Gravitation is a Gradient in the Velocity of Light

D.T. Froedge

Formerly Auburn University Phys-dtfroedge@glasgow-ky.com

V051015 @ http://www.arxdtf.org

ABSTRACT

It has long been known that a photon entering a gravitational potential follows a path identical to that of a photon in a variable speed of light defined by the Shapiro velocity for Minkowski flat space [1]. A spatially variable speed of light is implicitly present in General Relativity, and in fact has a long history starting in the pre GR efforts of Einstein and others. The difference in the approach presented in this paper is not that gravitation changes the speed of light, but that gravitation is **a change in the speed of light.** There are lots of implications in this including whether or not gravitons and gravitational waves exist.

It will be demonstrated that a pair of opposite going photons or massless bosoms, instantaneously located at a point have the mass and inertial properties of a massive particle, and transform under a change in the velocity of light exactly as a massive particle responds to a gravitational field. The mechanism for holding massless bosons together as a particle will not be addressed in this paper, but it is not necessary to illustrate confinement to demonstrate the transformation properties on photon momentum in a variable speed of light.

At least some massive particles are in reality a confinement of massless bosons, and if it can be shown that the collective momentum is accelerated by a gradient in c, then the premise vindicated. It is shown here that the center of mass of a particle defined as a pair of opposite going null vector photons, in a gradient light speed, is accelerated exactly as a massive particle is accelerated in a gravitational potential.

It is thus asserted that gravitation can be defined as a gradient in c produced by the presence of mass, and thus, Newton's apple falls not because of an increase in energy, but because the speed of light at the branch is higher than the speed of light at the ground.

Introduction

There are hundreds of papers on gravitation and a variable speed of light (VLS) some of which preexist General Relativity and many thereafter[3]. All papers so far reviewed by this author assert that gravitation alters the velocity of light as well as providing an attractive potential for mass particles. If however mass as a confinement of bosons in a volume of space is accelerated by a gradient in c equivalent to a gravitational potential, then asserting that gravitation is anything but a gradient in c is not necessary.

There has been a historical aversion to defining mass for speed of light particles, mass being only associated with a proper rest mass. This is the standard convention although currently used in some Bose condensate papers [9], the convention of using $m = E / c^2$ as photon mass seems rare. This paper is not going to actually deviate from the norm. For a pair of non collinear photons there is a less than light speed velocity of the center of mass, and thus a rest mass can be defined for a pair or group. If photons are confined such as in a reflective cavity or as a standing wave between two reflectors, then there is a rest mass rest equivalent.

Recently it has been noted by this author that locally contained speed of light massless bosons, have an acceleration of the center of mass by a gradient in c equivalent to the acceleration of a massive particle in a gravitational field. By theorizing a Proto-particle model consisting of two collinear opposite phase photons, it can be shown that the relativistic dynamics are the same as a massive particle [2]. It will be the assertion then that some real particles consist of bosons that are similarly accelerated by a gradient in c which effectuates gravitation.

The assertion in this development is then, not that gravitation alters the speed of light, but that gravitation **is an alteration in the speed of light**, and the gradient accelerates massive particles.

Appendix II will discuss the prospect that if gravitation is a gradient in c, how it could originate from the effects of Quantum Field Theory, thus making gravitation completely an electromagnetic phenomena.

Relativist Lagrangian

Earlier papers by the author have developed an alternate theory of gravitation in Minkowski space based on a locally conserved energy theory, in which a particle infalling a gravitational potential maintains constant energy. As the particle infalls, rest mass energy is converted to kinetic energy, but the total remains constant. A particle stopped at a position in the potential has a mass defect [4]. This theory will be the basis for the work here

From [4], the Lagrangian relation between the relativist, kinetic and rest mass including gravitation is based on the summation over point particle masses is:

$$m^{2}\left(1-\frac{v^{2}}{c^{2}}\right) = m_{0}^{2}\left(1-\frac{\mu}{r}\right)^{2}$$
(1)

The right of this relation is the rest mass as a function of the gravitational potential:

$$m_{\rm R} = m_0^2 \left(1 - \frac{\mu}{r}\right)^2, \qquad (2)$$

and the left side is the kinetic energy, and relativistic mass. m_0 , is the rest mass in gravitational free space.

