Abstract

A formula for the mass of the electron is derived from a lepton mass factor, a Koidean ratio for quarks and a dimensionless factor based on the fine-structure constant. If this formula were correct it would prove a profound relationship between the masses of leptons and quarks.

Keywords: lepton, quark, electron, muon, tau particle (tauon), up quark, down quark, strange quark, charm quark, bottom quark, top quark, generations of matter, fine-structure constant, Koide ratio, NIST.

1. Nomenclature

The following are the symbols for the constants I shall use in this paper

\[ m_e = \text{electron rest mass} \]
\[ m_\mu = \text{muon rest mass} \]
\[ m_\tau = \text{tau particle rest mass} \]
\[ m_{up} = \text{up quark nude mass} \]
\[ m_{down} = \text{down quark nude mass} \]
\[ m_{strange} = \text{strange quark nude mass} \]
\[ m_{charm} = \text{charm quark nude mass} \]
\[ m_{bottom} = \text{bottom quark nude mass} \]
\[ m_{top} = \text{top quark nude mass} \]
\[ R_{\text{quarks}} = \text{Koidean ratio for quarks} \]
\[ \alpha = \text{fine-structure constant (atomic structure constant)} \]
\[ f = \text{dimensionless factor} \]
\[ F_{\text{JMeV}} = 1.602176564 \times 10^{-13} \text{J/MeV} \text{ (conversion factor)} \]

2. The Formula for the Mass of the Electron in Terms of the Masses of Leptons and Quarks and the Fine Structure Constant

Let us define the Koide formula for quarks which is similar to the Koide formula for leptons

\[ R_{\text{quarks}} \equiv \frac{m_{up} + m_{down} + m_{strange} + m_{charm} + m_{bottom} + m_{top}}{\left( \sqrt{m_{up}} + \sqrt{m_{down}} + \sqrt{m_{strange}} + \sqrt{m_{charm}} + \sqrt{m_{bottom}} + \sqrt{m_{top}} \right)^2} \] (2.1)
We shall also define a dimensionless factor, $f$, as

$$f \equiv 1 + \frac{\alpha}{\pi} - \frac{\alpha}{\pi^2} + \frac{\alpha}{\pi^3} - \frac{\alpha}{\pi^4} + \frac{\alpha}{\pi^5} - \frac{\alpha}{\pi^6} + \frac{\alpha}{\pi^7} - \frac{\alpha}{\pi^8} + \frac{\alpha}{\pi^9} - \frac{\alpha}{\pi^{10}} + \frac{\alpha}{\pi^{11}} - \frac{\alpha}{\pi^{12}} + \frac{\alpha}{\pi^{13}} - \frac{\alpha}{\pi^{14}} \quad (2.2)$$

The following is the formula for the mass of the electron I discovered in 2012:

$$m_e \approx \left( \frac{m_{\mu}^2}{\pi^3 m_e} \right) \left( \frac{f}{R_{\text{quarks}}^2} \right) \quad (2.3)$$

The formula relates the masses of leptons with the masses of quarks. Using equation (2.2) we can expand equation (2.3). This expansion gives

$$m_e \approx \left( 1 + \frac{\alpha}{\pi} - \frac{\alpha}{\pi^2} + \frac{\alpha}{\pi^3} - \frac{\alpha}{\pi^4} + \frac{\alpha}{\pi^5} - \frac{\alpha}{\pi^6} + \frac{\alpha}{\pi^7} - \frac{\alpha}{\pi^8} + \frac{\alpha}{\pi^9} - \frac{\alpha}{\pi^{10}} + \frac{\alpha}{\pi^{11}} - \frac{\alpha}{\pi^{12}} + \frac{\alpha}{\pi^{13}} - \frac{\alpha}{\pi^{14}} \right) \left( \frac{m_{\mu}^2}{\pi^3 m_e} \right) \left( \frac{1}{R_{\text{quarks}}^2} \right) \quad (2.4)$$

We can also expressed the above formula in a more compact form as follows

$$m_e \approx \left( 1 + \sum_{i=1}^{14} (-1)^{i+1} \frac{\alpha}{\pi^i} \right) \left( \frac{m_{\mu}^2}{\pi^3 m_e} \right) \left( \frac{1}{R_{\text{quarks}}^2} \right) \quad (2.5)$$

The masses of the quarks: up, down and strange were taken from a web page published by phys.org [1] and based on the research of professor G. P. Lepage et al. [2]

“According to their results, the up quark weighs approximately 2 mega electron volts (MeV), which is a unit of energy, the down quark weighs approximately 4.8 MeV, and the strange quark weighs in at about 92 MeV.”

