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NEUTROSOPHIC REFINED SIMILARITY MEASURE BASED ON COTANGENT FUNCTION AND ITS APPLICATION TO MULTI-ATTRIBUTE DECISION MAKING

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ABSTRACT
In this paper, cotangent similarity measure of neutrosophic refined set is proposed and some of its properties are studied. Finally, using this refined cotangent similarity measure of single valued neutrosophic set, an application on educational stream selection is presented.

General Terms
Neutrosophic set, Cotangent similarity measure

Keywords
Cotangent similarity measure, refined set, neutrosophic set, neutrosophic refined set, indeterminacy-membership degree, 3D-vector space

1. INTRODUCTION
Smarandache [1] introduced the new philosophy called “neutrosophy”. The concept of neutrosophy reflects the study of neutral thoughts. Neutrosophic set [1] is originated from the neutrosophy. The concept of neutrosophic sets is the generalization of crisp set, fuzzy set [2], interval valued fuzzy sets [3, 4, 5], intuitionistic fuzzy set [6], interval valued intuitionistic fuzzy sets [7], vague sets [8], grey sets [9, 10] etc. Wang et al. [11] introduced the concept of single valued neutrosophic set (SVNS) in order to deal with realistic problems. It has been studied and applied in different fields such as medical diagnosis problem [12], decision making problems [13, 14, 15, 16, 17], social problems [18, 19], educational problem [20, 21], conflict resolution [22] and so on.

Several similarity measures have been proposed in the literature by researchers to deal with different type problems. In 2013 Broumi and Smarandache [23] studied the Hausdorff distance between neutrosophic sets and some similarity

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measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. In 2013, Broumi and Smarandache [24] also proposed the correlation coefficient between interval neutrosphic sets. Majumdar and Samanta [25] studied several similarity measures of single valued neutrosophic sets based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [26] proposed three vector similarity measure for SNSs, an instance of SVNSs and interval neutrosophic sets including the Jaccard, Dice, and cosine similarity and applied them to multicriteria decision-making problems with simplified neutrosophic information. Ye [27] and Ye and Zhang [28] further proposed the similarity measures of SVNSs for decision making problems. Ye [29] proposed improved cosine similarity measures of SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. Biswas et al. [30] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Recently, Pramanik and Mondal [31] proposed rough cosine similarity measure in rough neutrosophic environment.

Yager [32] introduced the notion of multisets which is the generalization of the concept of set theory. Sebastian and Ramakrishnan [33] studied a new concept called multifuzzy sets, which is the generalization of multiset. Since then, Sebastian and Ramakrishnan [34] established more properties on multi fuzzy set. Shinoj and John [35] extended the concept of fuzzy multisets by introducing intuitionistic fuzzy multisets(IFMS). An element of a multi fuzzy sets can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multisets is capable of having repeated occurrences of membership and non--membership values. However, the concepts of FMS and IFMS are not capable of dealing with indeterminatcy. In 2013, Smarandache [36] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively, T_1, T_2, ... T_m, and, I_1, I_2, ... I_p and F_1, F_2, ... F_r. Recently, Deli abd Broumi [37] introduced the concept of neutrosophic refined sets and studied some of their basic properties. The concept of neutrosophic refined set (NRS) [38] is a generalization of fuzzy multisets and intuitionistic fuzzy multisets. In 2014, Broumi and Smarandache [38] extended the improved cosine similarity of single valued neutrosophic set proposed by Ye [26] to the case of neutrosophic refined sets and proved some of their basic properties. Broumi and Smarandache [38] also presented an application of cosine similarity measure of neutrosophic refined sets in medical diagnosis problems. Ye and Ye [39] introduced the concept of single valued neutrosophic multiset (SVNM) and proved some basic operational relations of SVNMs. They proposed the Dice similarity measure and the weighted Dice similarity measure for SVNMs and investigated their properties and they applied the Dice similarity measure of SVNMs to medicine diagnosis under the SVNM environment.

Pramanik and Mondal [40] studied weighted fuzzy similarity measure based on tangent function and provided its application to medical diagnosis. Mondal and Pramanik [41] extended the concept to neutrosophic tangent similarity measure.

In this paper, motivated by the tangent similarity measure proposed by Pramanik and Mondal [40] and Mondal and Pramanik [41], we propose a new cotangent similarity measure called "refined cotangent similarity measure for single valued neutrosophic sets". The proposed refined cotangent similarity measure is applied to suitable educational stream selection problem.

