Correlation coefficients of single valued neutrosophic hesitant fuzzy sets and their applications in decision making

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Abstract

As a combination of the hesitant fuzzy set (HFS) and the single valued neutrosophic set (SVNS), the single valued neutrosophic hesitant fuzzy set (SVNHFS) is an important concept to handle uncertain and vague information existing in real-life, which consists of three membership functions including hesitancy, as the truth-hesitancy membership function, the indeterminacy-hesitancy membership function and the falsity-hesitancy membership function, and encompass the fuzzy set (FS), intuitionistic fuzzy set (IFS), hesitant fuzzy set (HFS), dual hesitant fuzzy set (DHFS) and single-valued neutrosophic set (SVNS). Correlation and correlation coefficient have been applied widely in many research domains and practical fields. This paper, motivated by the idea of correlation coefficients derived for HFSs, IFSs, DHFSs and SVNSs, focuses on the correlation and correlation coefficient of SVNHFSs and investigates their some basic properties in detail. By using the weighted correlation coefficient information between each alternative and the optimal alternative, a decision making method are established to handling the single valued neutrosophic hesitant fuzzy information. Finally, an effective example is used to demonstrate the validity and applicability of the proposed approach in decision making, and the relationship between the each existing method and the developed method is given as a comparison study.

Keywords

Single-valued neutrosophic set, hesitant fuzzy set, single-valued neutrosophic hesitant fuzzy set, correlation, correlation coefficient, multiple attribute decision making

1. Introduction

In 1965, Zadeh [1] initiated the approach of fuzzy sets (FS) and applied it in multi attribute decision making (MADM). The extensions of FS have been developed by some researchers, including the interval-valued fuzzy set (IVFS) proposed by Turksen [2], intuitionistic fuzzy set (IFS) pioneered by Atanassov [3], interval-valued intuitionistic fuzzy set (IVIFS) pointed out by Atanassov and Gargov [4], type-2 fuzzy set (TP-2 FS) pioneered by Dubois and Prade [5] and fuzzy multiset (FMS) introduced by Yager [6]. However, in realistic situations, due to time pressure, complexity of the problem, lack of information processing capabilities, poor knowledge of the public domain and information, decision makers cannot provide exact evaluation of decision-parameters involved in MADM problems. In such situation, preference information provided by the experts or decision makers may be incomplete or imprecise in nature. To deal with these cases, the hesitant fuzzy set (HFS) was defined by Torra [7] and Torra and Narukawa [8], whose the membership value of each element in a HFS includes a set of possible values between zero and one. On the other hand, as a generalization of HFSs, the dual hesitant fuzzy set (DHFS) was defined by Zhu et al. [9] and discussed the some related properties of DHFSs. Thus, the theory of DHFS allows the extension of FS, IFS, HFS, and FMS in view of logic.

The neutrosophic set (NS) proposed firstly by Smarandache [10,11] generalizes an IFS from philosophical point of view. NSs are characterized by truth, indeterminacy, and falsity membership functions which are independent in nature. In MADM context, the ratings of the alternatives provided by the decision maker can be expressed with NSs. These NSs can handle indeterminate and inconsistent information quite well, whereas IFSs and FSs can only handle incomplete or partial information. However, it is almost impossible the NSs to apply in concrete areas such as real engineering and scientific. Wang et al. [12] initiated the theory of single valued neutrosophic set (SVNS) and provided some definitions relating to set theoretic operators. Recently, many other research topics have also been discussed with the help of SVNSs [13–26].

Currently, based on the integration of SVNSs and HFSs, Ye [27] introduced the single valued neutrosophic hesitant fuzzy set (SVNHFS) which includes FSs, IFSs, HFSs, FMSs, DHFSs and SVNSs, and discussed the some properties of SVNHFSs. SVNHFSs are characterized by truth-hesitancy, indeterminacy-hesitancy, and falsity-hesitancy membership functions which are independent in nature. Therefore, it is not only more general than aforementioned set but only more suitable for handle the MADM problems due to considering much more information provided by decision makers. Also, it can provide richer expressions than a neutrosophic term a hesitant term, and can better address the vague and imprecise information, the form only permitting consecutive values cannot reflect common hesitance and divergence of decision makers.

Further, the concept of correlation is an important concept used to handling the uncertainty information and have been extensively applied in some practical decision-making problems related to pattern recognition, decision making, supply chain management, market prediction and machine learning and so on. For example, Chen et al. [28] derived a sequence of the correlation coefficients for HFSs and used them to two real world examples by combining the clustering analysis and hesitant fuzzy information. Xu and Xia [29] extended the correlation measures to HFSs and derived some new definition measures of HFSs. Xu [30] discussed the correlation measures of IFSs. Then, Wang et al. [31] introduced some correlation measures of DHFSs and applied them in MADM problems. Recently, Ye [20] derived a correlation coefficient of SVNSs and used it to solve a MADM problem under single valued neutrosophic environment. The correlation measures given in aforementioned studies, however, cannot be utilized to handle the single valued neutrosophic hesitant fuzzy information. Thus, we need to propose some new measures for SVNHFSs. Therefore, this paper mainly focuses on how to propose new definitions regarding the correlation of SVNHFSs, as a new extension of existing measures. The rest of this paper is represented as below. Section 2 give some concepts concerning the HFSs, DHFSs, NSs, SVNSs and SVNHFSs, and present correlation coefficients of HFSs and DHFSs. In Section 3, the concepts of informational energy, correlation and correlation coefficient of SVNHFSs are proposed based on an extension of the concepts provided for HFSs and DHFSs. Section 4 establishes a MADM using the proposed weighted correlation coefficient of SVNHFSs. In Section 5, a numerical example related the selection of desirable alternative is presented to illustrate the validity and efficiency of the derived correlation coefficients of SVNHFSs in decision making. Section 6 gives a comparison study between the developed method and the existing methods. Finally, some final results and further work are continued with a discussion given in Section 7.