I have calibrated the values of the masses of the charm, bottom and top quarks to produce the correct value of the mass of the electron. These three calibrated values are marked with an asterisk on table 1 and they fall within the mass range published by Wikipedia. Table 1 shows the quarks masses we adopted in this paper. The reader will surely realize that different calibration values could have been adopted instead of the values proposed by the author. However, only more accurate calculations of all quark masses will indicate the accuracy of the above formula.
<table>
<thead>
<tr>
<th>Quark Name</th>
<th>Symbol (Particle)</th>
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<th>Mass Range (MeV/c²) (source Wikipedia)</th>
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<td>u</td>
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<td>1237(*)</td>
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<tr>
<td>bottom</td>
<td>b</td>
<td>b̄</td>
<td>4,130 - 4,670</td>
<td>4670 (*)</td>
<td>[4]</td>
</tr>
<tr>
<td>top</td>
<td>t</td>
<td>t̄</td>
<td>169,100 - 173,300</td>
<td>171987.5 (*)</td>
<td>[5][6][7]</td>
</tr>
</tbody>
</table>

Table 1: The masses of quarks.

3. The Predicted Mass of the Electron

Let us calculate the value of the mass of the electron predicted by the formula (2.5). Let us start with the dimensionless factor, \( f \)

\[
f = 1.00176196752951
\]

The quark Koidean ratio is

\[
R_{\text{quarks}} = \frac{\left(2 + 4.8 + 92 + 1237 + 4670 + 171987.5\right)\text{MeV/c}^2}{\left(\sqrt{2} + \sqrt{4.8} + \sqrt{92} + \sqrt{1237} + \sqrt{4670} + \sqrt{171987.5}\right)^2\text{MeV/c}^2}
\]

\[
R_{\text{quarks}} = 0.630 \pm 247.316
\]

The inverse of the square of the quark Koidean ratio is

\[
\frac{1}{R_{\text{quarks}}^2} = 2.517 \pm 333.646
\]

According to NIST 2010, the values of the masses of the muon (second generation of matter) and tau particle (third generation of matter) are

\[
m_\tau = 3.16747(29) \times 10^{-27} \text{ Kg} \quad \text{(NIST 2010)}
\]

\[
m_\mu = 1.883531475(96) \times 10^{-28} \text{ Kg} \quad \text{(NIST 2010)}
\]

We shall define the value for the muon square mass over the tauon mass times \( \pi^3 \) as

\[
m_{\text{lepton}} \equiv \left(\frac{m_\mu^2}{\pi^3 m_\tau}\right) = \frac{\left(1.883531475(96) \times 10^{-28} \text{ Kg}\right)^2}{\pi^3 \times 3.16747 \times 10^{-27} \text{ Kg}} = 3.61229854 \times 10^{-31} \text{ Kg}
\]
Thus, the value of the predicted mass of the electron turns out to be

\[ m_e = 1.00176196753 \times 3.61229854 \times 10^{-31} \times 2.517333646 \text{ Kg} \]

\[ m_e = 9.109 \ 382 \ 859 \times 10^{-31} \text{ Kg} \]

4. The Measured Mass of the Electron

According to NIST [8], the measured value for the rest mass of the electron is

\[ m_{e_{\text{NIST 2010}}} = 9.109 \ 382 \ 91(40) \times 10^{-31} \text{ Kg} \quad \text{(NIST 2010)} \]

It is customary to express the mass of a particle in \( \text{MeV}/c^2 \). Thus we use the conversion factor, \( F_{\text{JMeV}} \), between Joules and MeV

\[ F_{\text{JMeV}} = 1.602 \ 176 \ 564 \times 10^{-13} \frac{J}{\text{MeV}} \]

This yields

\[ m_{e_{\text{exp}}} \frac{c^2}{F_{\text{JMeV}}} = \frac{9.109 \ 382 \ 91 \times 10^{-31} \times (299 \ 792 \ 458)^2 J}{1.602 \ 176 \ 564 \times 10^{-13} J/\text{MeV}} = 0.510 \ 998 \ 927 \ 9 \text{ MeV} \]

\[ m_{e_{\text{exp}}} = 0.510 \ 998 \ 927 \ 9 \text{ MeV}/c^2 \approx 0.511 \text{ MeV}/c^2 \]

5. Conclusions

When we calibrate the masses of the charm, the bottom and the top quark, the formula presented in this paper yields the value of the electron mass very accurately. Because all the masses used in this calculation fall within the mass range given in Table 1, it is reasonable to think that the formula is capable of yielding a very accurate value for the electron mass should we know the exact values of the quark masses. The reason of performing a calibration is due to the fact that we do not know the exact values of the naked quark masses. This hole in our knowledge tell us that a more refined calculation of the quark masses is needed to determine the degree of accuracy and validity of the above formula.
REFERENCES