Rest of the paper is structured as follows. Section 2 presents neutrosophic preliminaries. Section 3 is devoted to introduce refined cotangent similarity measure for single valued neutrosophic sets and some of its properties. Section 4 describes decision making based on refined cotangent similarity measure. Section 5 presents the application of refined cotangent similarity measure to the problem namely, neutrosophic decision making on educational stream selection. Finally, section 6 presents the concluding remarks and future scope of research.

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2. MATHEMATICAL PRELIMINARIES

2.1 Neutrosophic sets

Definition 2.1[1]

Let X be an universe of discourse then the neutrosophic set S is expressed in the form $S = \{< x: T_S(x), I_S(x), F_S(x)>, x \in X\}$, where the functions $T_S(x), I_S(x), F_S(x): X\rightarrow\ ]^-0,1^+[$ are defined respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set S satisfying the following the condition.

$^-0 \le \sup T_S(x) + \sup I_S(x) + \sup F_S(x) \le 3^+$
(1)

The neutrosophic set assumes the values from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ it assumes the values from the interval [0, 1] for practical situations, because $]^-0, 1^+[$ will be difficult to use in the real applications such as in scientific and engineering problems. For two neutrosophic sets, $S1 = \{< x: T_{S1}(x), I_{S1}(x), F_{S1}(x)> | x \in X\}$ and $S2 = \{< x, T_{S2}(x), I_{S2}(x), F_{S2}(x)> | x \in X\}$ the two relations are defined as follows:

(1) $S1 \subseteq S2$ if and only if $T_{S1}(x) \le T_{S2}(x), I_{S1}(x) \ge I_{S2}(x), F_{S1}(x) \ge F_{S2}(x)$

(2) $S1 = S2$ if and only if $T_{S1}(x) = T_{S2}(x), I_{S1}(x) = I_{S2}(x), F_{S1}(x) = F_{S2}(x)$

2.2 Single valued neutrosophic sets

Definition 2.2 [8]

Let X be a space of points with generic elements in X denoted by x. A single valued neutrosophic set S in X is characterized by a truth-membership function $T_S(x)$, an indeterminacy-membership function $I_S(x)$, and a falsity membership function $F_S(x)$, for each point x in X,

$T_S(x), I_S(x), F_S(x) \in [0, 1]$. When X is continuous, a single valued neutrosophic set S can be presented as follows:

$$S = \int_X \frac{<T_S(x), I_S(x), F_S(x)>}{x} : x \in X$$

When X is discrete, a single valued neutrosophic set S can be presented as follows:

$$S = \sum_{i=1}^{n} \frac{<T_S(x_i), I_S(x_i), F_S(x_i)>}{x_i} : x_i \in X$$

For two SVNSs, $S1_{SVNS} = \{<x: T_{S1}(x), I_{S1}(x), F_{S1}(x)> | x \in X\}$ and $S2_{SVNS} = \{<x, T_{S2}(x), I_{S2}(x), F_{S2}(x)> | x \in X\}$ the two relations are written as follows:

(1) $S1_{SVNS} \subseteq S2_{SVNS}$ if and only if $T_{S1}(x) \le T_{S2}(x), I_{S1}(x) \ge I_{S2}(x), F_{S1}(x) \ge F_{S2}(x)$

(2) $S1_{SVNS} = S2_{SVNS}$ if and only if $T_{S1}(x) = T_{S2}(x), I_{S1}(x) = I_{S2}(x), F_{S1}(x) = F_{S2}(x)$ for any $x \in X$