The connection to the standard relativistic Lagrangian can be demonstrated by taking the square root and dividing by the right side of this expression, and ignoring a third order terms.

$$L = m_0 c^2 = \left(mc^2 - m\frac{1}{2}v^2 + mc^2\frac{\mu}{r} \right)$$
(3)

The distinct difference is that the right side of Eq.(1), is the local rest mass and a function of r. The dynamics for the Proto-particle shown it this development will illustrate a conversion of rest energy to kinetic as a result of a change in c exactly as proposed by Eq. (1).

Variable Speed of light in flat Minkowski space

Blandford et.al [1], and others [4],[5], have shown that photons operating according to Fermat's principle, in a medium having a speed of light with an index of refraction defined by:

$$\mathbf{c} = \mathbf{c}_0 \left(1 - 2\frac{\mu}{r} \right),\tag{4}$$

follows a trajectory identical to that of a photon in a gravitation field. This velocity relation is arrived at by a projection of the photon path defined in the GR Schwarzschild metric tensor, onto flat time Minkowski space.

Fermat's principle, with an index of refraction defined by Eq.(4), yields the proper trajectories for photons in the presence of gravitational fields and is commonly used for making galactic lensing calculations.

A slightly different, but more symmetric relation is can be deduced from Eq.(1), [6] in a locally conserved gravitational potential paper, and is:

$$\mathbf{c} = \mathbf{c}_0 \left(1 - \frac{\mu}{r} \right)^2. \tag{5}$$

For the purposes of this paper, either form is suitable.

With the speed of light in a gravitational potential defined by Eq.(5), putting this into Eq.(1), the Lagrangian for the particle in a VLS is:

$$m^{2}\left(1-\frac{v^{2}}{c^{2}}\right) = m_{0}^{2}\frac{c}{c_{0}}$$
(6)

This is the relationship between the Lorentz relativistic mass and the velocity of light resulting from the relation between gravitation and the speed of light. It is expected that this relation is valid for any vacuum change in the speed of light, induced by gravitation or otherwise.

Two Photon Proto-particle

Premise

A particle, designated as a Proto-particle will be defined in terms of a photon pair, shown to be a Lorentz invariant, identical, in inertial properties, to a massive particle, and accelerated locally by a gradient of c.

It is not illogical that confining a photon to a volume of space results in the creation of rest mass. If the photon is absorbed by a particle, mass in increased, if a photon enters and is trapped in an apparatus the energy has increased; the inertial properties of the box should reflect the increase in rest mass. The increase in the mass of an apparatus trapping a photon in a box increases by, hv, and thus inertial rest mass increases by hv / c^2 . If a massless speed of light particle is confined to a volume of space there is little doubt that the inertial properties of that volume of space should reflect the energy as mass.

For two non-collinear confined photons, there is always a fame in which the momentum and energy of both are equal, and in opposite directions. If the photons are not collinear however there is also an angular momentum couple associated with the motion. For defining a simple mass, complications including confinement mechanisms, can be avoided, that do not contribute to understanding the issue. Photons will be assumed since the inertial dynamics are well understood, but any massless particle with energy and frequency related by Planck's constant would do as well.

Definition

We will define two collinear; opposite phase velocity photons located at the same point at an instant in time as a "Proto-particle".

From momentum considerations the motion of the center of mass for this Protoparticle for two opposite going photons is, as any other mass, defined by:

$$\mathbf{v}\left(v_{1}+v_{2}\right)=\left(v_{1}-v_{2}\right)\mathbf{c}$$
(7)

For a photon the frequency is presumed to represent the energy and the Planck constant will be set to one. So long as the relations are understood, the terms energy, and mass for photons can be used somewhat interchangeably.

Proto-particle Four Momentum

Photons moving along collinear vector paths in the opposite direction can be described by the null four-momentum in geometric algebra matrix as:

.

$$\vec{\mathbf{P}}_{1} = \frac{\mathbf{h}\mathbf{v}_{1}}{\mathbf{c}^{2}} \left(\gamma^{k}\mathbf{c}_{k} + \gamma^{0}\mathbf{c} \right), \tag{8}$$

and the same with an opposite phase velocity:

$$\overrightarrow{\mathbf{P}_{2}} = \frac{\mathbf{h}\mathbf{v}_{2}}{\mathbf{c}^{2}} \Big(-\gamma^{k}\mathbf{c}_{k}+\gamma^{0}\mathbf{c}\Big)$$
(9)

(The Weyl representation of the Dirac gamma matrix is assumed.)