2. Preliminaries

2.1 Neutrosophic set

Definition 1. (Smarandache [10]) Let X be a universe of discourse, then a neutrosophic set is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}, \tag{1}$$

which is characterized by a truth-membership function $T_A : X \to]0^-, 1^+[$, an indeterminacy-membership function $I_A : X \to]0^-, 1^+[$and a falsity-membership function $F_A : X \to]0^-, 1^+[$.

There is not restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Wang et al. [12] defined the single valued neutrosophic set which is an instance of neutrosophic set.

2.2. Single valued neutrosophic sets

Definition 2. (Wang et al. [12]) Let X be a universe of discourse, then a single valued neutrosophic set is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}, \tag{2}$$

where $T_A : X \to [0,1]$, $I_A : X \to [0,1]$ and $F_A : X \to [0,1]$ with $0 \le T(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$. The values $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

2.3. Hesitant fuzzy sets

Definition 3. (Torra [7]) Let X be a fixed set; a hesitant fuzzy set (HFS) M on X is defined as

$$M = \{\langle x_i, h_M(x_i)\rangle : x_i \in X\}, \tag{3}$$

where $h_M(x_i)$ is a set of some different values in [0,1], denoted by $h_M(x_i) = \{\gamma_{M1}(x_i), \gamma_{M2}(x_i), \dots, \gamma_{Ml_h}(x_i)\}$, representing the possible membership degrees of the element $x_i \in X$ to M. For convenience, the $h_M(x_i)$ is named a hesitant fuzzy element (HFE), denoted by $h = \{\gamma_{M1}, \gamma_{M2}, \dots, \gamma_{Ml_h}\}$, where l_h is the number of values in $h_M(x_i)$.

2.4. Dual Hesitant fuzzy sets

Definition 4. (Zhu [9]) Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed set then a dual hesitant fuzzy set (DHFS) D on X is described as;

$$D = \{\langle x_i, h_D(x_i), g_D(x_i)\rangle : x_i \in X\}$$

in which $h_D(x)$ and $g_D(x)$ are two sets of some different values in [0,1], representing the possible membership degrees and nonmembership degrees of the element $x_i \in X$ to D, respectively, with the conditions $0 \le \gamma, \eta \le 1$ and $0 \le \gamma^+ + \eta^+ \le 1$, where $\gamma \in h_D(x_i)$, $\eta \in g_D(x_i)$, $\gamma^+ \in h_D^+(x_i) = \cup_{\gamma \in h_D(x_i)} \max\{\gamma\}$, and $\eta^+ \in g_D^+(x_i) = \cup_{\eta \in g_D(x_i)} \max\{\eta\}$ for $x \in X$. For convenience, the $d(x_i) = \langle h_D(x_i), g_D(x_i)\rangle$ is named a dual hesitant fuzzy element (DHFE), denoted by $d = \{h, g\}$ such that $h = \{\gamma_{D1}, \gamma_{D2}, \dots, \gamma_{Dl_h}\}$ and $g = \{\eta_{D1}, \eta_{D2}, \dots, \eta_{Dl_g}\}$, where l_h and l_g are the number of values in $h_D(x_i)$ and $g_D(x_i)$, respectively.

2.5. Single-valued neutrosophic hesitant sets

Ye [26] proposed the following single-valued neutrosophic sets as a generalization of HFs, DHFSs and SVNSs.

Definition 5. Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed set, then a single-valued neutrosophic hesitant fuzzy set (SVNHFS) N on X is described as;

$$N = \{\langle x_i, (h_N(x_i), \iota_N(x_i), g_N(x_i))\rangle : x_i \in X\} \tag{4}$$

in which $h_N(x_i), \iota_N(x_i)$, and $g_N(x_i)$ are three sets of some different values in [0,1] with $h_N(x_i) = \{\gamma_{N1}(x_i), \gamma_{N2}(x_i), \dots, \gamma_{Nl_h}(x_i)\}$, $h_N(x_i) = \{\eta_{N1}(x_i), \eta_{N2}(x_i), \dots, \eta_{Nl_\iota}(x_i)\}$ and $h_N(x_i) = \{\eta_{N1}(x_i), \eta_{N2}(x_i), \dots, \eta_{l_g}(x_i)\}$, representing the possible truth-hesitant membership degree, indeterminacy-hesitant membership degree, and falsity- hesitant membership degree of the element $x_i \in X$ to N, respectively, with the conditions $0 \le \gamma, \delta, \eta \le 1$ and $0 \le \gamma^+ + \delta^+ + \eta^+ \le 3$, where $\gamma \in h_N(x_i)$, $\delta \in \iota_N(x_i)$, $\eta \in g_N(x_i)$, $\gamma^+ \in h_N^+(x_i) = \cup_{\gamma \in h_N(x_i)} \max\{\gamma\}$, $\delta^+ \in \iota_N^+(x_i) = \cup_{\delta \in \iota_N(x_i)} \max\{\delta\}$, and $\eta^+ \in g_N^+(x_i) = \cup_{\eta \in g_N(x_i)} \max\{\eta\}$ for $x_i \in X$.

