F_{A_2}(u)\right)$$

and

$$T_{B_1\|_\lambda B_2}(u)=\lambda\min\left(T_{B_1}(u),T_{B_2}(u)\right)+(1-\lambda)\max\left(T_{B_1}(u),T_{B_2}(u)\right),$$

$$I_{B_1\|_\lambda B_2}(u)=\lambda\min\left(I_{B_1}(u),I_{B_2}(u)\right)+(1-\lambda)\max\left(I_{B_1}(u),I_{B_2}(u)\right),$$

$$F_{B_1\|_\lambda B_2}(u)=\lambda\min\left(F_{B_1}(u),F_{B_2}(u)\right)+(1-\lambda)\max\left(F_{B_1}(u),F_{B_2}(u)\right),$$

where $\lambda\in[0,1]$. Then $A_1\|_\lambda A_2=(\delta_1*\delta_2)B_1\|_\lambda B_2$.

IV. δ-Equalities with respect to Neutrosophic Relations and Norms

In this section, δ-equalities of neutrosophic sets have been further extended to neutrosophic relations and neutrosophic norms respectively. These δ-equalities applied in the composition of neutrosophic relations, Cartesian product and neutrosophic triangular norms.

Proposition 4.1: Let X,Y and Z be initial universes, and Σ be the collection of all neutrosophic sets defined on $X\times Y$ and Π denote the collection of all neutrosophic sets defined on $Y\times Z$ respectively. Let $R,R^{'}\in\Sigma$ and $S,S^{'}\in\Pi$, i.e., $R,R^{'},S$ and $S^{'}$ are neutrosophic relations on $X\times Y$ and $Y\times Z$ respectively. Further, let $R\circ S$ and $R^{'}\circ S^{'}$ be their composition, and $T_{R\circ S}(x,y)$, $I_{R\circ S}(x,y)$, $F_{R\circ S}(x,y)$ and $T_{R^{'}\circ S^{'}}(x,y)$, $I_{R^{'}\circ S^{'}}(x,y)$ and $F_{R^{'}\circ S^{'}}(x,y)$ their truth membership functions, indeterminate membership functions and falsehood membership functions respectively, where

$$T_{R\circ S}(x,\mathrm{z})=\sup_{y\in Y}\min\left(T_R(x,y),T_S(y,z)\right),$$

$$I_{R\circ S}(x,\mathrm{z})=\sup_{y\in Y}\min\left(I_R(x,y),I_S(y,z)\right),$$

$$F_{R\circ S}(x,\mathrm{z})=\sup_{y\in Y}\min\left(F_R(x,y),F_S(y,z)\right),$$

and

$$T_{R^{'}\circ S^{'}}(x,\mathrm{z})=\sup_{y\in Y}\min\left(T_R(x,y),T_S(y,z)\right),$$

$$I_{R^{'}\circ S^{'}}(x,\mathrm{z})=\sup_{y\in Y}\min\left(I_R(x,y),I_S(y,z)\right),$$

$$F_{R^{'}\circ S^{'}}(x,\mathrm{z})=\sup_{y\in Y}\min\left(F_R(x,y),F_S(y,z)\right),$$

for all $x\in X$ and $z\in Z$. Suppose $R=(\delta_1)R^{'}$ and $S=(\delta_2)S^{'}$. Then

$$R\circ S=\left(\min(\delta_1,\delta_2)\right)R^{'}\circ S^{'}.$$

Proposition 4.2: Let $U_1,U_2,\cdots U_n$ be universes and A_j,B_j be neutrosophic sets defined on U_j, $j=1,2,\cdots,n$. Let $A_j=(\delta_j)B_j$, where $j=1,2,\cdots,n$.