Presuming these momentum vectors are the null vectors associated with the action of a photon function, then they represent photons with the phase moving in opposite directions at the speed of light, and instantaneously colocated.

The square of the difference of the two null vectors is not zero, but is independent of time and space and is an invariant under a Lorentz transformation of the vector sum. [7]

$$\left(\vec{P}_{1} - \vec{P}_{1}\right)^{2} = \left[\frac{v_{1} + v_{2}}{c\sqrt{3}}(\gamma_{1} + \gamma_{2} + \gamma_{3}) + (v_{1} - v_{2})\gamma_{4}\right]^{2}, \quad (10)$$

or:

$$\frac{\left(v_1 + v_2\right)^2}{c^2} - \frac{\left(v_1 - v_2\right)^2}{c^2} = \frac{4v_1v_2}{c^2} = \frac{v_0^2}{c_0^2}$$
(11)

7

Thus v_0 / c is the invariant fixed momentum associated with the pair of opposite going photons, independent of a Lorentz transformation jointly applied to the null vectors. (It will be shown later that this expression requires a slight modification in regard to having invariance in c. (See Eq.(33))

Factoring the sum of the two particle momentum from Eq.(11), gives:

$$\frac{\left(v_{1}+v_{2}\right)^{2}}{c^{2}}\left[1-\frac{\left(v_{1}-v_{2}\right)^{2}}{\left(v_{1}+v_{2}\right)^{2}}\right]=\frac{v_{0}^{2}}{c_{0}^{2}}$$
(12)

Noting that from Eq.(7), for two opposite going photons:

$$\frac{\mathbf{v}}{c} = \frac{(v_1 - v_2)}{(v_1 + v_2)}$$
(13)

Where \mathbf{v} is the velocity of the center of mass, or energy. Putting this into, Eq.(12), it is found that the relativistic Lagrangian for the Proto-particle is exactly equivalent to the relativistic velocity relation for massive particles:

$$p^{2}\left(1 - \frac{v^{2}}{c^{2}}\right) = p_{0}^{2}$$
(14)

With mass defined in the local frame as the total energy and $c_0 = 1$:

$$\mathbf{m} = (v_1 + v_2) / c_0^2 \to (v_1 + v_2)$$
(15)

Local Photon Energy Velocity Differentials

The local energy, velocity differentials for single photons under a transformation in v, and c can be evaluated.

For the photon transform, the relativistic Doppler relation, along the direction of motion, for frequency is, [12]:

$$v = v_0 \left[\left(1 - \frac{\mathbf{v}_{\mathrm{T}}}{c} \right) / \left(1 + \frac{\mathbf{v}_{\mathrm{T}}}{c} \right) \right]^{1/2}$$
(16)

 \mathbf{V}_{T} is the transform velocity.

Differentiating gives dv / dv for the single photon energy:

$$\frac{\mathrm{d}v}{\mathrm{d}\mathbf{v}_{\mathrm{T}}} = \frac{v}{\mathrm{c}} \tag{17}$$

In this expression \mathbf{V}_{T} is the transformation velocity and is opposite for opposite for opposite going photons.

Speed of Light - single photon c

The local total differential for the velocity of light in wavelength and frequency for an individual photon is:

$$\frac{d\lambda v}{dc} = v_0 \frac{d\lambda}{dc} + \lambda_0 \frac{dv}{dc},$$
(18)

or:

$$\frac{\mathrm{d}c}{v_0\lambda_0} = \frac{\mathrm{d}c}{c_0} = \frac{\mathrm{d}\lambda}{\lambda_0} + \frac{\mathrm{d}v}{v_0}$$
(19)

Observing the individual differentials:

$$dv = dc / \lambda_0 \qquad \qquad d\lambda = dc / v_0 \tag{20}$$

So that from Eq(20):

$$dc = \lambda_0 dv = v_0 d\lambda, \qquad (21)$$

Putting this into Eq.(19), gives dv / dc for the single photon dv / dc:

$$\frac{\mathrm{d}v}{\mathrm{d}c} = \frac{v}{2c} \tag{22}$$

This relation is independent of the direction of the photon.

