Nonlinear finite element model for transient heat transfer under soil with plastic mulch in agriculture applications

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Abstract

In this paper, transient heat transfer under soil with plastic mulch is modeled using quadratic finite elements in the Galerkin approach. The model is nonlinear because of the radiative heat exchange between the soil surface, the plastic mulch and the atmosphere. Model validation was done experimentally by measuring the temperature of the soil surface in a terrain in the Savanna of Bogotá in Colombia, giving good approximation of the model to the measured data.

Keywords: heat equation, finite element method, nonlinear boundary conditions, plastic mulch, atmospheric radiation

SYMBOLS

τ_l	Longwave mulch transmittance
ρ_l	Longwave mulch reflectivity
τ_s	Solar radiation mulch transmittance
ρ_s	Solar radiation mulch reflectivity
ϵ_m	Mulch emissivity
ϵ_s	Soil surface emissivity
a_s	Soil albedo
k	Soil thermal conductivity [W/m K]
ρc_p	Soil specific heat per volume unit [J/m^3 K]
h_i	Convective heat transfer coefficient between the soil surface and the mulch [W/m^2 K]
h_o	Convective heat transfer coefficient between the mulch and the ambient air [W/m^2 K]
R_s	Direct and diffuse solar shortwave radiation [W/m^2]
T_a	Ambient air temperature [K]
v	Wind velocity [m/s]
T_m	Mulch temperature [K]
T	Soil temperature [K]
q	Heat flux [W/m^2]
σ	Boltzmann constant, 5.67×10^{-8} W/m^2 K^4

1 INTRODUCTION

Plastic mulch has had a great rise among plant growth techniques in the last decades. It consists in covering the soil surface with a plastic film that retains humidity and heat, so soil temperature is elevated and soil properties are improved so a better quality of grown plants is obtained and some unwanted parasites are killed by the high temperatures achieved [1]. In Colombia, plastic mulches are used in the flower's industry among others, but there are not investigations dedicated to develop a general model to compute the temperature distribution in soil in order to determine the plastic mulch's properties needed to specific applications, so it is necessary to develop a model that helps make that choosing without trial-and-error procedures, and that helps visualize the influence of plastic mulch in the soil's thermal response under specific meteorological conditions, with a particular case being the protection of seeds in terrains in the Savanna of Bogotá where there are reached temperatures below zero at some times of the year.

2 METHODOLOGY

Mathematical model

It is considered that the heat absorbed by the soil surface is transmitted entirely to the bottom of the soil [2], [3], so the temperature distribution under soil is governed by the one-dimensional heat equation:

$$\rho c_p \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial z^2} = 0 \qquad (1)$$

The boundary conditions of equation (1) are the net heat flux absorbed at the soil surface and the temperature at the bottom of the soil. The net heat flux absorbed at the soil surface is:

$$-k\frac{\partial T}{\partial z} = h_i(T_m - T) + \frac{\tau_s(1-a_s)}{1-\rho_s a_s}R_s + \frac{\epsilon_s\sigma}{1-\rho_l+\rho_l\epsilon_s}\left[\tau_l\epsilon_{sky}T_a^4 + \epsilon_m T_m^4 - (1-\rho_l)T^4\right] \quad (2)$$

Sky emissivity is calculated by the Swinbank formula [4]:

$$\epsilon_{sky} = 9.2\times 10^{-6}T_a^2 \quad (3)$$

And the internal heat transfer coefficient was found experimentally [5] to be constant in the value $h_i = 7.2\ \mathrm{Wm^{-2}K^{-1}}$. The terms in equation (2) regarding radiation are found using a ray-diagram and geometric series [2]. The bottom of the soil is the depth at which there are no variations of temperature with time, that being a realistic assumption because soil is modeled as a semi-infinite body:

$$T(t, z_\infty) = T_\infty \quad (4)$$

The value of z_∞ has been found by [2] to be 80 cm and by [6] to be 30 cm. In the present work it was considered to be 100 cm for simulations purposes. Equation (2) shows that the temperature of the mulch must be known at each time step, which is achieved by the heat balance on the mulch. Because of the small thickness of the plastic films generally used, it is considered that the mulch does not store energy, so the heat balance is:

$$h_i(T - T_m) + h_o(T_a - T_m) + \left[1 - \rho_s - \frac{\tau_s(1-a_s+\tau_s a_s)}{1-\rho_s a_s}\right]R_s + \left[1 - \rho_l - \frac{\tau_l(\tau_l+\epsilon_s(1-\tau_l))}{1-\rho_l+\rho_l\epsilon_s}\right]\epsilon_{sky}\sigma T_a^4 - \left[2 - \frac{(1-\epsilon_s)(1-\tau_l-\rho_l)}{1-\rho_l+\rho_l\epsilon_s}\right]\epsilon_m\sigma T_m^4 + \left[1 - \frac{\tau_l+\rho_l}{1-\rho_l+\rho_l\epsilon_s}\right]\epsilon_s\sigma T^4 = 0 \quad (5)$$

The external convective heat transfer coefficient was found experimentally [5] to be:

$$h_o = 7.2 + 3.8v \quad (6)$$

Like in equation (2), the terms in equation (3) regarding radiation are found using a ray-diagram and geometric series [2].

Finite element model

In this section the conventions of tensor algebra are followed. Considering a uniform mesh of quadratic elements each one of length h with local coordinates ξ so that $0 < \xi < h$, we have that the solution of equation (1) is approximated on each element by:

$$T^e(\xi, t) \approx \psi_j(\xi)T_j^e(t) \quad (7)$$

Where $j = 1, 2, 3$ and the quadratic interpolation functions are:

$$\psi_1(\xi) = \frac{2}{h^2}(h-\xi)(0.5h-\xi) \quad (8)$$
$$\psi_2(\xi) = \frac{4}{h^2}(h-\xi)\xi \quad (9)$$
$$\psi_3(\xi) = -\frac{2}{h^2}\xi(0.5h-\xi) \quad (10)$$

The weak form of equation (1) is on each element:

$$\rho c_p\frac{\partial T_j^e}{\partial t}\int_0^h \psi_i\psi_j d\xi + kT_j^e\int_0^h \frac{\partial\psi_i}{\partial\xi}\frac{\partial\psi_j}{\partial\xi}d\xi = \psi_i(0)q(0) - \psi_i(h)q(h) \quad (11)$$

Or in explicit matrix form:

$$\frac{\rho c_p}{30}\begin{bmatrix}4 & 2 & -1\\ 2 & 16 & 2\\ -1 & 2 & 4\end{bmatrix}\begin{Bmatrix}\dot{T}_1^e\\ \dot{T}_2^e\\ \dot{T}_3^e\end{Bmatrix} + \frac{k}{3h}\begin{bmatrix}7 & -8 & 1\\ -8 & 16 & -8\\ 1 & -8 & 7\end{bmatrix}\begin{Bmatrix}T_1^e\\ T_2^e\\ T_3^e\end{Bmatrix} = \begin{Bmatrix}q_1\\ 0\\ -q_3\end{Bmatrix} \quad (12)$$

After assembly of the element matrices results the global matrix form of the finite element model:

$$[C]\{\dot{T}\} + [K]\{T\} = \{F\} \quad (13)$$

Approximating the time derivative by a backward finite-differences model, we have:

$$[\hat{K}]\{T\}^{n+1} = \{\hat{F}\}^{n+1} \quad (14)$$

Where $[\hat{K}] = [C] + \Delta t[K]$ and $\{\hat{F}\}^{n+1} = [C]\{T\}^n + \Delta t\{F\}^{n+1}$, and the superscript represents the time step. Equation (14) is a nonlinear algebraic system of equations because of the 4th power of the temperature terms in the boundary conditions. The system is solved by the fixed point iteration [8]:

$$\{T\}_{p+1}^{n+1} = [\hat{K}]^{-1}\{\hat{F}\}_p^{n+1} \quad (15)$$

Additionally, equation (5) is also solved by fixed point iteration in order to know the mulch temperature and have a complete solution of all of the variables involved. The solution was written in an algorithm in MATLAB that reads the input data imported from the weather station and fills and solves the matrices in equations (13) and (15).

Experimental validation

A mulch of low density polyethylene (LDPE) was constructed in a terrain at the Savanna of Bogotá. The mulch optical properties and the soil thermal properties were characterized and then meteorological data and soil surface temperature were measured simultaneously using the portable automated weather station Casella NOMAD and an OMEGA 871A NiCr-NiAl thermocouple.