2.3 Neutrosophic refined sets

Definition 2.3 [38] Let M be a neutrosophic refined set (NRS). Then,

$M = \{< x, (T_M^1(x_i), T_M^2(x_i), ..., T_M^t(x_i)), (I_M^1(x_i), I_M^2(x_i), ..., I_M^t(x_i)), (F_M^1(x_i), F_M^2(x_i), ..., F_M^t(x_i))>: x \in X\}$ where, $T_M^1(x_i)$, $T_M^2(x_i)$, ..., $T_M^t(x_i)$: $X \in [0, 1]$, $I_M^1(x_i)$, $I_M^2(x_i)$, ..., $I_M^t(x_i)$: $X \in [0, 1]$, and $F_M^1(x_i)$, $F_M^2(x_i)$, ..., $F_M^t(x_i)$: $X \in [0, 1]$, such that $0 \le \sup T_M^i(x_i) + \sup I_M^i(x_i) + \sup F_M^i(x_i) \le 3$, for i = 1, 2, …, t for any $x \in X$. Now, $(T_M^1(x_i), T_M^2(x_i), ..., T_M^t(x_i))$, $(I_M^1(x_i), I_M^2(x_i), ..., I_M^t(x_i))$, $(F_M^1(x_i), F_M^2(x_i), ..., F_M^t(x_i))$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, t is the dimension of neutrosophic refined sets M.

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3. COTANGENT SIMILARITY MEASURE FOR SINGLE VALUED REFINED NEUTROSOPHIC SETS

Let N = < $x(T_N^j(x_i), I_N^j(x_i), F_N^j(x_i))$> and P = < $x(T_P^j(x_i), I_P^j(x_i), F_P^j(x_i))$ > be two single valued refined neutrosophic numbers. Now refined cotangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as:

$COT_{NRS}(N, P)$=

$$\frac{1}{p}\sum_{j=1}^{p}\left[\frac{1}{n}\sum_{i=1}^{n}\left\langle \cot\left(\frac{\pi}{12}\left(3+\left|T_P^j(x_i)-T_Q^j(x_i)\right|+\left|I_P^j(x_i)-I_Q^j(x_i)\right|+\left|F_P^j(x_i)-F_Q^j(x_i)\right|\right)\right)\right\rangle\right]$$

(1)

Proposition 3.1. The defined refined cotangent similarity measure $COT_{NRS}(N, P)$ between NRSs N and P satisfies the following properties:

1. $0 \le COT_{NRS}(N, P) \le 1$
2. $COT_{NRS}(N, P) = 1$ if and only if $N = P$
3. $COT_{NRS}(N, P) = COT_{NRS}(P, N)$
4. If R is a NRS in X and $N \subset P \subset R$ then

   $COT_{NRS}(N, R) \le COT_{NRS}(N, P)$ and $COT_{NRS}(N, R) \le COT_{NRS}(P, R)$

Proofs:

(1)

As the membership, indeterminacy and non-membership functions of the NRSs and the value of the cotangent function are within [0 ,1], the refined similarity measure based on cotangent function also lies within [0,1].

Hence $0 \le COT_{NRS}(N, P) \le 1$

(2)

For any two NRS N and P if $N = P$, then the following relations hold $T_P^j(x) = T_P^j(x)$, $I_P^j(x) = I_P^j(x)$, $F_P^j(x) = F_P^j(x)$. Hence

$\left|T_N^j(x) - T_P^j(x)\right| = 0$, $\left|I_N^j(x) - I_P^j(x)\right| = 0$, $\left|F_N^j(x) - F_P^j(x)\right| = 0$, Thus $COT_{NRS}(N, P) = 1$

Conversely,

If $COT_{NRS}(N, P) = 1$, then $\left|T_N^j(x) - T_P^j(x)\right| = 0$, $\left|I_N^j(x) - I_P^j(x)\right| = 0$, $\left|F_N^j(x) - F_P^j(x)\right| = 0$, since tan(0) = 0. So we can write $T_P^j(x) = T_P^j(x)$, $I_P^j(x) = I_P^j(x)$, $\mathrm{F}_P^j(x) = \mathrm{F}_P^j(x)$.

Hence $N = P$**.**

(3)

This proof is obvious.

(4)

If $N \subset P \subset R$, then $T_N^j(x) \le T_P^j(x) \le T_R^j(x)$,

$I_N^j(x) \le I_P^j(x) \le I_R^j(x)$, $F_N^j(x) \le F_P^j(x) \le F_R^j(x)$ for $x \in X$.

Now we can write the following inequalities:

$|T_N^j(x) - T_P^j(x)| \le |T_N^j(x) - T_R^j(x)|$, $|T_P^j(x) - T_R^j(x)| \le |T_N^j(x) - T_R^j(x)|$;

$|I_N^j(x) - I_P^j(x)| \le |I_N^j(x) - I_R^j(x)|$, $|I_P^j(x) - I_R^j(x)| \le |I_N^j(x) - I_R^j(x)|$;

$|F_N^j(x) - F_P^j(x)| \le |F_N^j(x) - F_R^j(x)|$, $|F_P^j(x) - F_R^j(x)| \le |F_N^j(x) - F_R^j(x)|$.

