Quantized Capacitance and Energy of the Atom and Photon

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Abstract

By modeling both the atom and the photon as capacitors, the correct energy levels are easily produced via extrapolation from Maxwell's, Gauss', Coulomb's and Ohm's laws – without the need to inject Planck's constant into the equation ad-hoc. In the case of the photon, Einstein's photoelectric equation is formulated as a result, with Planck's constant consequently occurring as an aggregate of fundamental constants. Analysis of these equations lends credence to Planck's fervent and controversial personal dogma that the constant which he himself had discovered is nothing but "a mathematical trick[1]". Further analysis shows that this model reconciles the wave-particle duality; wherein the wave properties of light and matter produce the particle-like aspects as a result of the laws of electrical engineering in conjunction with the uncertainty principle and Schrödinger's wave equations.

Keywords: Wave-Particle Duality, Hydrogen, Ground State, Photoelectric Equation, Quantum Optics, Planck's Constant, Fine Structure Constant, Quantum Mechanics, Photon, Capacitor, Energy, QED, Quantum Transition, Electrodynamics, Electrical Engineering

Capacitance of the Atom

Modeling the atom as a spherical capacitor[Fig. (1)] seems logical at face value. Capacitors store potential energy. Atoms store potential energy. They both have positive and negative charge differentials separated by a distance. They both have a dielectric – which in the case of an atom, the dielectric is the vacuum and has the permittivity of free space[2].

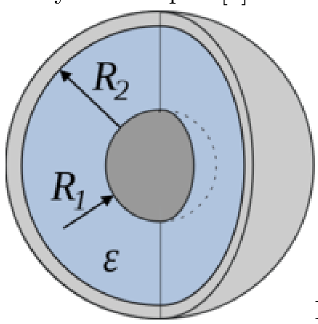

Fig. (1)

However, modeling the atom as an ideal isolated conducting sphere[Fig. (2)] makes even more sense, because at an intrinsic level, the shell that forms the sphere represents the radius of the ground state of the Bohr hydrogen model, which is also the radius of highest density of the ground state electron cloud[3]. When viewing the cloud at its thickest probability density, the electron forms a shell around the proton due to its wavefunction and inherent uncertainty in position. When the electron is interacted with, the wave of potential collapses and the electron is forced to further localize to a position in accordance with the probabilities dictated by the electron cloud formed by said wavefunction.

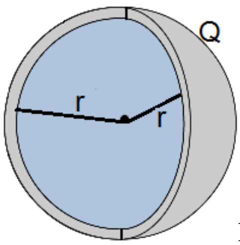

Fig. (2)

However, physical similarities aside – when the ground state hydrogen atom is modeled as an ideal isolated conducting sphere[Fig. (2)], the exact energy contained in the ground state can easily be calculated by knowing the radius. To do that, we shall start with the standard equation for the capacitance of an ideal isolated conducting sphere[4]:

$$C = 4\pi\varepsilon_0 r \tag{1}$$

Here, r is effectively the radius. The radius of ground state hydrogen, a_0, will be plugged in for r to calculate the capacitance of the atom:

$$C = 4\pi\varepsilon_0 a_0 \tag{2}$$

Energy of the Atom

A standard equation to express the potential energy of a capacitor in terms of its capacitance is [5]:

$$E = \frac{Q^2}{2C} \tag{3}$$

The proton has a charge of positive e and the electron has a charge of negative e[6]. In Eq. (3), Q represents the charge differential, so the charge differential between the proton and the electron in this case is the elementary charge, e. Inserting Eq. (2) into Eq. (3), and plugging e in for Q yields:

$$E = \frac{e^2}{8\pi\varepsilon_0 a_0} = Ry \tag{4}$$

E is exactly equal to the energy of the ground state hydrogen atom (also known as the Rydberg energy), symbolized Ry. The equation $Ry = \frac{e^2}{8\pi\varepsilon_0 a_0}$ is commonly known[7], but to the knowledge of the author, has never been formulated through the laws of capacitance prior to this document. A ground state hydrogen atom matches the physical characteristics of an ideal isolated spherical capacitor. The equations perfectly translate between the energy and the ground state radius. By inserting a factor of n for the principal quantum number in the capacitance equation $C = 4\pi\varepsilon_0 a_o n^2$ to account for the accurate size of the Bohr radius, the correct energy calculation for all electron orbitals is produced by the equation: $E_n = \frac{e^2}{8\pi\varepsilon_0 a_0 n^2}$.