- *Mulch and soil characterization*

The spectrophotometer NICOLET 380 FTIR was used to characterize the used LDPE properties to longwave radiation and the spectrophotometer Perkin Elmer Lambda 3 UV/VIS was used to characterize the used LDPE mulch properties to solar shortwave radiation. According to Wien's Displacement Law, the infrared wavelengths of interest in this case are in the range of 8700 nm to 10600 nm, also, the solar radiation sensor of the weather station measures in the range of 400 nm to 1100 nm. The optical properties of the used LDPE in those ranges were found by numerical integration and are shown in Table 1.

Table 1. Measured optical properties of the LDPE mulch.

Property	Value
τ_l	0.6
ρ_l	0.398
τ_s	0.733
ρ_s	0.265

The soil thermal properties were calculated using a 75 W electric resistance and measuring the variation of temperature in a small sample of soil in a range of 10 min. The estimated soil thermal properties are shown in Table 2.

Table 2. Estimated soil thermal properties.

Property	Value
k	2.2 W/mK
ρc_p	1.01×10^6 J/m^3

3 RESULTS

Data were measured each 5 min between 09:45 and 14:40 of December 5th of 2008. The meteorological input data measured by the automated weather station is shown in Figures 1 to 3.

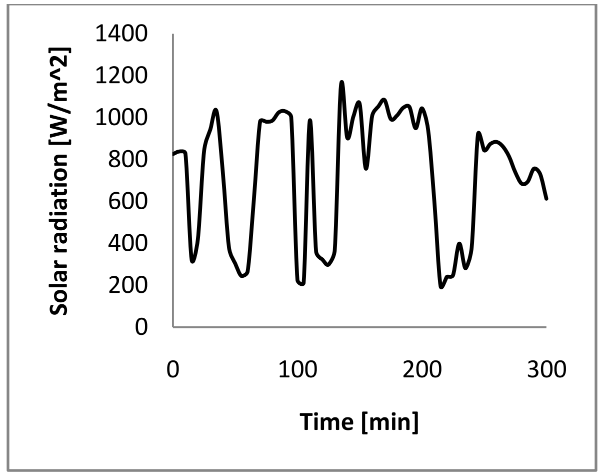

Figure 1. Solar radiation.

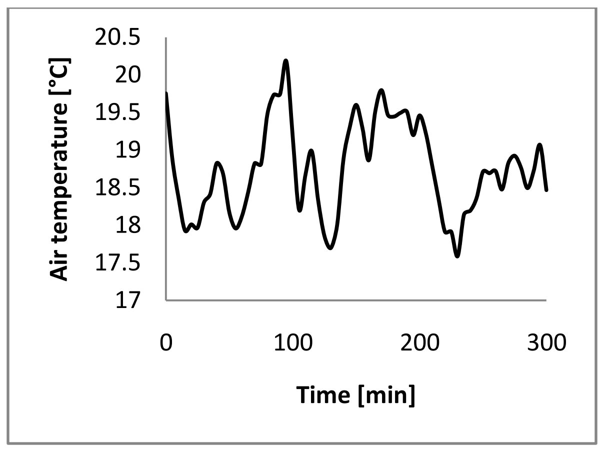

Figure 2. Air temperature.

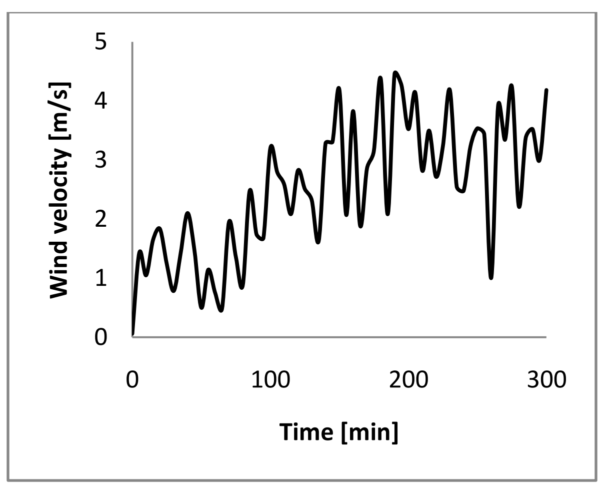

Figure 3. Wind velocity.