For convenience, the $n(x_i) = \langle h_N(x_i), \iota_N(x_i), g_N(x) \rangle$ is named a single-valued neutrosophic hesitant element (SVNHFE), denoted by $n = \{ h, \iota, g\}$ such that $h = \{\gamma_{N1}, \gamma_{N2}, \dots, \gamma_{Nl_h}\}$, $\iota = \{\eta_{N1}, \eta_{N2}, \dots, \eta_{Nl_\iota}\}$ and $g = \{\eta_{N1}, \eta_{N2}, \dots, \eta_{l_g}\}$, where l_h,l_ι and l_g are the number of values in $h_N(x_i), \iota_N(x_i)$ and $g_N(x_i)$, respectively.

From Definition 5, we can see that a SVNHFS is an effective and flexible model to determine values for each element in the domain, and can deal with three kinds of hesitancy in this case**.**

3. Correlation coefficient of single valued neutrosophic hesitant fuzzy sets

3.1. Correlation coefficient of hesitant fuzzy sets

The values of a hesitant fuzzy element are usually given a disorder, so we need to arrange them in a decreasing order. For a hesitant fuzzy element h, let $\sigma: (1,2,\dots,n) \to (1,2,\dots,n)$ be a permutation satisfying $h_{\sigma(j)} \geq h_{\sigma(j+1)}$ for $j = 1,2,\dots,n$, and $h_{\sigma(j)}$ be the jth largest value in h. Sometimes the cardinality of two HFEs are different. In such cases, as to Chen *et al.* [28]'s methodology, we need to make the lengths of the two HFEs be the same. There are many different regulations to extend the shorter HFE to the same length as the longer one. The most representative regulations are the pessimistic principle and the optimistic principle. For two HFEs h_A and h_B, let $l = \max\{l_{h_A}, l_{h_B}\}$, where l_{h_A} and l_{h_B} are the number of values in h_A and h_B, respectively. When $l_{h_A} \neq l_{h_B}$, one can extend the short HFE by adding some values in it until it has the same length with the other. In terms of the pessimistic principle, the short HFE is extended by adding the minimum value in it until it has the same length with the other HFE; while as to the optimistic principle, the maximum value of the short HFE should be added till the HFE has the same length as the longer one. In Chen *et al.* [28]'s definition, they used the former case and thus the correlation coefficient between two HFSs was defined as:

Definition 6. Let A be a hesitant fuzzy set on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ denoted by $A = \{\langle x_i, h_A(x_i)\rangle : x_i \in X\}$. Then, the informational energy of A is defined as

$$E_{HFS}(A) = \sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i}\gamma_{A\sigma(j)}^2(x_i)\right) \tag{5}$$

where $l_i = l(h_A(x_i))$ is the number of values in $h_A(x_i)$, and $\gamma_{A\sigma(j)}(x_i)$ the jth value in $h_A(x_i)$, $x_i \in X$.

Definition 7. Let A and B be two hesitant fuzzy sets on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ denoted by $A = \{\langle x_i, h_A(x_i)\rangle : x_i \in X\}$ and $B = \{\langle x_i, h_B(x_i)\rangle : x_i \in X\}$, respectively. Then, the correlation between A and B is defined by

$$C_{HFS}(A,B) = \sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i}\gamma_{A\sigma(j)}(x_i)\gamma_{B\sigma(j)}(x_i)\right) \tag{6}$$

Definition 8. Let A and B be two hesitant fuzzy sets on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, then the correlation coefficient between A and B is defined by

$$\rho_{HFS}(A,B) = \frac{C_{HFS}(A,B)}{\sqrt{C_{HFS}(A,A)}\sqrt{C_{HFS}(B,B)}} = \frac{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i}\gamma_{A\sigma(j)}(x_i)\gamma_{B\sigma(j)}(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i}\gamma_{A\sigma(j)}^2(x_i)\right)}\sqrt{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i}\gamma_{B\sigma(j)}^2(x_i)\right)}} \quad (7)$$

Let A and B be any two DHFSs, the correlation coefficient defined by Equation (7) should satisfy the following properties:

(1) $\rho_{HFS}(A,B) = \rho_{HFS}(B,A)$;

(2) $0 \le \rho_{HFS}(A,B) \le 1$;

(3) $\rho_{HFS}(A,B) = 1$, if $A = B$.

3.2. Correlation coefficient of dual hesitant fuzzy sets

Let us consider the two DHFSs $A = \{\langle x_i, h_A(x_i), g_A(x_i)\rangle : x_i \in X\}$ and $B = \{\langle x_i, h_B(x_i), g_B(x_i)\rangle : x_i \in X\}$with $h_A = \{\gamma_{A1}, \gamma_{A2}, \dots, \gamma_{Ak_i}\}$, $g_A = \{\eta_{A1}, \eta_{A2}, \dots, \eta_{Al_i}\}$ and $h_B = \{\gamma_{B1}, \gamma_{B2}, \dots, \gamma_{Bk_i}\}$, $g_B = \{\eta_{B1}, \eta_{B2}, \dots, \eta_{Bl_i}\}$, where $k_i = k(h_A(x_i))$ and $l_i = l(g_A(x_i))$ are the number of values in $h_A(x_i)$ and $g_A(x_i)$, and $\gamma_{A\sigma(s)}(x_i)$ and $\eta_{A\sigma(t)}(x_i)$ are the sth and tth values in $h_A(x_i)$ and $g_A(x_i)$, respectively.

