Pi Formulas , Part 23

Edgar Valdebenito

abstract

In this note we show some formulas related with the constant Pi

FÓRMULAS QUE CONTIENEN LA CONSTANTE π

EDGAR VALDEBENITO V.
(2000)

Resumen. Se muestran fórmulas que contienen la constante π.

1. INTRODUCCIÓN.

En esta nota se muestran algunas fórmulas clásicas que involucran el número π:

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.14159265...$$

$$\pi = 3.14159265358979323846264338327950288419716939937 51...$$

2. FÓRMULAS.

2.1.

$$\pi + 2\ln(2) = 8\sqrt{2}\sum_{n=0}^{\infty}\left(\frac{\left(\sqrt{2}-1\right)^{8n+1}}{8n+1} - \frac{\left(\sqrt{2}-1\right)^{8n+5}}{8n+5}\right) - 16\sum_{n=0}^{\infty}\left(\frac{\left(\sqrt{2}-1\right)^{8n+4}}{8n+4} + \frac{\left(\sqrt{2}-1\right)^{8n+6}}{8n+6}\right)$$

2.2.

$$\pi + 3\ln\left(1+\frac{\sqrt{3}}{2}\right) = 12\sqrt{2}\sum_{n=0}^{\infty}\left(\frac{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{6}}\right)^{8n+1}}{8n+1} - \frac{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{6}}\right)^{8n+5}}{8n+5}\right) - 24\sum_{n=0}^{\infty}\left(\frac{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{6}}\right)^{8n+4}}{8n+4} + \frac{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{6}}\right)^{8n+6}}{8n+6}\right)$$

2.3.

$$\pi+4\,ln\left(1+\frac{1}{\sqrt{2}}\right)=16\sqrt{2}\sum_{n=0}^{\infty}\left(\frac{\left(1-\sqrt{2-\sqrt{2}}\right)^{8n+1}}{8n+1}-\frac{\left(1-\sqrt{2-\sqrt{2}}\right)^{8n+5}}{8n+5}\right)$$
$$-32\sum_{n=0}^{\infty}\left(\frac{\left(1-\sqrt{2-\sqrt{2}}\right)^{8n+4}}{8n+4}+\frac{\left(1-\sqrt{2-\sqrt{2}}\right)^{8n+6}}{8n+6}\right)$$

2.4.

$$\pi+6\,ln(2)+4\,ln(3)-4\,ln(5)=$$
$$=8\sqrt{2}\sum_{n=0}^{\infty}\left(\frac{\left(\frac{2\sqrt{2}-\sqrt{5}}{3}\right)^{8n+1}}{8n+1}-\frac{\left(\frac{2\sqrt{2}-\sqrt{5}}{3}\right)^{8n+5}}{8n+5}\right)$$
$$-16\sum_{n=0}^{\infty}\left(\frac{\left(\frac{2\sqrt{2}-\sqrt{5}}{3}\right)^{8n+4}}{8n+4}+\frac{\left(\frac{2\sqrt{2}-\sqrt{5}}{3}\right)^{8n+6}}{8n+6}\right)+$$
$$+8\sqrt{2}\sum_{n=0}^{\infty}\left(\frac{\left(\frac{3\sqrt{2}-\sqrt{10}}{4}\right)^{8n+1}}{8n+1}-\frac{\left(\frac{3\sqrt{2}-\sqrt{10}}{4}\right)^{8n+5}}{8n+5}\right)$$
$$-16\sum_{n=0}^{\infty}\left(\frac{\left(\frac{3\sqrt{2}-\sqrt{10}}{4}\right)^{8n+4}}{8n+4}+\frac{\left(\frac{3\sqrt{2}-\sqrt{10}}{4}\right)^{8n+6}}{8n+6}\right)$$

2.5.

$$\frac{2\pi}{3}+ln\left(1+\frac{\sqrt{3}}{2}\right)=$$
$$=\sum_{n=0}^{\infty}\frac{\left(\sqrt{3}-1\right)^{8n+1}}{2^{4n}}\left(\frac{4}{8n+1}-\frac{2\left(\sqrt{3}-1\right)^{3}}{8n+4}-\frac{\left(\sqrt{3}-1\right)^{4}}{8n+5}-\frac{\left(\sqrt{3}-1\right)^{5}}{8n+6}\right)$$

2.6. Para $1\le m\le 1+\frac{1}{\sqrt{2}}$, se tiene:

$$ln\left(\frac{2m^2-1}{2m^2-2m+1}\right)^2+\pi-2\tan^{-1}\left(\frac{2m\left(2m^3-m-1\right)}{2m^2\left(2m+1\right)-1}\right)=$$
$$=\sum_{n=0}^{\infty}\left(\frac{(4/m)\left(1/2m^2\right)^{4n}}{8n+1}-\frac{(4/m)\left(1/2m^2\right)^{4n+2}}{8n+5}-\frac{8\left(1/2m^2\right)^{4n+2}}{8n+4}-\frac{8\left(1/2m^2\right)^{4n+3}}{8n+6}\right)$$

2.7. Para $m\ge 1+\frac{1}{\sqrt{2}}$, se tiene:

$$ln\left(\frac{2m^2-1}{2m^2-2m+1}\right)^2+\frac{\pi}{2}-2\tan^{-1}\left(\frac{4m^4-4m^3-4m^2-2m+1}{4m^4+4m^3-2m-1}\right)=$$
$$=\sum_{n=0}^{\infty}\left(\frac{(4/m)\left(1/2m^2\right)^{4n}}{8n+1}-\frac{(4/m)\left(1/2m^2\right)^{4n+2}}{8n+5}-\frac{8\left(1/2m^2\right)^{4n+2}}{8n+4}-\frac{8\left(1/2m^2\right)^{4n+3}}{8n+6}\right)$$

3. REFERENCIAS.

1) Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover , 1965.
2) I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey) , Academic Press, New York, London, and Toronto, 1980.
3) M. R. Spiegel, Mathematical Handbook, McGraw-Hill Book Company, New York, 1968.
4) E. Valdebenito, Pi Handbook, manuscript, unpublished, 1989 , (20000 fórmulas).