Let $A=A_1\times A_2\times\cdots\times A_n$ and $B=B_1\times B_2\times\cdots\times B_n$ and $T_A(u_1,u_2,\cdots,u_n)$, $I_A(u_1,u_2,\cdots,u_n)$, $F_A(u_1,u_2,\cdots,u_n)$ and $T_B(u_1,u_2,\cdots,u_n)$, $I_B(u_1,u_2,\cdots,u_n)$, $F_B(u_1,u_2,\cdots,u_n)$, be their truth membership functions, indeterminacy membership functions and falsity membership functions respectively, where

$$T_A(u_1,u_2,\cdots,u_n)=\min\left(T_{A_1}(u),T_{A_2}(u),\cdots,T_{A_n}(u)\right),$$

$$I_A(u_1,u_2,\cdots,u_n)=\min\left(I_{A_1}(u),I_{A_2}(u),\cdots,I_{A_n}(u)\right),$$

$$F_A(u_1,u_2,\cdots,u_n)=\min\left(F_{A_1}(u),F_{A_2}(u),\cdots,F_{A_n}(u)\right),$$

and

$$T_B(u_1,u_2,\cdots,u_n)=\min\left(T_{B_1}(u),T_{B_2}(u),\cdots,T_{B_n}(u)\right),$$

$$I_B(u_1,u_2,\cdots,u_n)=\min\left(I_{B_1}(u),I_{B_2}(u),\cdots,I_{B_n}(u)\right),$$

$$F_B(u_1,u_2,\cdots,u_n)=\min\left(F_{B_1}(u),F_{B_2}(u),\cdots,F_{B_n}(u)\right).$$

Then $A=\left(\inf_{1\le j\le n}\delta_j\right)B$.

Proposition 4.3: Let A_1,B_1,C_1 and A_2,B_2,C_2 be neutrosophic sets on U such that $T_{A_1}(u)\le T_{B_1}(u)\le T_{C_1}(u)$, $I_{A_1}(u)\le I_{B_1}(u)\le I_{C_1}(u)$, $F_{A_1}(u)\le F_{B_1}(u)\le F_{C_1}(u)$, and $T_{A_2}(u)\le T_{B_2}(u)\le T_{C_2}(u)$, $I_{A_2}(u)\le I_{B_2}(u)\le I_{C_2}(u)$, $F_{A_2}(u)\le F_{B_2}(u)\le F_{C_2}(u)$, for all $u\in U$. Also, let $A_1=(\delta_a)A_2$, $C_1=(\delta_c)C_2$ and $A_1=(\delta_{ac})C_1$. Then $B_1=\left(\delta_{ac}*\min(\delta_a,\delta_c)\right)B_2$.

Definition 4.1: A neutrosophic triangular norm N_t is a function $T_N:\left(\left]0^-,1^+\right[\times\left]0^-,1^+\right[\times\left]0^-,1^+\right[\right)^2\to\left]0^-,1^+\right[\times\left]0^-,1^+\right[\times\left]0^-,1^+\right[$ defined by $N_t\left(x(T_1,I_1,F_1),y(T_2,I_2,F_2)\right)=\left(N_tT(x,y),N_tI(x,y),N_tF(x,y)\right)$, where $N_tT(x,y),N_tI(x,y),N_tF(x,y)$ are truth membership, indeterminacy membership and falsity membership components respectively. N_t satisfies the following conditions:

i. $N_t(0,0)=0;\ N_t(x,1)=N_t(1,x)=x$;

ii. $N_t(x,y)\le N_t(w,z)$, whenever $x\le w$ and $y\le z$;

iii. $N_t(x,y)=N_t(y,x)$;

iv. $N_t\left(N_t(x,y),z\right)=N_t\left(x,N_t(y,z)\right)$.

Proposition 4.4: Let U be a universe of discourse, and A,A' and B,B' be neutrosophic sets on U. Let N_t be a neutrosophic triangular norm and let C,C' be neutrosophic sets define on U via N_t,

$T_C(u)=N_t\left(T_A(u),T_B(u)\right)$, $I_C(u)=N_t\left(I_A(u),\mathrm{I}_B(u)\right)$, $F_C(u)=N_t\left(F_A(u),F_B(u)\right)$, and $T_{C'}(u)=N_t\left(T_{A'}(u),T_{B'}(u)\right)$, $I_{C'}(u)=N_t\left(I_{A'}(u),\mathrm{I}_{B'}(u)\right)$, $F_{C'}(u)=N_t\left(F_{A'}(u),F_{B'}(u)\right)$.