Wang *et al*. [31] proposed some definitions related to correlation of dual hesitant fuzzy sets as follows:

Definition 9. Let A be a DHFS on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ then, the informational energy of A is defined as

$$E_{DHFS}(A) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}^2(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}^2(x_i)\right) \quad (8)$$

Definition 10. Let A and B be two DHFSs on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, then the correlation between A and B is defined by

$$C_{DHFS}(A,B) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\right). \quad (9)$$

Let A and B be any two DHFSs, the correlation defined by Eq. (9) should satisfy the following properties:

(1) $C_{DHFS}(A,A) = E_{DHFS}(A)$;

(2) $C_{DHFS}(A,B) = C_{DHFS}(B,A)$.

Definition 11. Let A and B be two DHFSs on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, then the correlation coefficient between A and B is defined by

$$\rho_{DHFS}(A,B) = \frac{C_{DHFS}(A,B)}{\sqrt{C_{DHFS}(A,A)}\sqrt{C_{DHFS}(B,B)}}$$

$$= \frac{\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}^2(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\gamma_{A\sigma(t)}^2(x_i)\right)} \times \sqrt{\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\eta_{B\sigma(s)}^2(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{B\sigma(t)}^2(x_i)\right)}} \tag{10}$$

Theorem 1. Let A and B be any two DHFSs, the correlation coefficient defined by Eq. (10) should satisfy the following properties:

(1) $\rho_{DHFS}(A,B) = \rho_{DHFS}(B,A)$;

(2) $0 \le \rho_{DHFS}(A,B) \le 1$;

(3) $\rho_{DHFS}(A,B) = 1$, if $A = B$.

In next section, we will propose a new correlation coefficient along with some related concepts for SVNHFSs.

3.3. Correlation coefficient of single valued neutrosophic hesitant fuzzy sets

Similar to HFS, in most of the cases, the number of values in different SVNHFEs might be different, i.e., $l_{h_A}(x_i) \neq l_{h_B}(x_i)$, $l_{\iota_A}(x_i) \neq l_{\iota_B}(x_i)$ and $l_{g_A}(x_i) \neq l_{g_B}(x_i)$. Let $k_i(x_i) = \max\{l_{h_A}(x_i), l_{h_B}(x_i)\}$, $p_i(x_i) = \max\{l_{\iota_A}(x_i), l_{\iota_B}(x_i)\}$ and $l_i(x_i) = \max\{l_{g_A}(x_i), l_{g_B}(x_i)\}$ for each $x_i \in X$. To operate correctly, we should extend the shorter one until both of them have the same length when we compare them. The selection of this operation mainly depends on the decision makers' risk preferences. Pessimists expect unfavorable outcomes and may add the minimum of the truth-membership degree and maximum value of indeterminacy-membership degree and falsity-membership degree. Optimists anticipate desirable outcomes and may add the maximum of the truth-membership degree and minimum value of indeterminacy-membership degree and falsity-membership degree That is, according to the pessimistic principle, if $l_{h_A}(x_i) < l_{h_B}(x_i)$, then the least value of $h_A(x_i)$ or $h_B(x_i)$ will be added to $h_A(x_i)$. Moreover, if $l_{\iota_A}(x_i) < l_{\iota_B}(x_i)$, then the largest value of $l_{\iota_A}(x_i)$ or $l_{\iota_B}(x_i)$ will be inserted in $\iota_A(x_i)$ for $x_i \in X$. Similarity, if $l_{g_A}(x_i) < l_{g_B}(x_i)$, then the largest value of $l_{g_A}(x_i)$ or $l_{g_B}(x_i)$ will be inserted in $g_A(x_i)$ for $x_i \in X$.

By motived the definitions of Chen *et al.* [28] and Wang *et al.* [31], we extend the concepts of informational energy, correlation and correlation coefficients to SVNHFSs, and obtain the following definitions.

Let us consider the two SVNHFSs $A = \{\langle x_i, h_A(x_i), \iota_A(x_i), g_A(x_i)\rangle : x_i \in X\}$ and $B = \{\langle x_i, h_B(x_i), \iota_B(x_i), g_B(x_i)\rangle : x_i \in X\}$ with $h_A = \{\gamma_{A1}, \gamma_{A2}, \dots, \gamma_{Ak_i}\}$, $\iota_A = \{\delta_{A1}, \delta_{A2}, \dots, \delta_{Ap_i}\}$, and $g_A = \{\eta_{A1}, \eta_{A2}, \dots, \eta_{Al_i}\}$, and $h_B = \{\gamma_{B1}, \gamma_{B2}, \dots, \gamma_{Bk_i}\}$, $\iota_B = \{\delta_{B1}, \delta_{B2}, \dots, \delta_{Bp_i}\}$, and $g_B = \{\eta_{B1}, \eta_{B2}, \dots, \eta_{Bl_i}\}$, where $k_i = l(h_A(x_i))$, $p_i = l(\iota_A(x_i))$ and $l_i = l(g_A(x_i))$ are the number of values in $h_A(x_i)$, $\iota_A(x_i)$ and

$g_A(x_i)$, and $\gamma_{A\sigma(s)}(x_i)$, $\delta_{A\sigma(z)}(x_i)$ and $\eta_{A\sigma(t)}(x_i)$ are the sth, zth and tth values in $h_A(x_i)$, $\iota_A(x_i)$ and $g_A(x_i)$, respectively, $x_i \in X$.

