Open Letter on Hilbert's Fifth Problem

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Abstract

Hilbert's Fifth Problem, in English translation, [1], is as follows :
"How far Lie's concept of continuous groups of transformations is approachable in our investigations without the assumption of the differentiability of the functions ?"
followed by :
"In how far are the assertions which we can make in the case of differentiable functions true under proper modifications without this assumptions ?"
Lately, in the American mathematical literature, due to unclear reasons, it has often been distorted and truncated as follows, [3] :
"Hilbert's fifth problem, like many of Hilbert's problems, does not have a unique interpretation, but one of the most commonly accepted accepted interpretations ..."
A recent letter in this regard, sent to Terence Tao, and the editors of [3], Dan Abramovich, Daniel S Freed, Rafe Mazzeo and Gigliola Staffilani can be found below.

The Letter

Dear Terence

This is about a rather serious issue, so that as to keep to some balance, let me please start in a more jokingly amusing manner ...
Thank you ...

I was quite glad to see your latest book on Hilbert's Fifth Problem, [3], and it is indeed a nice and valuable performance !

Related to its title, however, it may be more than somewhat misleading ...
Indeed, as you well know, no doubt, when one takes the oath in a court of law, one is supposed to promise "to tell the truth, the whole truth, and nothing but the truth" ...

Well, strangely enough, it has seemingly been a long long time since a mathematician in America, be it American or not, did tell the *whole truth* about Hilbert's Fifth Problem. Instead, just like you yourself in [3], they come up front with the excuse-cum-diversion, and here I cite you yourself from page 10 of your mentioned book, that:

"Hilbert's fifth problem, like many of Hilbert's problems, does not have a unique interpretation, but one of the most commonly accepted accepted interpretations ..."

And so sorry, this is far from being the whole truth about Hilbert's fifth problem ...
Thus, in a court of law, it would simply and instantly qualify as a ... , you guessed what ...

It not clear why American mathematicians keep doing that for quite a while by now. And honestly, I am not interested to get into any related assumptions ...

The last time I happened to see a proper, and full presentation by an American mathematician of the original formulation by Hilbert himself of his fifth problem, was in the 1976 book [1] :

F.E. Browder (Ed) : "Mathematical Developments Arising from Hilbert Problems", Proceedings of Symposia in Pure Mathematics, Vol. XXVIII, AMS, Providence

And there, on pages 12-14, the mentioned proper and full formulation, translated into English, goes as follows :

"How far Lie's concept of continuous groups of transformations is approachable in our investigations without the assumption of the differentiability of the functions ?"

followed by :

"In how far are the assertions which we can make in the case of differentiable functions true under proper modifications without this assumptions ?"

Further related details can also be found in my book [2] :

"Parametric Lie Group Actions on Global Generalized Solutions of Nonlinear PDEs, Including a Solution to Hilbert's Fifth Problem", Mathematics and Its Applications, Kluwer Academic Publishers, Dordrecht, Boston, London, 1998, ISBN 0-7923-5232-7,

on pages 163-172.

And needless to say, in the above original formulation of Hilbert's Fifth Problem, there is not one single word about ... locally Euclidean ... whatever ...

So then, getting a bit less ... jokingly amusing ..., could you, please, try and take a short moment, and explain how do you ever get to what you call "one of the most commonly accepted interpretations" ?

And while you may, hopefully, be doing so, do, please, also try at long last to explain and excuse yourself why you - together with your American colleagues - are seemingly not brave enough to deal with the *genuine, original, full* version of Hilbert's Fifth Problem, and instead keep belittling yourselves by focusing merely on ... one of its most commonly accepted interpretations ... ?

And perhaps more importantly, why do you not mention even once the proper and full formulation of Hilbert's Fifth Problem ?

Were you simply in such a hurry that, like seemingly just about all American mathematicians have for long by now been doing, you, too, got stuck with that alleged "one of the most commonly accepted accepted interpretations" ?

And now, if you do not mind, also to some more specific issues :

My mentioned book [2], regardless of what it may contain about Lie groups and generalized solutions of nonlinear PDEs, has - as far as I see it - three ideas worth considering :

1) The parametric approach to Lie groups turns out to be so simple and natural, as to allow for the first time their most general definition globally, and their corresponding most general applications to PDEs also globally. Amusingly, this parametric approach could have been used from the very beginning by Lie himself, since it is but elementary Calculus. Well, it is but one of those ... missed opportunities ...

