

Short note n°4: Number Pi

Una fórmula clásica para la constante Pi

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Abstract

We show a formula for constant Pi:

$$\pi = 4 \sum_{n=0}^{\infty} (-1)^n (2n+1)^{-1} = 3.141592 \dots$$

Keyword: Number Pi, Infinite Sum

Resumen

En esta nota mostramos una fórmula para la constante Pi : $\pi = 3.141592 \dots$, la fórmula involucra una suma de determinantes de 3×3 .

I. Fórmula

$$\pi = 2 + 2 \sum_{n=1}^{\infty} \sum_{m=1}^{2^{n-1}} |d(n, m)| = 2 - 2 \sum_{n=1}^{\infty} \sum_{m=1}^{2^{n-1}} d(n, m) \quad (1)$$

donde $d(n, m)$ es un determinante de 3×3 definido como sigue :

$$d(n, m) = \begin{vmatrix} x_{n,m,1} & \sqrt{1-x_{n,m,1}^2} & 1 \\ x_{n,m,2} & \sqrt{1-x_{n,m,2}^2} & 1 \\ x_{n,m,3} & \sqrt{1-x_{n,m,3}^2} & 1 \end{vmatrix}, \quad n \in \mathbb{N}, m = 1, 2, \dots, 2^{n-1} \quad (2)$$

y los puntos $x_{n,m,k}$, $k = 1, 2, 3$, se definen como :

$$x_{n,m,k} = \frac{2m+k-3}{2^n}, \quad n \in \mathbb{N}, m = 1, 2, \dots, 2^{n-1}. \quad (3)$$

II. Referencias

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