Definition 12. Let A be a SVNHFS on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. Then, the informational energy of A is defined as

$$E_{SVNHFS}(A) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}^2(x_i) + \frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{A\sigma(z)}^2(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}^2(x_i)\right) \quad (11)$$

Definition 13. Let A and B be two SVNHFSs on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. Then, the correlation between A and B is defined by

$$C_{SVNHFS}(A,B) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) + \frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{A\sigma(z)}(x_i)\delta_{B\sigma(z)}(x_i)\right.$$
$$\left. + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\right). \quad (12)$$

Assume that A and B are any two SVNHFSs, then we have the following properties:

1) $C_{SVNHFS}(A,A) = E_{SVNHFS}(A)$;
2) $C_{SVNHFS}(A,B) = C_{SVNHFS}(B,A)$.

Definition 14. Let A and B be two SVNHFSs on a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. Then, the correlation coefficient between A and B is defined by

$$\rho_{SVNHFS}(A,B) = \left[\frac{C_{SVNHFS}(A,B)}{\sqrt{C_{SVNHFS}(A,A)}\sqrt{C_{SVNHFS}(B,B)}}\right]$$

$$= \left[\frac{\sum_{i=1}^{n}\begin{pmatrix}\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i)\\ +\frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{A\sigma(z)}(x_i)\delta_{B\sigma(z)}(x_i)\\ +\frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\end{pmatrix}}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}^2(x_i) + \frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{A\sigma(z)}^2(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}^2(x_i)\right)} \times \sqrt{\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{B\sigma(s)}^2(x_i) + \frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{B\sigma(z)}^2(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{B\sigma(t)}^2(x_i)\right)}}\right].$$

(13)

Theorem 2. For two SVNHFSs A and B, the correlation coefficient defined by Eq. (13) should satisfy the following properties:

(1) $0 \le \rho_{SVHHFS}(A,B) \le 1$;

(2) $\rho_{SVHHFS}(A,B) = \rho_{SVHHFS}(B,A)$;

(3) $\rho_{SVHHFS}(A,B) = 1$, if $A = B$.

Proof:

(1) The inequality $0 \le \rho_{SVHHFS}(A,B)$ is clear. Now, let us prove that $\rho_{SVHHFS}(A,B) \le 1$;

$$C_{SVNHFS}(A,B) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) + \frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{A\sigma(z)}(x_i)\delta_{B\sigma(z)}(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\right)$$

$$= \frac{1}{k_1}\sum_{s=1}^{k_1}\gamma_{A\sigma(s)}(x_1)\gamma_{B\sigma(s)}(x_1) + \frac{1}{k_2}\sum_{s=1}^{k_2}\gamma_{A\sigma(s)}(x_2)\gamma_{B\sigma(s)}(x_2) + \cdots + \frac{1}{k_n}\sum_{s=1}^{k_n}\gamma_{A\sigma(s)}(x_n)\gamma_{B\sigma(s)}(x_n)$$
$$+ \frac{1}{p_1}\sum_{z=1}^{p_1}\delta_{A\sigma(z)}(x_1)\delta_{B\sigma(z)}(x_1) + \frac{1}{p_2}\sum_{z=1}^{p_2}\delta_{A\sigma(z)}(x_2)\delta_{B\sigma(z)}(x_2) + \cdots + \frac{1}{p_n}\sum_{z=1}^{p_n}\delta_{A\sigma(z)}(x_n)\delta_{B\sigma(z)}(x_n)$$
$$+ \frac{1}{l_1}\sum_{t=1}^{l_1}\eta_{A\sigma(t)}(x_1)\eta_{B\sigma(t)}(x_1) + \frac{1}{l_2}\sum_{t=1}^{l_2}\eta_{A\sigma(t)}(x_2)\eta_{B\sigma(t)}(x_2) + \cdots + \frac{1}{l_n}\sum_{t=1}^{l_n}\eta_{A\sigma(t)}(x_n)\eta_{B\sigma(t)}(x_n)$$
$$= \sum_{s=1}^{k_1}\frac{\gamma_{A\sigma(s)}(x_1)}{\sqrt{k_1}}\cdot\frac{\gamma_{B\sigma(s)}(x_1)}{\sqrt{k_1}} + \sum_{s=1}^{k_2}\frac{\gamma_{A\sigma(s)}(x_2)}{\sqrt{k_2}}\cdot\frac{\gamma_{B\sigma(s)}(x_2)}{\sqrt{k_2}} + \cdots + \sum_{s=1}^{k_n}\frac{\gamma_{A\sigma(s)}(x_n)}{\sqrt{k_n}}\cdot\frac{\gamma_{B\sigma(s)}(x_n)}{\sqrt{k_n}}$$
$$+ \sum_{z=1}^{p_1}\frac{\delta_{A\sigma(z)}(x_1)}{\sqrt{p_1}}\cdot\frac{\delta_{B\sigma(z)}(x_1)}{\sqrt{p_1}} + \sum_{z=1}^{p_2}\frac{\delta_{A\sigma(z)}(x_2)}{\sqrt{p_2}}\cdot\frac{\delta_{B\sigma(z)}(x_2)}{\sqrt{p_2}} + \cdots + \sum_{z=1}^{p_n}\frac{\delta_{A\sigma(z)}(x_n)}{\sqrt{p_n}}\cdot\frac{\delta_{\sigma\varphi(z)}(x_n)}{\sqrt{p_n}}$$
$$+ \sum_{t=1}^{l_1}\frac{\eta_{A\sigma(t)}(x_1)}{\sqrt{l_1}}\cdot\frac{\eta_{B\sigma(t)}(x_1)}{\sqrt{l_1}} + \sum_{t=1}^{l_2}\frac{\eta_{A\sigma(t)}(x_2)}{\sqrt{l_2}}\cdot\frac{\eta_{B\sigma(t)}(x_2)}{\sqrt{l_2}} + \cdots + \sum_{t=1}^{l_n}\frac{\eta_{A\sigma(t)}(x_n)}{\sqrt{l_n}}\cdot\frac{\eta_{B\sigma(t)}(x_n)}{\sqrt{l_n}}$$