2) Based on that parametric approach, one obtains for the first time a complete solution of Hilbert's Fifth Problem in the sense of its original formulation mentioned above.

3) What seems, however, to be by far most important is that the parametric approach allows the introduction of *genuine Lie semi-groups*, that is, Lie semi-groups which are *not* sub-semi-groups of Lie groups. And that does immensely extend the study of symmetries, called accordingly "semi-symmetries", since they can contain this time transformations which are *not* invertible. Briefly, we can go from C^∞-smooth diffeomorphisms of a given manifold M, to absolutely all C^∞ transformations of M into itself. Quite an enlargement indeed ...

Amusingly, you do not seem to be aware of any of that ...
At least as far as your mentioned book [3] is concerned ...

Well, this is what the story I wanted to tell you is all about ...
And I hope, you would not much mind that I may circulate it among a larger number of mathematicians ...

Last and not least, since you are so young, quite likely you are not aware of the following historical amusement :

The Soviets, before they went down in history in 1991, had the ridiculous habit to claim that just about absolutely every major scientific or technological discovery in the last three centuries, that is, since their Tsar Peter the Great, was made either by Russians before communism, or by Russians in the Soviet Union ...

And needless to say, any number of outstanding jokes circulated about that around the world, further contributing to making the Soviets ridiculous ...

Well, are perhaps the Americans trying to make a sort of ... dual ... story ?
Namely, to disregard just about absolutely everything in science and technology which was not discovered in America ?
Or at best, to distort it, as with Hilbert's Fifth Problem ?

Your mentioned book [3], I am afraid, you can consider it to make already one more step in this regard ...

Anyhow, do you happen to consider that book as being, indeed, a contribution markedly above of what you have done so far in mathematics ?
Perhaps, you do, and it may indeed happen to be so ...
In which case, let me please apologize ...

With all the best wishes to you on the occasion of the New Year,

Yours the same as always,

Elemér

Comment on the Fields Medal ...

On occasion, ever since its inception back in 1936, the Fields Medal in mathematics has been subjected to one or another criticism, as it usually happens with all such distinctions and, naturally, not only in mathematics ...

What may, however, be indeed a genuine and more and more worrisome negative phenomenon is that the clause which restricts the medal to mathematicians under the age of 40 (forty), has over the years proved to inflict more and more harm upon the recipients of the Fields Medal.
And this harm on the respective recipients - typically suffered in silence - is going to affect more and more relatively young mathematicians, as the Fields Medal tends to be awarded by now to no less than 4 (four) mathematicians at a time ...

A classic - and by now, publicly known - example in this regard is the case of René Thom (1923-2002), recipient of the 1958 Fields Medal.

Briefly, this harm is mainly about forcing the recipient into a merciless competition with himself or herself, aimed to come up with new mathematics which is significantly better that the one for which he or she received the Fields Medal ...
In this regard, the Fields Medal seems to be the only more prestigious distinction, and not only in Mathematics, which has such a harmful age restriction clause ...

Consequently, would it be totally impossible, or even inconceivable, simply to remove that age restriction clause ?
And how about the possibility of receiving more than once the Fields Medal, and without any age restrictions ?
After all, a few times in its history, the Nobel Prize was already awarded to the same person more than once ...

Meanwhile, hardly any Fields Medalists have ever managed to do a genuinely better mathematics, than the one for which they were given that distinction. And then, silently, they may have to suffer that unenviable feeling for three or four, or more decades, until the end of their lives ...

References

[1] F.E. Browder (Ed) : "Mathematical Developments Arising from Hilbert Problems", Proceedings of Symposia in Pure Mathematics, Vol. XXVIII, AMS, Providence

[2] E.E. Rosinger : "Parametric Lie Group Actions on Global Generalized Solutions of Nonlinear PDEs, Including a Solution to Hilbert's Fifth Problem", Mathematics and Its Applications, Kluwer Academic Publishers, Dordrecht, Boston, London, 1998, ISBN 0-7923-5232-7

[3] T. Tao : Hilbert's Fifth Problem and Related Topics. Graduate Studies in Mathematics, Vol. 153, AMS, Providence, Rhode Island

[4] R. Thom : "Parables, Parabolas and Catastrophes: Conversations on Mathematics, Science and Philosophy"