Thus $COT_{NRS}(N, R) \le COT_{NRS}(N, P)$ and $COT_{NRS}(N, R) \le COT_{NRS}(P, R)$, since cotangent function is decreasing in the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

4. DECISION MAKING UNDER SINGLE VALUED NEUTROSOPHIC SETS BASED ON COTANGENT SIMILARITY MEASURE

Let $A_1, A_2, ..., A_m$ be the discrete set of candidates, $C_1, C_2, ..., C_n$ be the set of criteria of each candidate, and $B_1, B_2, ..., B_k$ are the alternatives of each candidates. The decision-maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performances of candidates A_i (i = 1, 2,..., m) with respect to the criteria C_j (j = 1, 2, ..., n). The values associated with the alternatives for multi-attribute decision making problem can be presented in the following decision matrix (see the Table 1 and the Table 2).

Table 1: The relation between candidates and attributes

	C_1	C_2	...	C_n
A_1	$\langle d_{11}^1, d_{11}^2, \cdots, d_{11}^t \rangle$	$\langle d_{12}^1, d_{12}^2, \cdots, d_{12}^t \rangle$	...	$\langle d_{1n}^1, d_{1n}^2, \cdots, d_{1n}^t \rangle$
A_2	$\langle d_{21}^1, d_{21}^2, \cdots, d_{21}^t \rangle$	$\langle d_{22}^1, d_{22}^2, \cdots, d_{22}^t \rangle$	...	$\langle d_{2n}^1, d_{2n}^2, \cdots, d_{2n}^t \rangle$
...	...	...	...	...
A_m	$\langle d_{n1}^1, d_{n1}^2, \cdots, d_{n1}^t \rangle$	$\langle d_{n2}^1, d_{n2}^2, \cdots, d_{n2}^t \rangle$	...	$\langle d_{mn}^1, d_{mn}^2, \cdots, d_{mn}^t \rangle$

Table 2: The relation between attributes and alternatives

	B_1	B_2	...	B_k
C_1	δ_{11}	δ_{12}	...	δ_{1k}
C_2	δ_{21}	δ_{22}	...	δ_{2k}
...	...	...	...	...
C_n	δ_{n1}	δ_{n2}	...	δ_{nk}

Here d_{ij}^t and δ_{ij} are all single valued neutrosophic numbers.

The steps corresponding to refined neutrosophic numbers based on tangent function are presented as follows.

Step 1: Determination the relation between candidates and attributes

The relation between candidate A_i (i = 1, 2, ..., m) and their attribute C_j (j = 1, 2, ..., n) in NRS can be presented as follows (see the Table 3):

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Table 3: Relation between candidates and attributes in terms of NRSs

$$
\begin{array}{c|cccc}
 & C_1 & C_2 & \ldots & C_n \\
\hline
A_1 & \left\{\begin{array}{l} \langle T_{11}^1, I_{11}^1, F_{11}^1\rangle, \\ \langle T_{11}^2, I_{11}^2, F_{11}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{11}^t, I_{11}^t, F_{11}^t\rangle \end{array}\right\} & \left\{\begin{array}{l} \langle T_{12}^1, I_{12}^1, F_{12}^1\rangle, \\ \langle T_{12}^2, I_{12}^2, F_{12}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{12}^t, I_{12}^t, F_{12}^t\rangle \end{array}\right\} & \ldots & \left\{\begin{array}{l} \langle T_{1n}^1, I_{1n}^1, F_{1n}^1\rangle, \\ \langle T_{1n}^2, I_{1n}^2, F_{1n}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{2n}^t, I_{2n}^t, F_{2n}^t\rangle \end{array}\right\} \\
A_2 & \left\{\begin{array}{l} \langle T_{21}^1, I_{21}^1, F_{21}^1\rangle, \\ \langle T_{21}^2, I_{21}^2, F_{21}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{21}^t, I_{21}^t, F_{21}^t\rangle \end{array}\right\} & \left\{\begin{array}{l} \langle T_{22}^1, I_{22}^1, F_{22}^1\rangle, \\ \langle T_{22}^2, I_{22}^2, F_{22}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{22}^t, I_{22}^t, F_{22}^t\rangle \end{array}\right\} & \ldots & \left\{\begin{array}{l} \langle T_{2n}^1, I_{2n}^1, F_{2n}^1\rangle, \\ \langle T_{2n}^2, I_{2n}^2, F_{2n}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{1n}^t, I_{1n}^t, F_{1n}^t\rangle \end{array}\right\} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
A_m & \left\{\begin{array}{l} \langle T_{m1}^1, I_{m1}^1, F_{m1}^1\rangle, \\ \langle T_{m1}^2, I_{m1}^2, F_{m1}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{m1}^t, I_{m1}^t, F_{m1}^t\rangle \end{array}\right\} & \left\{\begin{array}{l} \langle T_{m2}^1, I_{m2}^1, F_{m2}^1\rangle, \\ \langle T_{m2}^2, I_{m2}^2, F_{m2}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{m2}^t, I_{m2}^t, F_{m2}^t\rangle \end{array}\right\} & \ldots & \left\{\begin{array}{l} \langle T_{mn}^1, I_{mn}^1, F_{mn}^1\rangle, \\ \langle T_{mn}^2, I_{mn}^2, F_{mn}^2\rangle, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots\ \ldots\ldots\ldots, \\ \langle T_{mn}^t, I_{mn}^t, F_{mn}^t\rangle \end{array}\right\}
\end{array}
$$