According to the Cauchy–Schwarz inequality:

$$(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 \le (x_1^2 + x_2^2 + \cdots + x_n^2)\cdot(y_1^2 + y_2^2 + \cdots + y_n^2),$$

where $(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ and $(y_1, y_2, \dots, y_n) \in \mathbb{R}^n$, we get:

$$\left(C_{SVNHFS}(A,B)\right)^2 \le$$
$$\Bigg[\frac{1}{k_1}\sum_{s=1}^{k_i}\gamma^2_{A\sigma(s)}(x_1) + \frac{1}{k_2}\sum_{s=1}^{k_2}\gamma^2_{A\sigma(s)}(x_2) + \cdots + \frac{1}{k_n}\sum_{s=1}^{k_n}\gamma^2_{A\sigma(s)}(x_n)$$
$$+ \frac{1}{p_1}\sum_{z=1}^{p_1}\delta^2_{A\sigma(z)}(x_1) + \frac{1}{p_2}\sum_{z=1}^{p_2}\delta^2_{A\sigma(z)}(x_2) + \cdots + \frac{1}{p_n}\sum_{z=1}^{p_n}\delta^2_{A\sigma(z)}(x_n)$$

$$+\frac{1}{l_1}\sum_{t=1}^{l_1}\eta^2_{A\sigma(t)}(x_1)+\frac{1}{l_2}\sum_{t=1}^{l_2}\eta^2_{A\sigma(t)}(x_2)+\cdots+\frac{1}{l_n}\sum_{t=1}^{l_n}\eta^2_{A\sigma(t)}(x_n)\Bigg]$$

$$\times\Bigg[\frac{1}{k_1}\sum_{s=1}^{k_i}\gamma^2_{B\sigma(s)}(x_1)+\frac{1}{k_2}\sum_{s=1}^{k_2}\gamma^2_{B\sigma(s)}(x_2)+\cdots+\frac{1}{k_n}\sum_{s=1}^{k_n}\delta^2_{B\sigma(s)}(x_n)$$

$$+\frac{1}{p_1}\sum_{z=1}^{p_1}\delta^2_{B\sigma(z)}(x_1)+\frac{1}{p_2}\sum_{z=1}^{p_2}\delta^2_{B\sigma(z)}(x_2)+\cdots+\frac{1}{p_n}\sum_{z=1}^{p_n}\delta^2_{B\sigma(z)}(x_n)$$

$$+\frac{1}{l_1}\sum_{t=1}^{l_1}\eta^2_{B\sigma(t)}(x_1)+\frac{1}{l_2}\sum_{t=1}^{l_2}\eta^2_{B\sigma(t)}(x_2)+\cdots+\frac{1}{l_n}\sum_{t=1}^{l_n}\eta^2_{B\sigma(t)}(x_n)\Bigg]$$

$$=\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma^2_{A\sigma(s)}(x_i)+\frac{1}{p_i}\sum_{z=1}^{p_i}\delta^2_{A\sigma(z)}(x_i)+\frac{1}{l_i}\sum_{t=1}^{l_i}\eta^2_{A\sigma(t)}(x_i)\right)$$

$$\times\sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma^2_{B\sigma(s)}(x_i)+\frac{1}{p_i}\sum_{z=1}^{p_i}\delta^2_{B\sigma(t)}(x_i)+\frac{1}{l_i}\sum_{t=1}^{l_i}\eta^2_{B\sigma(z)}(x_i)\right)$$

$$=C_{SVNHFS}(A,A)\cdot C_{SVNHFS}(B,B).$$

Therefore, $C_{SVNHFS}(A,B)\le\sqrt{\left(C_{SVNHFS}(A,A)\right)}\cdot\sqrt{C_{SVNHFS}(B,B)}$. Thus, $0\le\rho_{SVHHFS}(A,B)\le 1$;

(2) It is straightforward.

(3) $A=B\Rightarrow\gamma_{A\sigma(s)}(x_i)=\gamma_{B\sigma(s)}(x_i),\delta_{A\sigma(z)}(x_i)=\delta_{B\sigma(z)}(x_i)\ and\ \eta_{A\sigma(t)}(x_i)=\eta_{B\sigma(t)}(x_i),x_i\in X\Rightarrow$ $\rho_{SVHHFS}(A,B)=1$.

However, the differences of importance are considered in the elements in the universe. Therefore, we need to take the weights of the elements $x_i(i=1,2,\ldots,n)$ into account. In the following, we develop the weighted correlation coefficient between SVNHFSs.

