Formulas Involving Constant Pi

Edgar Valdebenito

July 5 , 2009

Abstract

Some formulas are shown for the constant Pi

$\pi = 3.141592...$

Introduction

In this paper we show some formulas for constant Pi:

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...\right) = 3.141592... \tag{1}$$

In all formulas the matrix

$$A = \begin{pmatrix} 0 & 6 & 0 & 2 & 0 & 6 & 0 & -2 \\ 1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & 2 & 0 & -2 & 0 & 2 \\ 1 & 0 & 3 & 0 & -1 & 0 & 3 & 0 \\ 0 & -6 & 0 & 2 & 0 & 6 & 0 & 2 \\ -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 2 & 0 & 2 \\ 1 & 0 & -3 & 0 & 1 & 0 & 3 & 0 \end{pmatrix} \tag{2}$$

Is essential for the calculation of numbers s_n and c_n .

Formulas

$$\pi = 4\sum_{n=1}^{\infty}\frac{s_{2n-1}}{(2n-1)\,4^{2n-1}} \tag{3}$$

$$\pi = 24\sum_{n=1}^{\infty}\frac{(-1)^{n-1} s_n}{n\,4^n} \tag{4}$$

$$\pi = \frac{24}{11}\sum_{n=1}^{\infty}\frac{s_n}{n\,4^n} \tag{5}$$

$$\pi = 8\left(\sqrt{6}-\sqrt{2}\right)\sum_{n=1}^{\infty}\frac{n\,s_{2n}}{\left(4n^2-1\right)4^{2n}} \tag{6}$$

$$\pi = \left(\sqrt{2}+\sqrt{6}\right)\left(2-4\sum_{n=1}^{\infty}\frac{c_{2n}}{\left(4n^2-1\right)4^{2n}}\right) \tag{7}$$

$$\pi^2 = \frac{48}{5}\sum_{n=1}^{\infty}\frac{c_{2n-1}}{\left(2n-1\right)^2 4^{2n-1}} \tag{8}$$

$$\pi^2 = \frac{576}{47}\sum_{n=1}^{\infty}\frac{(-1)^{n-1} c_n}{n^2\,4^n} \tag{9}$$

where

$$s_n = \alpha_n + \beta_n\sqrt{6} + \gamma_n\sqrt{3} + \delta_n\sqrt{2} \quad , n \in \mathbb{N} \tag{10}$$

$$c_n = \eta_n + \kappa_n\sqrt{6} + \lambda_n\sqrt{3} + \mu_n\sqrt{2} \quad , n \in \mathbb{N} \tag{11}$$

$$\alpha_n, \beta_n, \gamma_n, \delta_n, \eta_n, \kappa_n, \lambda_n, \mu_n \in \mathbb{N} \cup \{0\} \tag{12}$$

$$X_{n+1} = A\,X_n \quad , n \in \mathbb{N} \tag{13}$$

$$X_n = \begin{pmatrix}\alpha_n & \beta_n & \gamma_n & \delta_n & \eta_n & \kappa_n & \lambda_n & \mu_n\end{pmatrix}^T \tag{14}$$

$$X_1 = \begin{pmatrix}0 & 1 & 0 & -1 & 0 & 1 & 0 & 1\end{pmatrix}^T \tag{15}$$

Of (13) gives

$$X_{n+1} = A^n\,X_1 \quad , n \in \mathbb{N} \tag{16}$$

for $n \in \mathbb{N}$, has

$$s_n = 4^n \sin\left(\frac{n\pi}{12}\right) \quad , \quad c_n = 4^n \cos\left(\frac{n\pi}{12}\right) \tag{17}$$

Some values for s_n and c_n

$$s_n = \begin{pmatrix} \sqrt{6}-\sqrt{2} \\ 8 \\ 32\sqrt{2} \\ 128\sqrt{3} \\ 256\left(\sqrt{2}+\sqrt{6}\right) \\ 4096 \\ 4096\left(\sqrt{2}+\sqrt{6}\right) \\ 32768\sqrt{3} \\ 131072\sqrt{2} \\ 524288 \end{pmatrix} \quad , \quad c_n = \begin{pmatrix} \sqrt{2}+\sqrt{6} \\ 8\sqrt{3} \\ 32\sqrt{2} \\ 128 \\ 256\left(\sqrt{6}-\sqrt{2}\right) \\ 0 \\ 4096\left(\sqrt{2}-\sqrt{6}\right) \\ -32768 \\ -131072\sqrt{2} \\ -524288\sqrt{3} \end{pmatrix} \tag{18}$$

References

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