Inserting Local Single Photon differentials in Proto-particle

Lorentz velocity transform

For the Lorentz velocity transformation, inserting the differentials into the mass for the Proto-particle Eq.(15), gives:

$$dm = d(v_1 + v_2) = \left(v_1 \frac{d\mathbf{v}_T}{c} - v_2 \frac{d\mathbf{v}_T}{c}\right) = (v_1 - v_2) \left(\frac{d\mathbf{v}_T}{c}\right), \quad (23)$$

or:

$$d\mathbf{m} = \left(v_1 + v_2\right) \frac{v_1 - v_2}{\left(v_1 + v_2\right)} \left(\frac{d\mathbf{v}_T}{c}\right) = \mathbf{m} \frac{\mathbf{v}}{c} \frac{d\mathbf{v}_T}{c}$$
(24)

The change in the mass as a function of the Lorentz transforms velocity for the Proto-particle is then:

$$\frac{\mathrm{dm}}{\mathrm{d}\mathbf{v}_{\mathrm{T}}} = \mathrm{m}_{\mathrm{0}} \frac{\mathrm{v}}{\mathrm{c}^{2}} \tag{25}$$

Now the same relation can be found to compare with the change in the relativistic mass of a real particle as a function of velocity.

$$d\left[m\left(1-\frac{v^2}{c^2}\right)\right] = dm_0 \quad , \tag{26}$$

or:

10

$$mv\frac{dv}{c^2} + \left(1 - \frac{v^2}{2c^2}\right)dm = 0$$
(27)

or:

$$\frac{\mathrm{dm}}{\mathrm{dv}} = \mathrm{m}_0 \frac{\mathrm{v}}{\mathrm{c}^2} \tag{28}$$

Comparing, Eq.(25), and Eq.(28), shows that the total energy change for the two photons in the Proto-particle is the same as the energy change of the relativistic mass. The Proto-particle mass energy thus transforms properly under a Lorentz velocity transformation.

This match of Lorentz transform properties for the Proto-particle and a massive particle constitutes a proof of concept for defining a rest mass for confined photons.

Speed of Light transform

For observing the change in the mass of the Proto-particle, as a result of a change in c the local dv / dc, from Eq.(22), can be put into the mass.

Thus the local photon differential for a two photon energy Proto-particle as a function of a change in c from Eq.(22). is:

$$dm^{2} = d(v_{1} + v_{2})^{2} = 2(v_{1} + v_{2})\left(v_{1}\frac{dc}{2c_{0}} + v_{2}\frac{dc}{2c_{0}}\right) = (v_{1} + v_{2})^{2}\left(\frac{dc}{c}\right),$$
(29)

or:

$$\frac{dm^2}{dc} = \frac{(v_1 + v_2)^2}{c}$$
(30)

This is a clear indication that the Proto-particle mass as stated is not invariant under a change in the velocity of light.

More importantly, neither is the momentum defined in Eq.(11),

$$p^{2} = \frac{\left(v_{1} + v_{2}\right)^{2}}{c^{2}}$$
(31)

This is not an altogether unexpected. It would be true for a fixed velocity of light, but since c is variable this is no longer true. It is easily shown however that:

$$p^{2} = \frac{\left(v_{1} + v_{2}\right)^{2}}{cc_{0}},$$
(32)

is an invariant under c as well as v, and thus Eq.(11), should properly be stated as:

$$\frac{\left(v_{1}+v_{2}\right)^{2}}{cc_{0}} - \frac{\left(v_{1}-v_{2}\right)^{2}}{cc_{0}} = \frac{4v_{1}v_{2}}{c_{0}^{2}} = \frac{v_{0}^{2}}{c_{0}^{2}}$$
(33)

and Eq.(12), as:

$$\frac{\left(v_{1}+v_{2}\right)^{2}}{cc_{0}}\left[1-\frac{\left(v_{1}-v_{2}\right)^{2}}{\left(v_{1}+v_{2}\right)^{2}}\right]=\frac{v_{0}^{2}}{c_{0}^{2}}$$
(34)

This relation is derived from the magnitude of two opposite going photons with an invariant momentum under a local instantaneous Lorentz velocity or VSL transformation, and is independent of any reference to gravitation.