Capacitance of the Photon

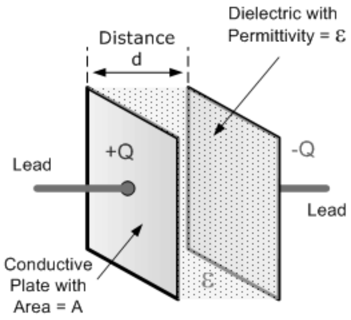

Fig. (3)

Next, the photon shall be modeled as a parallel plate capacitor[Fig. (3)]. Undoubtedly, the photon has no net charge, but as it oscillates harmonically between positive and negative electric fields[8][Fig. (4)], it can be thought to have a virtual charge equal to the elementary charge, which also oscillates between positive and negative states in a sinusoidal fashion[9], matched in phase with the wave of the electric field. The photon also displays another characteristic inherent to capacitors – it is essentially connected to positive and negative terminals; the atom emitting energy (positive lead) and the atom absorbing energy (negative lead). Despite these two similarities, it may seem illogical that this fundamental aspect of nature could or should be compared to a plate capacitor. At face value, a photon does not seem to exhibit any of the characteristics of a set of plates. This misunderstanding is because the photon is usually visualized from the frame of reference of an outside observer, where it propagates through both time and space. It is necessary to visualize what the photon itself experiences, not how the outside observer experiences the photon.

From the frame of reference of the photon, many things change, and the system can then be examined electrostatically. For instance, the photon is traveling at the speed of light, therefore because of the implications of special relativity and the Lorentz transformation, the photon does not experience time[10]. Because there is no time from the photon's frame of reference, this means the photon experiences both the positive and negative states simultaneously. This fits the first aspect of a plate capacitor; having a charge differential.

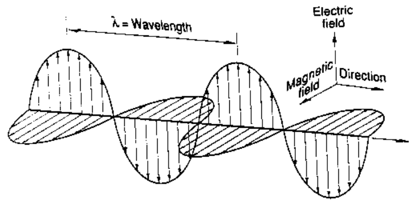

Fig. (4)

Another misconception is that a photon is a one dimensional particle traveling in a line. Fig. (4) Is a horrible representation of reality, although it does display some of the physical attributes and is an easy model for the Luddite stuck in the Newtonian regime to digest. The uncertainty principle tells us the point-particle model is not accurate; that a particle has neither a perfectly defined history nor a perfectly defined location[11]. Single slit Fraunhofer diffraction shows that a photon does indeed have a width, as when the slit is closed tighter than one wavelength, the wave interferes with itself, and produces strange but mathematically logical results[12]. As the uncertainty in position is decreased, the uncertainty in momentum is increased[11]. Therefore, the photon not only has a wavelength, but it also has a width. It also has a distance between the positive and negative components[Fig. (4)]. The distance between the positive and negative charge peaks of the sine wave is half a wavelength[Fig. (4)]. Particles are essentially smeared out via the uncertainty principle to form gradiently fuzzy plates. An apt analogy to describe how the plates gain their third dimension – the distance of separation between them – would be the worldsheets encountered in string theory, in which a one-dimensional string propagates through time, forming a two-dimensional sheet[Fig. (5)].

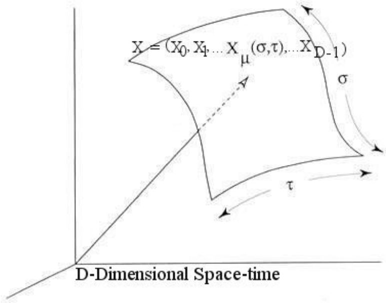

Fig. (5)