Suppose that $A=(\delta_1)B$ and $A'=(\delta_2)B'$. Then $C=\left(\delta*\min(\delta_1,\delta_2)\right)C'$, where

$$\delta=1-\sup_{u:\max\left(T_A(u),T_{B(u)}\right)\ne1}\min\left(T_A(u),T_B(u)\right),$$

$$\delta=1-\sup_{u:\max\left(I_A(u),\mathrm{I}_{B(u)}\right)\ne1}\min\left(I_A(u),\mathrm{I}_B(u)\right),$$

$$\delta=1-\sup_{u:\max\left(F_A(u),F_{B(u)}\right)\ne1}\min\left(F_A(u),\mathrm{F}_B(u)\right).$$

V. APPLICATIONS OF δ-EQUALITIES

In this section, the applications and utilizations of δ-equalities have been presented. In this regards, δ-equalities have been successfully applied in Fault Tree Analysis, Canonical Computer Reliability and Neutrosophic Reliability (generalization of Profust Reliability).

Fault Tree Analysis.

A fault tree can be seen as following in the Fig. 1 [3,4]. We mainly concerned with the relationship between the probability of top events and bottom events in fault tree analysis. Suppose that $P_1,P_2,\cdots,P_5$ denote the occurrence probabilities of bottom events $e_1,e_2,\cdots,e_5$ and P_t represent of top event. Further, suppose that $e_1,e_2,\cdots,e_5$ are independent events. Therefore,

$$P_t=1-(1-P_1P_2)(1-P_3)(1-P_4P_5).$$

On the other hand, in conventional fault tree analysis $P_1,P_2,\cdots,P_5$ are assumed to accurate numbers, but in [3,4], $P_1,P_2,\cdots,P_5$ are treated as fuzzy numbers. Cai [3,4] utilized the concept of δ-equalities by applying it in fault tree analysis by considering $P_1,P_2,\cdots,P_5$ as fuzzy numbers.

Here, we consider $P_1,P_2,\cdots,P_5$ as neutrosophic numbers instead of fuzzy numbers. In engineering, an expert presents his judgement on P_j as $a_j\le P_j\le a'_j$ for truth membership, $b_j\le P_j\le b'_j$ for indeterminacy membership and $c_j\le P_j\le c'_j$ for falsehood membership respectively. Since we suppose that P_j's can be treated as neutrosophic number, therefore, we can define the following truth membership functions $T_{P_j}(u)$, $I_{P_j}(u)$ and $F_{P_j}(u)$ can be defined as following.

$$T_{P_j}(u)=\begin{cases}\dfrac{u-a_j}{a_{j_1}-a_j}\mathrm{T}_{P_j}, & a_j\le u<a_{j_1},\\ \mathrm{T}_{P_j}, & a_{j_1}\le u<a_{j_2},\\ \dfrac{a'_j-u}{a'_j-a_{j_2}}\mathrm{T}_{P_j}, & a_{j_2}\le u<a'_j\\ 0, & \text{otherwise.}\end{cases}$$

$$I_{P_j}(u)=\begin{cases}\dfrac{b_{j_1}-u+I_{P_j}(u-b_j)}{b_{j_1}-b_j}, & b_j\le u<b_{j_1},\\ I_{P_j}, & b_{j_1}\le u<b_{j_2},\\ \dfrac{u-b_{j_2}+I_{P_j}(b'_j-u)}{b'_j-b_{j_2}}, & b_{j_2}\le u<b'_j\\ 1, & \text{otherwise.}\end{cases}$$

$$F_{P_j}(u)=\begin{cases}\dfrac{c_{j_1}-u+F_{P_j}(u-c_j)}{c_{j_1}-c_j}, & c_j\le u<c_{j_1},\\ F_{P_j}, & c_{j_1}\le u<c_{j_2},\\ \dfrac{u-c_{j_2}+F_{P_j}(c'_j-u)}{c'_j-c_{j_2}}, & c_{j_2}\le u<c'_j\\ 1, & \text{otherwise}.\end{cases}$$

Suppose there is another expert who presents his judgement for P_j as $a''_j \le P_j \le a'''_j$ for truth membership, $b''_j \le P_j \le b'''_j$ for indeterminacy membership and $c''_j \le P_j \le c'''_j$ for falsehood membership respectively.