Step 2: Determination the relation between attributes and alternatives

The relation between attributes C_i (i = 1, 2, ..., n) and alternatives B_r (r = 1, 2, ..., k) is presented as follows (see the Table 4):

Table 4: The relation between attributes and alternatives in terms of NRSs

$$
\begin{array}{c|cccc}
 & B_1 & B_2 & \ldots & B_k \\
\hline
C_1 & \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \ldots & \langle T_{1k}, I_{1k}, F_{1k}\rangle \\
C_2 & \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \ldots & \langle T_{2k}, I_{2k}, F_{2k}\rangle \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
C_n & \langle T_{n1}, I_{n1}, F_{n1}\rangle & \langle T_{n2}, I_{n2}, F_{n2}\rangle & \ldots & \langle T_{nk}, I_{nk}, F_{nk}\rangle
\end{array}
$$

Step 3: Determination the correlation measure between attributes and alternatives

Determine the correlation measure between the Table 3 and the Table 4 using $COT_{NRS}(N, P)$ (Equation 1).

Step 4: Ranking the alternatives

Ranking of alternatives is prepared based on the descending order of correlation measures. Highest value of correlation measure reflects the best alternative.

Step 5: End

5. EXAMPLE: EDUCATIONAL STREAM SELECTION

Let us consider an illustrative example which is very important for students (after higher secondary examination) to select suitable educational stream for higher education. After higher secondary examination it is very important to select proper stream of education for a student. If the chosen branch is improper to the student, then a bad impact may occur to his/her future career. So it is necessary to use a suitable mathematical method or strategy for decision making. In some practical situations, indeterminacy is inherently involved. So information is characterized by truth membership, indeterminate and falsity membership function. The proposed similarity measure among the student Vs attributes and attributes Vs educational streams give the proper selection of educational stream of students. The main feature of the proposed method is that it considers single valued neutrosophic values of each attribute provided by the decision makers (experts) for the candidates. Descriptions of students, their attributes, possible educational streams are given below (see the Table 5). Our solution is to examine the students and make decision to choose suitable educational stream for the students (see the Table 6, 7). The decision making procedure is presented as the following steps:

Table 5: Description of students, their attributes and educational streams

Symbols	Descriptions
A_1	First student (rank -1) of ''Birnagar High School'' after HS examination (West Bengal, India)
A_2	Second student (rank -2) of ''Birnagar High School'' after HS examination (West Bengal, India)
A_3	Third student (rank -3) of ''Birnagar High School'' after HS examination (West Bengal, India)
S_1	Depth in languages (English and Bengali)
S_2	Depth in Mathematics and basic computers knowledge
S_3	Depth in Sciences
S_4	Concentration
S_5	Laborious
D_1	Mathematics Honors
D_2	Physics Honors
D_3	Engineering
D_4	Computer Science
D_5	Bio-chemistry

Step 1: Determination the relation between candidates and attributes:

The relation between student and their attributes was collected by three independent decision makers. Setting those three relational values (refined neutrosophic sets) for each student is presented as follows (see the Table 6).