When the photon is viewed as frozen in time, it does display the characteristics of a plate capacitor; two oppositely charged surfaces both with a length, a width, and separated by a distance. It must be noted that the photon is positive for half of its wavelength, and negative for half of its wavelength[Fig. (4)], so the length of each plate is half a wavelength. It must also be noted that the photon does not become even semi-localized until it interacts with a particle[13]. From emission, it travels through spacetime as nothing more than a superposition of possibilities, propagating according to Schrödinger's wave equation[14] until the wavefunction collapses upon absorption; the stage wherein these three-dimensional properties take shape. Knowing the dimensions of length, width, separating distance, and that the dielectric in this case is spacetime itself[15], with a vacuum permittivity of ε_0, we can calculate the capacitance of the photon using the standard equation for an ideal parallel plate capacitor[16].

$$C = \frac{\varepsilon_0 A}{D} \tag{5}$$

Plugging in $\frac{1}{2}\lambda$ for length, λ for width, and $\frac{1}{2}\lambda$ for distance, and then canceling, we produce:

$$C = \varepsilon_0 \lambda \tag{6}$$

Energy of the Photon

As was shown in Eq. (3), the standard equation for calculating the potential energy stored in an ideal capacitor in terms of capacitance is[5]:

$$E = \frac{Q^2}{2C}$$

Again, the charge differential (Q) is the elementary charge (e), which is the maximum virtual-charge of the photon during its sinusoidal peaks. It must be emphasized that the photon has no measurable properties until it interacts with another particle[13]. We are not trying to solve for the energy of a photon in free space. Such a photon cannot be measured, therefore, according to the Copenhagen Interpretation, it does not truly exist except as a superset of possibilities[17]. We are solving for the energy of a photon that is interacting with an electron – the wavefunction-collapsed photon – through the action of electromagnetic coupling[18]. Therefore we must take into account the strength of coupling – the electromagnetic coupling constant which denotes the strength of the interaction between light and matter, also known as the fine structure constant,

α[18]. We are solving for the energy of the coupled photon, so we must place α into the equation to solve for the coupled energy.

Inserting Eq. (6) into Eq. (3), placing α on the energy side of the equation, and plugging in e for the charge Q, produces:

$$E\alpha = \frac{e^2}{2\varepsilon_0 \lambda} \tag{7}$$

It would suffice to just solve for E to calculate the correct energy levels for the entire electromagnetic spectrum, but something very interesting happens when we put this equation in terms of frequency, which is possible because $f = \frac{c}{\lambda}$[19].

Solving for E, and converting wavelength to frequency by substituting $\lambda = \frac{c}{f}$, it produces:

$$E = \left[\frac{e^2}{2\varepsilon_0 c\alpha}\right] f \tag{8}$$

The terms in the brackets equals Planck's constant, h. The equation is equivalent to the photoelectric equation; $E = hf$.

The Rydberg Constant

When analyzing the energy of the photon formulated via capacitance in contrast to the energy of the atom calculated through capacitance, further information is gleaned. In Eq. (4) the Rydberg energy was produced when calculating the ground state energy of the hydrogen atom. The Rydberg energy, Ry, is directly related to the Rydberg constant, R_∞, by a factor of hc in the equation[20]:

$$Ry = R_\infty hc \tag{9}$$

The Rydberg constant, R_∞, is the wavenumber related to the spectral emissions of hydrogen, giving the highest possible wavenumber for a photon that can be emitted by hydrogen[21]. The wavenumber is simply the inverse of a wavelength. In other words, R_∞ is the inverse of the photon wavelength that would be emitted by a hydrogen atom if the atom was at a maximum pre-ionization energy and transitioned in one fell swoop down to the ground state. Let us analyze Eq. (9) by inserting the fundamental form of h which was formulated in Eq. (8), giving us:

$$Ry = R_\infty c \left[\frac{e^2}{2\varepsilon_0 c\alpha}\right] \tag{10}$$

Because R_∞ is an inverse wavelength, we can convert it to a real wavelength by inverting it, which places it in the numerator. To signify that it is now indeed a real wavelength, the inverse of R_∞ shall be renamed λ_R. Doing so and simultaneously canceling the c's yields:

$$Ry = \frac{e^2}{2\varepsilon_0 \lambda_R \alpha} \tag{11}$$

This equation should look familiar. Because Ry is simply an energy, and λ_R is just a wavelength, Eq. (11) is exactly the same as Eq. (7) would be after solving for E, which incidently produces:

$$E = \frac{e^2}{2\varepsilon_0 \lambda\alpha} \tag{12}$$

Eq. (11) and (12) are saying the exact same thing – that the energy in the photon comes from the photon's capacitance which is determined by the wavelength. This is interesting, because the Rydberg energy is both the rest energy of the ground state hydrogen atom (as per the relationship $E_n = -Ry(\frac{1}{n})^2$ for the rest energy[22]) as well as the highest possible energy photon able to be emitted by said atom (as per the relationship noted in the equation $\Delta E = Ry(\frac{1}{n_f^2} - \frac{1}{n_i^2})$[22]). Eq (8) through (15) show this relationship. This author proposes that because Planck's constant is not truly fundamental, the best expression to describe $Ry = R_\infty hc$ is actually:

$$Ry = \frac{R_\infty e^2}{2\varepsilon_0 \alpha} \tag{13}$$

The von Klitzing Constant and Novel Related Quanta

It is worth noting that the *quantum of resistance* (called the von Klitzing constant) which is observed in the quantum Hall effect is simply our formula for Planck's constant ($h = \frac{e^2}{2\varepsilon_0 c\alpha}$) but without the charge (e^2), yielding $R_K = \frac{1}{2\varepsilon_0 c\alpha}$[23], and turning Planck's quantum of *action* into a quantum of *resistance*, whilst sharing the same formulaic structure. If we take Eq. (6), which states $C = \varepsilon_0\lambda$ and by using the relation $c = \lambda f$ to convert λ to $\frac{c}{f}$ – and also insert the strength of electromagnetic coupling, it yields:

$$C = \frac{[\varepsilon_0 c\alpha]}{f} \tag{14}$$

The term in the brackets can be held as the *quantum of capacitance*, which is related to voltage, because of the equation $V = \frac{Q}{C}$, and because – once again – the elementary charge is the charge in question. This can be expressed:

$$V = \left[\frac{e}{\varepsilon_0 c\alpha}\right] f \tag{15}$$

The terms in the brackets can be held as the *quantum of voltage*. It is apparent that quanta of capacitance, voltage, action, light, and resistance are all fundamentally related through the laws of electrical engineering.

Conclusion

That the correct energy levels for *both* the atom and the photon can be calculated via modeling them as capacitors seems beyond the realm of coincidence. Applying the analysis that both of these fundamental devices store and release energy, so they *should* be modeled as capacitors lends credence to this supposition. Nature is written with mathematics. Math is all nature knows[24]. When equations work out to produce accurate predictions, they describe something about nature. It is up to us to sit down, listen, and try to figure out what the math is saying; to decipher what we learn and put it into the syntax of human language. These equations seem to be saying that both the atom and the photon are forms of capacitors.

When physics equations accurately describe nature, it is never purely coincidence. This is the reason why string theory took off in 1970[25]; when the equations predicted accurate results by modeling particles as reverberations on a string. This case is no different.

In this regime it becomes clear what gives rise to Planck's constant in the famous photoelectric equation. As Planck ardently argued[1], it is not a fundamental property of nature – it is an aggregate constant, birthed by the relationship of capacitance and energy and the uncertain location of the photon, in conjunction with relativity, the permittivity of free space, and the strength of electromagnetic coupling. This reconciles the wave-particle duality[26], as now we can see that the energy of the photon is not caused by its frequency, or its *particle nature*; it is simply caused by its capacitance, and its corresponding voltage ($V = \frac{Q}{C}$[27], where Q is the elementary charge) which is determined by its wavelength. The smaller the wavelength, the smaller the capacitance, which means a larger voltage, and therefore more energy in the system. The voltage of the photon is the amplitude in this wave, because it is an electrical system. The energy of a photon *does* relate to the wave's amplitude – just in an unsuspected way – because it is an electrical wave, and must be analyzed as such.

The energy of the photon *is* caused by its wave nature after all, just as the probability of its location is caused by its wave nature. This is the explanation that Planck knew in his heart *must* exist, although he was unable to find it. This author concludes that what is considered a particle after the wavefunction collapses is nothing more than a semi-localized smeared-out wave-field that imparts its energy through the voltage of the three dimensional nature of the wave, as it cannot be totally localized but has a constant charge differential due to the elementary charge. Bohr's rule of complementarity therefore does not apply to the Copenhagen interpretation regarding the wave-particle duality[28].

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