Conjecture that states that any Carmichael number is a cm-composite

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Abstract. In two of my previous papers I defined the notions of c-prime respectively m-prime. In this paper I will define the notion of cm-prime and the notions of c-composite, m-composite and cm-composite and I will conjecture that any Carmichael number is a cm-composite.

Introduction:

Though, as I mentioned in Abstract, I already defined the notions of c-prime and m-prime in previous papers, in order to be, this paper, self-contained, I shall define them here too.

Definition 1:

We name a c-prime a positive odd integer which is either prime either semiprime of the form \( p(1)q(1) \), \( p(1) < q(1) \), with the property that the number \( q(1) - p(1) + 1 \) is either prime either semiprime \( p(2)q(2) \) with the property that the number \( q(2) - p(2) + 1 \) is either prime or semiprime with the property showed above... (until, eventually, is obtained a prime).

Example: 4979 is a c-prime because 4979 = 13*383, where 383 - 13 + 1 = 371 = 7*53, where 53 - 7 + 1 = 47, a prime.

Definition 2:

We name a m-prime a positive odd integer which is either prime either semiprime of the form \( p(1)q(1) \), with the property that the number \( p(1)q(1) - 1 \) is either prime either semiprime \( p(2)q(2) \) with the property that the number \( p(2)q(2) - 1 \) is either prime or semiprime with the property showed above... (until, eventually, is obtained a prime).

Example: 5411 is a m-prime because 5411 = 7*773, where 7 + 773 - 1 = 779 = 19*41, where 19 + 41 - 1 = 59, a prime.
Definition 3:

We name a cm-prime a number which is both c-prime and m-prime.

Definition 4:

We name a c-composite the composite number with three or more prime factors $n = p(1)*p(2)*...*p(m)$, where $p(1)$, $p(2)$, ..., $p(m)$ are the prime factors of $n$, which has the following property: there exist $p(k)$ and $p(h)$, where $p(k)$ is the product of some distinct prime factors of $n$ and $p(h)$ the product of the other distinct prime factors such that the number $p(k) - p(h) + 1$ is a c-prime.

Definition 5:

We name a m-composite the composite number with three or more prime factors $n = p(1)*p(2)*...*p(m)$, where $p(1)$, $p(2)$, ..., $p(m)$ are the prime factors of $n$, which has the following property: there exist $p(k)$ and $p(h)$, where $p(k)$ is the product of some distinct prime factors of $n$ and $p(h)$ the product of the other distinct prime factors such that the number $p(k) + p(h) - 1$ is a m-prime.

Definition 6:

We name a cm-composite a number which is both c-composite and m-composite.

Note: We will consider the number 1 to be a prime in the six definitions from above; we will not discuss the controverted nature of number 1, just not to repeat in definitions “a prime or number 1”.

Conjecture: Any Carmichael number is a cm-composite.

Verifying the conjecture(for the first 11 Carmichael numbers):

For $561 = 3*11*17$ we have:
: the number $3*17 - 11 + 1 = 41$, a prime;
: the number $3*17 + 11 - 1 = 61$, a prime.

For $1105 = 5*13*17$ we have:
: the number $5*17 - 13 + 1 = 73$, a prime;
: the number $5*17 + 13 - 1 = 97$, a prime.

For $1729 = 7*13*19$ we have:
: the number $7*13 - 19 + 1 = 73$, a prime;
: the number $7*13 + 19 - 1 = 109$, a prime.

For $2465 = 5*17*29$ we have:
: the number $5*17 - 29 + 1 = 57 = 3*19$, a c-prime because $19 - 3 + 1 = 17$, a prime;
the number 5*17 + 29 - 1 = 113, a prime.

For 2821 = 7*13*31 we have:
: the number 7*31 - 13 + 1 = 205 = 5*41, a c-prime
because 41 - 5 + 1 = 37, a prime;
: the number 7*31 + 13 - 1 = 229, a prime.

For 6601 = 7*23*41 we have:
: the number 23*41 - 7 + 1 = 937, a prime;
: the number 23*41 + 7 - 1 = 949 = 13*73, a m-prime
because 13 + 73 - 1 = 85 = 5*17 and 5 + 17 - 1 = 21
= 3*7 and 3 + 7 - 1 = 9 = 3*3 and 3 + 3 - 1 = 5, a prime.

For 8911 = 7*19*67 we have:
: the number 7*19 - 67 + 1 = 67, a prime;
: the number 7*19 + 67 - 1 = 199, a prime.

For 10585 = 5*29*73 we have:
: the number 5*29 - 73 + 1 = 73, a prime;
: the number 5*29 + 73 - 1 = 217 = 7*31, a m-prime
because 7 + 31 - 1 = 37, a prime.

For 15841 = 7*31*73 we have:
: the number 7*31 - 73 + 1 = 145 = 5*29, a c-prime
because 29 - 5 + 1 = 25 and 5 - 5 + 1 = 1;
: the number 7*31 + 73 - 1 = 289, a m-prime because 17
+ 17 - 1 = 33 = 3*11 and 3 + 11 - 1 = 13, a prime.

For 29341 = 13*37*61 we have:
: the number 13*37 - 61 + 1 = 421, a prime;
: the number 13*37 + 61 - 1 = 541, a prime.

For 41041 = 7*11*13*41 we have:
: the number 11*41 - 7*13 + 1 = 361, a c-prime because 19 - 19 + 1 = 1;
: the number 11*41 + 7*13 - 1 = 541, a prime.