$$T'_{P_j}(u)=\begin{cases}\dfrac{u-a''_j}{a''_{j_1}-a''_j}\mathrm{T}'_{P_j}, & a''_j\le u<a''_{j_1},\\ \mathrm{T}'_{P_j}, & a''_{j_1}\le u<a''_{j_2},\\ \dfrac{a'''_j-u}{a'''_j-a''_{j_2}}\mathrm{T}'_{P_j}, & a''_{j_2}\le u<a''_j\\ 0, & \text{otherwise}.\end{cases}$$

$$I'_{P_j}(u)=\begin{cases}\dfrac{b''_{j_1}-u+I'_{P_j}(u-b''_j)}{b''_{j_1}-b''_j}, & b''_j\le u<b''_{j_1},\\ I'_{P_j}, & b''_{j_1}\le u<b''_{j_2},\\ \dfrac{u-b''_{j_2}+I'_{P_j}(b'''_j-u)}{b'''_j-b''_{j_2}}, & b''_{j_2}\le u<b'''_j\\ 1, & \text{otherwise}.\end{cases}$$

$$F'_{P_j}(u)=\begin{cases}\dfrac{c''_{j_1}-u+F'_{P_j}(u-c''_j)}{c''_{j_1}-c''_j}, & c''_j\le u<c''_{j_1},\\ F'_{P_j}, & c''_{j_1}\le u<c''_{j_2},\\ \dfrac{u-c''_{j_2}+F'_{P_j}(c'''_j-u)}{c'''_j-c''_{j_2}}, & c''_{j_2}\le u<c'''_j\\ 1, & \text{otherwise}.\end{cases}$$

Since $P_1,P_2,\cdots,P_5$ are treated as neutrosophic numbers, therefore, P_t is a neutrosophic set. The effect of estimation errors of in truth membership functions, indeterminacy membership functions and falsity membership functions of $P_1,P_2,\cdots,P_5$ on P_t can be formulated as following.

Let $\delta_j=1-\sup_{0\le u\le 1}\left|T_{P_j}(u)-T'_{P_j}(u)\right|$, $\delta_j=1-\sup_{0\le u\le 1}\left|I_{P_j}(u)-I'_{P_j}(u)\right|$ and $\delta_j=1-\sup_{0\le u\le 1}\left|F_{P_j}(u)-F'_{P_j}(u)\right|$. Let $T_{P_t}(u)$, $I_{P_t}(u)$ and $F_{P_t}(u)$ be the truth membership function, indeterminacy membership function and falsity membership function of P_t corresponding to $\{T_{P_j}(u),I_{P_j}(u),F_{P_j}(u)\}$ and $T'_{P_t}(u)$, $I'_{P_t}(u)$ and $F'_{P_t}(u)$ be the truth membership function, indeterminacy membership function and falsity membership function of P_t corresponding to $\{T'_{P_j}(u),I'_{P_j}(u),F'_{P_j}(u)\}$ respectively. Then from Proposition 4.3, we have

$$\sup_{0\le u\le 1}\left|T_{P_t}(u)-T'_{P_t}(u)\right|\le 1-\min_{1\le j\le 5}(\delta_j),$$
$$\sup_{0\le u\le 1}\left|I_{P_t}(u)-I'_{P_t}(u)\right|\le 1-\min_{1\le j\le 5}(\delta_j),$$
$$\sup_{0\le u\le 1}\left|F_{P_t}(u)-F'_{P_t}(u)\right|\le 1-\min_{1\le j\le 5}(\delta_j).$$

It is to be noted that δ_j is a function of $a_j,b_j,c_j,a'_j,b'_j,c'_j,a''_j,b''_j,c''_j,a'''_j,b'''_j,c'''_j$.

Neutrosophic Reliability (Generalization of Profust Reliability).

Cai [3,4] uses successfully the concept of δ-equalties in profust reliability which is based on the probability assumption and fuzzy-state assumption. For more detail, we refer the readers to [3,4]. But there is another state called neutrosophic-state assumption. In neutrosophic-state assumption, we have three state, i.e. the membership state, indeterminate membership state and non-membership state. In other words, state of success, state of failure and a state of neither failure nor success. From this we can say that neutrosophic Reliability is more general framework than the profust reliability. A neutrosophic reliability is based on neutrosophic probability and neutrosophic-state assumption. Now we can say that a system of order n (n-component) is referred to as a neutrosophic system if it satisfies the following conditions.