Table 6: (Relation-1) The relation between students and their attributes

Relation-1	S_1	S_2	S_3	S_4	S_5
A_1	(0.7, 0.2, 0.2) (0.6, 0.3, 0.3) (0.6, 0.3, 0.1)	(0.6, 0.2, 0.4) (0.5, 0.2, 0.4) (0.5, 0.1, 0.2)	(0.6, 0.3, 0.2) (0.6, 0.5, 0.2) (0.7, 0.3, 0.4)	(0.7, 0.2, 0.4) (0.7, 0.4, 0.4) (0.6, 0.3, 0.3)	(0.7,0.6, 0.4) (0.6,0.4, 0.5) (0.6,0.5, 0.4)
A_2	(0.8, 0.4, 0.4) (0.7, 0.6, 0.4) (0.8, 0.6, 0.4)	(0.5, 0.5, 0.2) (0.5, 0.4, 0.1) (0.6, 0.6, 0.3)	(0.8, 0.2, 0.2) (0.8, 0.2, 0.5) (0.8, 0.2, 0.4)	(0.6, 0.6, 0.2) (0.6, 0.7, 0.5) (0.5, 0.5, 0.4)	(0.6, 0.6, 0.4) (0.7, 0.6, 0.4) (0.8, 0.6, 0.3)
A_3	(0.7, 0.2, 0.2) (0.6, 0.4, 0.1)	(0.6, 0.1, 0.1) (0.6, 0.2, 0.4)	(0.6, 0.6, 0.4) (0.6, 0.5, 0.5)	(0.8, 0.5, 0.1) (0.6, 0.5, 0.5)	(0.6, 0.5, 0.5) (0.8, 0.5, 0.3)

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	(0.5, 0.3, 0.3)	(0.6, 0.1, 0.3)	(0.7, 0.4, 0.6)	(0.6, 0.6, 0.3)	(0.8, 0.3, 0.4)

Step 2: Determination the relation between attributes and alternatives:

The relation between student-attributes S_i (i = 1, 2, 3, 4, 5) and educational streams D_r (r = 1, 2, 3, 4, 5) is presented as follows (see the Table 7).

Table 7: (Relation-2) The relation among student-attributes and educational streams

Relation-2	D_1	D_2	D_3	D_4	D_5
S_1	(0.5, 0.2, 0.4)	(0.4, 0.5, 0.4)	(0.4, 0.4, 0.4)	(0.5, 0.3, 0.3)	(0.5, 0.4, 0.4)
S_2	(0.9, 0.2, 0.2)	(0.8, 0.4, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.4, 0.2)	(0.7, 0.4, 0.2)
S_3	(0.6, 0.4, 0.3)	(0.8, 0.2, 0.4)	(0.6, 0.3, 0.4)	(0.8, 0.4, 0.2)	(0.8, 0.4, 0.3)
S_4	(0.6, 0.2, 0.4)	(0.6, 0.1, 0.3)	(0.6, 0.5, 0.3)	(0.6, 0.5, 0.4)	(0.6, 0.5, 0.4)
S_5	(0.7, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.7, 0.6, 0.4)	(0.7, 0.2, 0.2)

Step 3: Determination the correlation measure between attributes and alternatives:

Using the proposed cotangent similarity measure ($COT_{NRS}(N,P)$) from equation 1, we form up the values as follows (see the Table 8).

Table 8: The correlation measure between Relation-1 and Relation-2

Refined cotangent similarity measure	D_1	D_2	D_3	D_4	D_5
A_1	0.8155	0.7821	**0.8458**	0.8395	0.7884
A_2	0.8122	0.7965	**0.8123**	0.8368	0.8150
A_3	0.7917	0.7619	**0.8357**	0.8103	0.7949

Step 4: Ranking the alternatives:

The highest correlation measure (see the Table 8) reflects the suitable educational stream selection after higher secondary examination. Therefore, all students A_1, A_2, and A_3 choose the engineering stream.

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6. CONCLUSION

In this paper, we have proposed a refined cotangent similarity measure approach of single valued neutrosophic set and proved some of their basic properties. We have presented an application of cotangent similarity measure of neutrosophic single valued sets in a decision making problem for educational stream selection. The concept presented in this paper can be extended to the other decision making problems involving neutrosophic refined sets.

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