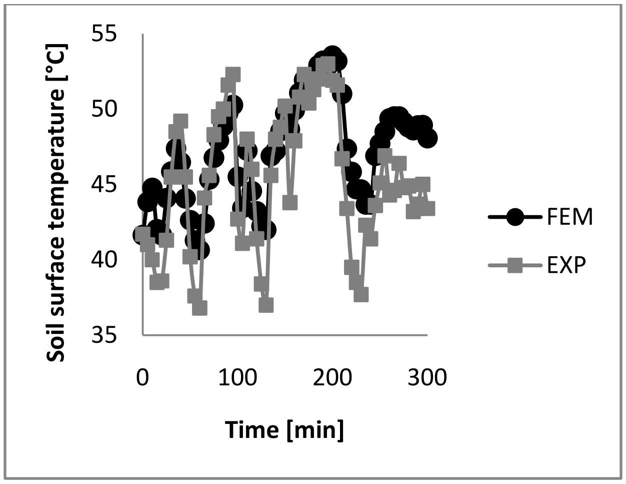

Figure 4. Comparison between measured (EXP) and simulated (FEM) soil surface temperature.

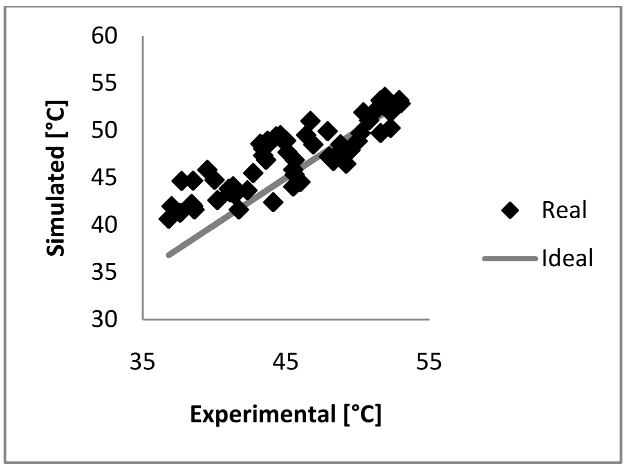

Figure 5. Correlation between experimental and simulated temperatures.

Simulated temperature was calculated using 100 elements. It can be seen from Figure 4 that the simulated temperature responds to input variations in the same manner that measured data does. Figure 5 shows the correlation between simulated and experimental temperature, and it shows that there exists good agreement of the model and the measured values. It is worth noting that the weather station gives a 5 min average of meteorological data, so it is possible that the errors seen when large changes of solar radiation occur are due to the way data is acquired and not because of the model, that is better seen in Figure 4 after 200 min of experimentation, where rapid changes of solar radiation were seen due to cloud movement. The mean relative error between measured and simulated soil surface temperature was 6.03%. Figure 6 shows the variation of temperature at different depths. In this case, the temperature variation ceased at a depth of 70 cm, which is closest to the result found by [2].

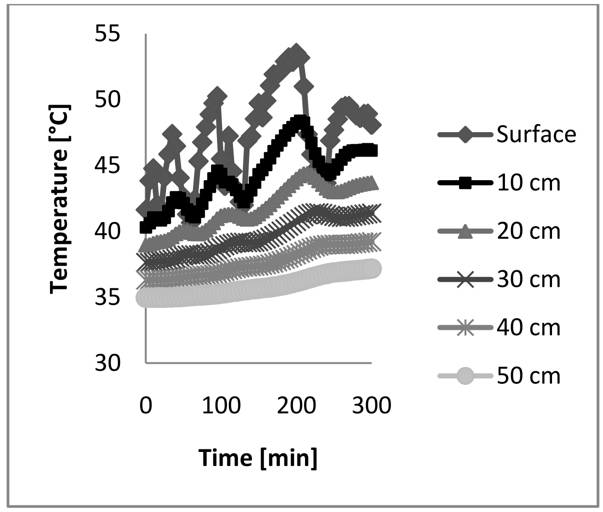

Figure 6. Simulated temperatures at different depths.

4 CONCLUSIONS

The one-dimensional nonlinear finite element model developed has good agreement with experimental data, so it is inferred that it is useful to predictions of temperature of soil with different kinds of plastic mulches, and will help to develop curves of mulch selection to specific applications. The future work will be pointed to develop non-dimensional curves to help select mulches to specific applications; another work will be done to analyze possible solutions regarding plastic mulches to protect the flower seeds when there are freezing temperatures at the Savanna of Bogotá.

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