$\mu_F=\prod_{j=1}^{n}\mu^{(j)}{}_F$, $\mu_S=1-\prod_{j=1}^{n}\left(1-\mu^{(j)}{}_S\right)$ and $\mu_I=\prod_{j=1}^{n}\left(\mu^{(j)}{}_F+\mu^{(j)}{}_S\right)$,

where μ_F, μ_S and μ_I are the false membership function of neutrosophic failure, truth membership function of neutrosophic success and indeterminate membership function of neutrosophic indeterminacy (both failure and success at the same time) of the neutrosophic system, respectively, and $\mu^{(j)}{}_F$, $\mu^{(j)}{}_S$ and $\mu^{(j)}{}_I=\mu^{(j)}{}_F+\mu^{(j)}{}_S$ are false membership, truth membership and indeterminate membership functions of failure, success and indeterminate of the component j.

From here it is clear that neutrosophic system is the generalization of the parallel system as by setting the indeterminate component μ_I equals to 0, the neutrosophic system reduced to parallel system.

Now suppose that there are estimations errors in $\mu^{(j)}{}_F$, $\mu^{(j)}{}_S$, $\mu^{(j)}{}_I$, or we have $\mu'^{(j)}{}_F$, $\mu'^{(j)}{}_S$, $\mu'^{(j)}{}_I$ and suppose that

$$\sup_u\left|\mu^{(j)}{}_F(u)-\mu'^{(j)}{}_F(u)\right|\le 1-\delta_j{}^F,$$
$$\sup_u\left|\mu^{(j)}{}_S(u)-\mu'^{(j)}{}_S(u)\right|\le 1-\delta_j{}^S,$$
$$\sup_u\left|\mu^{(j)}{}_I(u)-\mu'^{(j)}{}_I(u)\right|\le 1-\delta_j{}^I.$$

Then by using Proposition 3.3 and 3.5, we have

$$\sup_u \left|\mu_F(u) - \mu'_F(u)\right| \le 1 - \left(\delta_1^{\,F} * \delta_2^{\,F} * \cdots * \delta_n^{\,F}\right),$$

$$\sup_u \left|\mu_S(u) - \mu'_S(u)\right| \le 1 - \left(\delta_1^{\,S} * \delta_2^{\,S} * \cdots * \delta_n^{\,S}\right),$$

$$\sup_u \left|\mu_I(u) - \mu'_I(u)\right| \le 1 - \left(\delta_1^{\,I} * \delta_2^{\,I} * \cdots * \delta_n^{\,I}\right),$$

where $\mu'_F(u)$ corresponds to $\mu'^{(1)}_F(u), \cdots, \mu'^{(n)}_F(u)$, $\mu'_S(u)$ corresponds to $\mu'^{(1)}_S(u), \cdots, \mu'^{(n)}_S(u)$ and $\mu'_I(u)$ corresponds to $\mu'^{(1)}_I(u), \cdots, \mu'^{(n)}_I(u)$.

vi. CONCLUSION

New type of union, intersection and δ -equalities of neutrosophic sets presented in this paper. Two neutrosophic sets are said to be δ-equal if they are equal to some degree of δ. Later, these δ -equalities applied in several set theoretic operations such as union, intersection, complement, product, probabilistic sum, bold sum, bold intersection, bounded difference, symmetrical difference, and convex linear sum of min and max. It has also shown that how δ varies with different operation. These δ-equalities of neutrosophic sets have been further extended to neutrosophic relations and neutrosophic norms respectively. In this paper, δ -equalities also applied in the composition of neutrosophic relations, Cartesian product and neutrosophic triangular norms. The applications and utilizations of δ -equalities have been presented in this paper. In this regards, δ -equalities have been successfully applied in Fault Tree Analysis and Neutrosophic Reliability (generalization of Profust Reliability).

The significance of δ-equality can be justified in theory as well as in practice. On the one hand, this paper shows that the δ -equalities of neutrosophic sets can be defined and investigated in a general framework by introducing δ which is basically a difference between the truth membership functions T, a difference between the indeterminate membership functions I and a difference between the falsehood membership functions F of two neutrosophic sets respectively. On the other hand, as shown in the application section 5, the concept of δ-equalities of neutrosophic sets may be useful in various applications where errors of membership functions, non-membership functions and indeterminate membership function of neutrosophic sets are of concern.

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