It is clear that the right and left side of Eq.(34), are invariant under a Lorentz velocity or c velocity change. (The invariance of v/c in regard to VSL is presented in Appendix I). Rearranging Eq.(34):

$$\left(v_{1}+v_{2}\right)^{2}\left[1-\frac{\left(v_{1}-v_{2}\right)^{2}}{\left(v_{1}+v_{2}\right)^{2}}\right]=v_{0}^{2}\frac{c}{c_{0}}$$
(35)

This is then the relativistic mass or energy relation for the motion of the center of mass of two opposite phase velocity photons as a result a transformation in v or c. having no reference to gravitation.

With identification of mass Eq.(15), and velocity Eq.(13), this is identical to Eq.(6)

$$m^{2}\left(1-\frac{v^{2}}{c^{2}}\right) = m_{0}^{2}\frac{c}{c_{0}}$$
(36)

Solving this for v then gives:

$$v = c \sqrt{1 - \frac{m_0^2}{m^2} \frac{c}{c_0}},$$
 (37)

the velocity as a function of c.

Replacing c with its value in gravitation gives the standard value of velocity for a particle falling from infinity:

. .

$$v \approx c \sqrt{\frac{2\mu}{r}}$$
 (38)

Putting in the Shapiro dependence of the speed of light of gravitation gives back the original, Eq.(1), gravitational Lagrangian. Now particle mass and velocity is a function of c as defined by the Shapiro velocity:

$$m^{2}\left(1-\frac{v^{2}}{c^{2}}\right) = m_{0}^{2}\left(1-\frac{\mu}{r}\right)^{2}$$
(39)

If gravitation is only a change in the velocity of light, the Proto-particle motion will be same as a massive particle in a gravitational field, thus if the constituents of real particles transform the same under a change in c, there is no need for any other postulated mechanism.

Newton's apple falls not because of an increase in energy, but because the speed of light at the branch is higher than the speed of light at the ground.

Results

Confined photons are accelerated in a gradient in c just as a massive particle with the same mass.

Black holes could not exist in this theory since photon energy is not diminished by a gravitational field. The presentation of the alternative neutron star, to a black hole is developed in [11], and the alternative image is simulated in [12].

If gravitation is a gradient in c then there would not be gravitational radiation. To account for the energy loss in a pulsar binary, a time varying gradient in c must couple into the electromagnetic spectrum. If this is true gravitational waves would not be found, but electromagnetic radiation signatures of gravitational energy loss should.

Conclusion

It has been shown that the effects of gravitation attraction can be effectuated on a mass by a gradient in the speed of light without need of any other mechanism.

References:

- 1. Roger Blandford, Kip S. Thorne, in Applications of Classical Physics, (in preparation, 2004), Chapter 26 http://pmaweb.caltech.edu/Courses/ph136/yr2012/1227.1.K.pdf
- 2. DT Froedge, The Concept of Mass as Interfering Photons, V032615 http://www.arxdtf.org/css/interfering.pdf
- 3. Jan Broekaert, , 2008 A spatially-VSL gravity model with 1-PN limit of February 5mhttp://arxiv.org/pdf/gr-qc/0405015v4.pdf
- 4 DT Froedge, The Gravitational Theory with Local conservation of Energy, V020914, http://www.arxdtf.org/css/grav2a.pdf
- F. Karimi, S. Khorasani, Ray-tracing and Interferometry in Schwarzschild Geometry, arXiv:1001.2177 [gr-qc] arXiv:1206.1947v1 [gr-qc] 9 Jun 2012
- 6. DT Froedge, The Velocity of Light in a Locally Conserved Gravitational Field, V020914, http://www.arxdtf.org/css/velocity.pdf
- 7. Chris Doran, 2003 Geometric Algebra for Physicists, Cambridge University Press, pp.137
- 8. Feynman, Hibbs, 1965, Quantum Mechanics and Path Integrals McGraw-Hill.
- 3. D. Kharzeeva, K. Tuchinb, Vacuum Self–Focusing of Very Intense Laser Beams, arXiv:hep-ph/0611133v2
- 9. Alex Kruchkov Bose-Einstein condensation of light in a cavity http://arxiv.org/pdf/1401.0520v2.pdf