Let $w=(w_1,w_2,\ldots,w_n)^T$ be the weighting vector of $x_i(i=1,2,\ldots,n)$ with $w_i\ge 0$ and $\sum_{i=1}^{n}w_i=1$. As a generalization of Eq. (13), the weighted correlation coefficient is defined as follows:

$$\rho_{SVNHFS_w}(A,B)=\left[\frac{C_{SVNHFS_w}(A,B)}{\sqrt{C_{SVNHFS_w}(A,A)}\sqrt{C_{SVNHFS_w}(B,B)}}\right]$$

$$= \left[\frac{\sum_{i=1}^{n} w_i \begin{pmatrix} \frac{1}{k_i} \sum_{s=1}^{k_i} \gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) \\ + \frac{1}{p_i} \sum_{z=1}^{p_i} \delta_{A\sigma(z)}(x_i)\delta_{B\sigma(z)}(x_i) \\ + \frac{1}{l_i} \sum_{t=1}^{l_i} \eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i) \end{pmatrix}}{\sqrt{\sum_{i=1}^{n} w_i \left(\frac{1}{k_i} \sum_{s=1}^{k_i} \gamma_{A\sigma(s)}^2 (x_i) + \frac{1}{p_i} \sum_{z=1}^{p_i} \delta_{A\sigma(z)}^2 (x_i) + \frac{1}{l_i} \sum_{t=1}^{l_i} \eta_{A\sigma(t)}^2 (x_i) \right)} \times \sqrt{\sum_{i=1}^{n} w_i \left(\frac{1}{k_i} \sum_{s=1}^{k_i} \gamma_{B\sigma(s)}^2 (x_i) + \frac{1}{p_i} \sum_{z=1}^{p_i} \delta_{B\sigma(z)}^2 (x_i) + \frac{1}{l_i} \sum_{t=1}^{l_i} \eta_{B\sigma(t)}^2 (x_i) \right)}} \right]. \quad (14)$$

Specially, when $w_i = 1/n$ ($i = 1,2,\ldots,n$) Eq. (14) reduce to Eq. (13).

Moreover, for two SVNHFSs A and B, the weighted correlation coefficient defined by Eq. (14) should satisfy the following properties:

(1) $0 \le \rho_{SVNHFS_w}(A,B) \le 1$;

(2) $\rho_{SVNHFS_w}(A,B) = \rho_{SVNHFS_w}(B,A)$;

(3) $\rho_{SVNHFS_w}(A,B) = 1$, if $A = B$.

4. Decision-making method based on the single-valued neutrosophic hesitant fuzzy information

In this section, we use the developed correlation coefficient to find the best alternative in MADM with SVNHFSs.

For the MADM problem, let $A = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ be a discrete set of alternatives, $G = \{\beta_1, \beta_2, \ldots, \beta_n\}$ be a set of attributes for a MADM with SVNHFSs. The decision maker provides his decision as a SVNHFN $n_{ij} = \{h_{ij}, \iota_{ij}, g_{ij}\}$ ($j = 1,2,\ldots,n; i = 1,2,\ldots,m$) for the alternative $\alpha_i (i = 1,2,\ldots,m)$ under the attribute β_j ($j = 1,2,\ldots,n$). Suppose that $N = [n_{ij}]_{m\times n}$ is the decision matrix, where n_{ij} is expressed by single-valued neutrosophic hesitant fuzzy element.

In MADM process, we can utilize the concept of ideal point to determine the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives. Therefore, we propose each ideal SVNHFN in the ideal alternative $n_j^* = \{h_j^*, \iota_j^*, g_j^*\} = \{\{1\},\{0\},\{0\}\}$ ($j = 1,2,\ldots,n$) in the ideal alternative $\alpha^* = \{\langle \beta_j, n_j^* \rangle : \beta_j \in G\}$.

The procedure for the selection of best alternative is described as follows:

Step1. *Compute the* $\rho_{SVNHFS_w}(\alpha^*, \alpha_i)$ *between an alternative* $\alpha_i (i = 1,2,\ldots,m)$ *and the ideal alternative* α^* *by using Eq.(14).*

Step 2. *Rank all of the alternative with respect to the values of the* $\rho_{SVNHFS_w}(\alpha^*, \alpha_i) (i = 1,2,\ldots,m)$.

Step 3. *Choose the best alternative with respect to the maximum value of the* $\rho_{SVNHFS_w}(\alpha^*, \alpha_i)(i = 1,2,\ldots,m)$.

Step4. *End.*

5. Numerical example

Here, we take the example, from Ye [20, 27], to illustrate the utility of the proposed weighted correlation coefficient.

Example 11. Suppose that an investment company that wants to invest a sum of money in the best option. There is a panel with four possible alternatives in which to invest the money: (1) α_1 is a car company, (2) α_2 is a food company, (3) α_3 is a computer company, and (4) α_4 is an arms company. The investment company must make a decision according to the three attributes: (1) β_1 is the risk analysis; (2) β_2 is the growth analysis, and (3) β_3 is the environmental impact analysis. Suppose that $w = (0.35, 0.25, 0.40)$ is the attribute weight vector. The four possible alternatives α_i $(i = 1,2,3,4)$ are to be evaluated using the single valued neutrosophic hesitant fuzzy information by decision maker under three attributes β_j $(j = 1,2,3)$, and the decision matrix N is presented as follows:

Table 1: Decision matrix N

$$N = \begin{pmatrix} \{\{0.3, 0.4, 0.5\}, \{0.1\}, \{0.3, 0.4\}\} & \{\{0.5, 0.6\}, \{0.2, 0.3\}, \{0.3,0.4\}\} & \{\{0.2, 0.3\}, \{0.1, 0.2\}, \{0.5, 0.6\}\} \\ \{\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.2, 0.3\}\} & \{\{0.6, 0.7\}, \{0.1\}, \{0.3\}\} & \{\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.1, 0.2\}\} \\ \{\{0.5, 0.6\}, \{0.4\}, \{0.2, 0.3\}\} & \{\{0.6\}, \{0.3\}, \{0.4\}\} & \{\{0.5, 0.6\}, \{0.1\}, \{0.3\}\} \\ \{\{0.7, 0.8\}, \{0.1\}, \{0.1, 0.2\}\} & \{\{0.6, 0.7\}, \{0.1\}, \{0.2\}\} & \{\{0.3, 0.5\}, \{0.2\}, \{0.1, 0.2, 0.3\}\} \end{pmatrix}$$

To get the best alternative(s), the following steps are involved:

Step 1. Using Eq. (14), the $\rho_{SVNHFS_w}(\alpha_i, \alpha^*)$ $(i = 1,2,3,4)$ between the alternative α_i and the ideal alternative α^* were been calculated as follows:

$$\rho_{SVNHFS_w}(\alpha_1, \alpha^*) = 0.6124, \rho_{SVNHFS_w}(\alpha_2, \alpha^*) = 0.9210,$$
$$\rho_{SVNHFS_w}(\alpha_3, \alpha^*) = 0.7986, \rho_{SVNHFS_w}(\alpha_4, \alpha^*) = 0.8905.$$

Step 2. The ranking order of alternatives according to the $\rho_{SVNHFS_w}(\alpha_i, \alpha^*)$ $(i = 1,2,3,4)$ was been obtained as: $\alpha_2 \succ \alpha_4 \succ \alpha_3 \succ \alpha_1$, which have the same ranking result of Ye's [27].

Step 3. With respect to the increasing value among the $\rho_{SVNHFS_w}(\alpha_i, \alpha^*)$ $(i = 1,2,3,4)$, the alternative α_2 was been selected as the best alternative.

Example 1 clearly demonstrate that the developed method in this paper provides an effective and applicable way to solve MADM problems with single-valued neutrosophic hesitant fuzzy information. Since the SVNHFS is a further generalization of FS, IFS, HFS, FMS, DHFS and SVNS, the correlation coefficients of aforementioned sets are special cases of the correlation coefficient of SVNHFSs developed in this paper. That is, the proposed method can be used for not only the correlation coefficient with single valued neutrosophic hesitant fuzzy information but also the hesitant fuzzy information, intuitionistic fuzzy information and dual hesitant fuzzy information and single valued neutrosophic information, whereas the methods given in Xu and Xia [29], Xu [30] ,Wang et al. [31] and Ye [20] are only suitable for the problem with HFSs, IFSs, DHFSs and SVNSs, respectively.

6. A comparison analysis and discussion

In order to verify the validity of the developed method, we consider the method based on the cosine measure proposed by Ye [27], which is the first method developed to find the best alternative under single valued neutrosophic hesitant fuzzy environment; to rank this example, we can get the ranking as $\alpha_2 \succ \alpha_4 \succ \alpha_3 \succ \alpha_1$. Obviously, these two methods have the same ranking result.

With regard to the four methods in [29,30,31, 20], the weighted correlation coefficient between each alternative and the optimal alternative was computed and used to determine the final ranking sequence of all the alternatives, in which attribute values according to alternatives are evaluated by using hesitant information, intuitionistic fuzzy information, dual hesitant fuzzy information and single valued neutrosophic information, respectively. However, in our method, the uncertainty or vagueness presented, i.e. the indeterminacy case is handled independently from truth-hesitancy membership and falsity-hesitancy membership factors, whereas the incorporated uncertainty is based on the hesitant degree of membership of HFSs and the hesitant degrees of membership and non- membership of DHFs (or IFSs). On the other hand, our method is more general than Ye [20]'s method, because his method does not take into account the hesitant cases of truth, indeterminacy and falsity memberships. Therefore, this leads to the theory that the MADMs obtained by using HFSs, IFSs, DHFSs and SVNSs are a special case of the method using SVNHFSs. That is, the method developed in here can avoid losing and distorting the preference information provided which makes the final results better correspond with real life decision-making problems.

7. Conclusions

The SVNHFS is a generalized form that allows extension of FSs, IFSs, HFSs, FMSs, DHFSs and SVNSs, in which its truth-hesitancy membership value, indeterminacy-hesitancy membership value and falsity-hesitancy membership value are characterized by three sets of possible values. Therefore, it is a more flexible and more efficient set than aforementioned sets, considering more comprehensive information provided by experts in decision process. In this study, we defined first the informational energy of a SVNHFS and then proposed the concepts of correlation, and correlation coefficient of SVNHFSs, as a new generalization of FSs, IFSs, HFSs, FMSs, DHFSs and SVNSs. Further, the correlation coefficient are then applied to a MADM under single valued neutrosophic hesitant fuzzy environment. In order to determine the ranking sequence of all alternatives and choose the best alternative, the weighted correlation coefficient between each alternative and the optimal alternative has been utilized. Finally, a numerical and practical example have been given to support the findings and illustrate the validation and efficiency of the proposed correlation coefficient between SVNHFSs. The approach proposed in this paper has much application potential in dealing with MADM problems using single valued neutrosophic hesitant fuzzy information, and also can be effectively used in the real applications of decision making, pattern recognition, supply management, data mining and etc. in the future research.

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