Formula involving primorials that produces from any prime \( p \) probably an infinity of semiprimes \( qr \) such that \( r-q+1=np \)

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Abstract. In this paper I make a conjecture involving primorials which states that from any odd prime \( p \) can be obtained, through a certain formula, an infinity of semiprimes \( qr \) such that \( r + q - 1 = np \), where \( n \) non-null positive integer.

Conjecture:

For any odd prime \( p \) there exist an infinity of positive integers \( m \) such that \( p + m*\pi = q*r \), where \( \pi \) is the product of all primes less than \( p \) and \( q, r \) are primes such that \( r - q = n*p \), where \( n \) is non-null positive integer.

Note that, for \( p = 3 \), the conjecture states that there exist an infinity of positive integers \( m \) such that \( 3 + 2*m = q*r \), where \( q \) and \( r \) primes and \( r - q = n*p \), where \( n \) is non-null positive integer; for \( p = 5 \), the conjecture states that there exist an infinity of positive integers \( m \) such that \( 5 + 6*m = q*r \) (...); for \( p = 7 \), the conjecture states that there exist an infinity of positive integers \( m \) such that \( 7 + 30*m = q*r \) (...); for \( p = 11 \), the conjecture states that there exist an infinity of positive integers \( m \) such that \( 11 + 210*m = q*r \) (...) etc.

Note also that \( m \) can be or not divisible by \( p \).

Examples:

For \( p = 3 \) we have the following relations:

\[
\begin{align*}
3 + 2*11 &= 25 = 5*5, \text{ where } 5 + 5 - 1 = 9 = 3*3; \\
3 + 2*18 &= 39 = 3*13, \text{ where } 3 + 13 - 1 = 15 = 3*5;
\end{align*}
\]

The sequence of \( m \) is: 11, 18 (...). Note that \( m \) can be or not divisible by \( p \).

For \( p = 5 \) we have the following relations:

\[
\begin{align*}
5 + 6*25 &= 155 = 5*31, \text{ where } 5 + 31 - 1 = 35 = 7*5; \\
5 + 6*33 &= 203 = 7*29, \text{ where } 7 + 29 - 1 = 35 = 7*5;
\end{align*}
\]

The sequence of \( m \) is: 25, 33 (...)

For $p = 7$ we have the following relations:
\[ 7 + 30 \cdot 34 = 1027 = 13 \cdot 79, \text{ where } 13 + 79 - 1 = 91 = 7 \cdot 13; \]
\[ 7 + 30 \cdot 49 = 1477 = 7 \cdot 211, \text{ where } 7 + 211 - 1 = 217 = 7 \cdot 31. \]
The sequence of $m$ is: 34, 49 (…)

For $p = 13$ we have the following relations:
\[ 13 + 2310 \cdot 5 = 11563 = 31 \cdot 373, \text{ where } 31 + 373 - 1 = 403 = 31 \cdot 13; \]
\[ 13 + 2310 \cdot 17 = 39283 = 163 \cdot 241, \text{ where } 163 + 241 - 1 = 403 = 31 \cdot 13. \]
The sequence of $m$ is: 5, 17 (…)

For $p = 17$ we have the following relation:
\[ 17 + 30030 \cdot 4 = 120137 = 19 \cdot 6323, \text{ where } 19 + 6323 - 1 = 6341 = 373 \cdot 17. \]
The sequence of $m$ is: 4 (…)