- 10. John 10 David Jackson, Classical Electrodynamics Second Edition pp 364
- 11. DT Froedge, Black Hole vs. Variable Rest Mass Neutron Star Field,, **v041912**, http://www.arxdtf.org/css/BHvsVRM.pdf
- 12. DT Froedge, Image Comparisons of Black Hole vs. Neutron Dark Star by Ray Tracing,, V111514, http://www.arxdtf.org/css/Image%Comparisons.pdf

Appendix I

Ratio v/c

The v/c ratio for the Proto-particle, can be shown to be independent of a gradient in c. From Eq.(13), this ratio is:

$$d\left(\frac{v}{c}\right) = d\frac{(v_1 - v_2)}{(v_1 + v_2)} = \frac{(dv_1 - dv_2)}{(v_1 + v_2)} - \frac{(v_1 - v_2)}{(v_1 + v_2)} \frac{(dv_1 + dv_2)}{(v_1 + v_2)}$$
(1.1)

Making substitutions from Eq(22), into this gives:

$$d\left(\frac{v}{c}\right) = \frac{\left(v_{1}\frac{dc}{2c} - v_{2}\frac{dc}{2c}\right)}{\left(v_{1} + v_{2}\right)} - \left(v_{1} - v_{2}\right)\frac{\left(v_{1}\frac{dc}{2c} + v_{2}\frac{dc}{2c}\right)}{\left(v_{1} + v_{2}\right)^{2}} = \frac{v}{c}\frac{dc}{2c} - \frac{v}{c}\frac{dc}{2c} = 0 \quad 1.2$$

Thus:

$$\frac{\mathrm{d}}{\mathrm{dc}}\left(\frac{\mathrm{v}}{\mathrm{c}}\right) = 0 \tag{1.3}$$

QED.

Appendix II

QFT Origin of Gravitation?

This appendix is a bit of speculation, but indicated by the state of the art.

From the above discussion: Gravitation could be defined as the presence of mass altering the local velocity of light.

Since there are well-known processes defined in QFT and path integral formulations of QM, that alter the velocity of light in the proximity of moving particles,[8] it is speculated that the processes of QFT could be the progenitor of gravitation.

Consider an apparatus having a cavity with opposing mirrors and having photons trapped between the mirrors. From conservation of energy the apparatus has more mass and generates more gravitational attraction than the cavity without the photons. There is not speculated to be interaction between the photons, so the photons that are bouncing back and forth must be generating gravitation.



Fig1 Photons trapped between mirrors of an apparatus increase the mass and thus the gravitational attraction of the apparatus.

The increase in energy of the system is hv so the mass of the apparatus increase as a result of a trapped photon is:

16

$$m = \frac{\hbar\omega}{c^2}$$
(2.1)

The gravitational potential due to a photon is:

$$\frac{\mu}{r} = \frac{G\hbar\omega}{c^4 r}$$
(2.2)

Putting this into Eq.(5), then gives:

$$\mathbf{c} = \mathbf{c}_0 \left(1 - 2 \frac{\mathbf{G}\hbar\omega}{\mathbf{c}^4 \mathbf{r}} \right) \tag{2.3}$$

or:

$$\Delta c = 2 \frac{G\hbar}{c^3 r} \omega \tag{2.4}$$

Noting that the square of the Planck radius is $G\hbar/c^3$ this can be stated as:

$$\Delta c = 2 \frac{r_{\rm P}^2}{r} \omega , \qquad (2.5)$$

which if the motion of the photons generates gravitation, has to be the change in c at a distance r induced by a photon The fact that the Planck radius is the constant in the equation is quite curious.

By the methods of path integrals noted by Feynman the probability for the particle moving from point a to point b, exist throughout spaces, it has already been shown by the methods of Quantum Electrodynamics that a photon beam induces a change in the velocity of light in the vicinity of the beam.



Fig.2.This illustration shows the path actions induced by a pair of oscillating photons.

From the work of D. Kharzeeva, et.al, [9] it is shown that for an intense laser beam the QFT effects related to electron–positron loops induce vacuum "self-focusing" which is a vacuum alteration of the index of refraction in the speed of light in the vicinity of the beam

A particle model being a reciprocating bosons in a massless box, as asserted here, constitutes an intense, highly energetic back and forth reciprocal motion, orders of magnitude greater than a laser.

It is suggested here that the multiple path integration of the photon action over all space would alter the velocity of light near the path as a function of r, and if Eq.(2.5), is realized the connection between gravitation and QFT would be established.