The Fourth Electromagnetic Induction

Abstract

Different variants of electromagnetic induction are considered. The type of induction caused by changes of electromagnetic induction flow is separated. The dependence of this induction on the flow density of electromagnetic energy emf and on the parameters of the wire is explored.

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1. Introduction

There is the following known law of electromagnetic induction

\[ e = \frac{\partial \Phi}{\partial t}, \]  

(1)

where \( \Phi \) is the magnetic flow, \( e \) - emf. It is known also [1], that this electromagnetic magnetic induction – the appearance of emf in the conductor, may appear as a consequence of the following two laws:

\[ F = q(v \times B), \]  

(2)

\[ \nabla \times E = -\frac{\partial B}{\partial t}. \]  

(3)

In accordance to this fact two types of electromagnetic induction can be determined –

the first type - case (3), when emf in the conductor appears as a consequence of the magnetic flow change, - electromagnetic induction caused by the electromagnetic flow change;

the second type - case (2), when emf in the conductor appears under the influence of the Lorentz magnetic force due to the mutual displacement of the wire and the magnetic field, without changes in the magnetic flow, - electromagnetic induction caused by the Lorentz force.
There is also a known \textbf{third type} of electromagnetic induction, which appears in a unipolar Faraday generator – \textit{unipolar electromagnetic induction}. In this generator the motor rotates a permanent magnet, and on the radius of the magnet appears emf, which is determined according to the formula of the form
\[
e = \frac{\omega BL^2}{2},
\]  
where  
\(B\) - the induction of the permanent magnet,  
\(L\) - the length of magnet’s radius,  
\(\omega\) - the angular velocity of rotation.

This formula was obtained by different methods: in \cite{2} using the relativity theory, and in \cite{3} based of the law of momentum conservation.

There is also a widely known fact, that the current is inducted in a conductor located in the flow of electromagnetic wave energy flow. Let us give the \textit{electromagnetic induction caused by the change of electromagnetic energy flow} the name of the fourth type of electromagnetic induction. Further we shall determine the emf of this induction depending on the flow density.

In \cite{4} the following fact is proved: if the body is located in a uniform flow of electromagnetic energy  
\[
S = E \times H,
\]  
then the following force acts on it (hereinafter the SI system is being used)  
\[
F = V \left( \frac{\partial}{\partial t} \left( \frac{S \mu}{c^2} \right) + \frac{S \sqrt{\mu}}{c} \right),
\]  
where  
\(V\) - the volume of the body in which the electromagnetic field interacts with the charges and currents,  
\(\varepsilon\) - the relative permittivity of the body,  
\(\mu\) - the relative magnetic permeability of the body,  
\(c\) - The speed of light in vacuum.

In the electromagnetic energy flow an electron may be found. We can assume that this flow inside the electron’s body is always uniform (due to its small size). Then the electron will be subjected to the force (6).

\section*{2. The Own Energy Flow}

It is known that the power of the heat loss in the wire equal to the flux of the Poynting vector through the surface of the wire, and the density of the flow is determined by the electrical and magnetic tensity generated on the surface of the wire by the current in the wire.
Let us consider now the part of the wire in which an alternative current is flowing with a certain density
\[ j = j_o \sin(\omega t). \] (8)

Then the current and intensity in the wire are
\[ J = \frac{\pi d^2}{4} j, \] (9)
\[ E = j\rho, \] (10)
\[ H = J/(\pi d) = 0.25dj, \] (11)

and the density of electromagnetic flow entering the wire from all the sides (we shall call this flow “the own flow”) is
\[ S_1 = EH = 0.25d\rho j^2. \] (12)

Here
- \(d\) - the diameter of the wire,
- \(\rho\) - the resistivity of the wire.

The flow of electromagnetic energy entering a wire of length \(L\), is
\[ S_L = S \cdot \pi dL. \] (13)

Then
\[ S_{li} = 0.25\pi Ld^2 \rho j^2. \] (14)

The thermal power dissipated in the wire of volume
\[ V = 0.25\pi Ld^2 \] (15)
is determined in the same way.

In this case the force (6) takes the form
\[ F = 0.25d\rho V \left( \frac{\partial}{\partial t} \left( \frac{j_o^2 \sin^2(\omega t) \sqrt{\mu}}{c^2} \right) + \frac{j_o^2 \sin^2(\omega t) \sqrt{\mu}}{c} \right) \] (16)

This force acts on all the charges (electrons) in is directed towards current (i.e., it does not act on the wire as a whole). It allows to overcome the resistance to movement, or more precisely, it performs the work that is converted into heat.

Let the current density is
\[ j = j_o \sin(\omega t). \] (17)

Then
\[ F = 0.25d\rho V \left( \frac{\partial}{\partial t} \left( \frac{j_o^2 \sin^2(\omega t) \mu}{c^2} \right) + \frac{j_o^2 \sin^2(\omega t) \mu}{c} \right) \]
or
\[ F = \frac{0.25d\rho j_o^2 V}{c} \sqrt{\mu} \sin(\omega t) \left( \frac{2\omega \cos(\omega t) \sqrt{\mu}}{c} + \sin(\omega t) \right) \] (18)
With sufficiently low frequencies
\[
\left( \omega \ll \frac{c}{2\sqrt{\epsilon\mu}} \right)
\]
we may assume that
\[
F = \frac{0.25d\rho_j^2V\sqrt{\epsilon\mu}}{c} \sin^2(\omega t)
\]
i.e. the average force is
\[
F = \left( 0.25d\rho_j^2V\sqrt{2\epsilon\mu} \right)/c
\]

3. The External Energy Flow

Now we shall consider the case when the wire is located in the region of outside, external flow of electromagnetic energy, i.e. the flow created in the absence of current in this wire.

Let us now assume that the external electric energy flow permeates the wire along the diameter (rather not enters into it from all sides, as in the previous case). Let the density of the external flow be equal to
\[
S = S_o \sin^2(\omega t).
\]
In this case (6) takes the form
\[
F = VS_o \left( \frac{\partial}{\partial t} \left( \frac{\sin^2(\omega t)\epsilon\mu}{c^2} \right) + \frac{\sin^2(\omega t)\sqrt{\epsilon\mu}}{c} \right)
\]
or
\[
F = \frac{VS_o \sqrt{\epsilon\mu}}{c} \sin(\omega t) \left( \frac{2\omega\cos(\omega t)\sqrt{\epsilon\mu}}{c} + \sin(\omega t) \right)
\]
For sufficiently low frequencies (19) we may assume that
\[
F = \frac{VS_o \sqrt{\epsilon\mu}}{c} \sin^2(\omega t)
\]
i.e. the average force is
\[
F = VS_o \sqrt{2\epsilon\mu}/c
\]
From comparing (21) and (26) it follows that the force (26) exceeds the force (21) when
\[
S_o > 0.25d\rho_j^2,
\]
and the current excited by the external flow is,
\[
j_o = 2 \sqrt{\frac{S_o}{d\rho}}
\]
Given that
\[ S_o = EH = E^2 \sqrt{\frac{\varepsilon \varepsilon}{\mu_0 \mu}}, \] (29)

from (28) we find:
\[ j_o = 2E \sqrt{\frac{1}{d\rho}} \sqrt{\frac{\varepsilon \varepsilon}{\mu_0 \mu}} = \frac{2E}{\sqrt{d\rho}} \sqrt{\frac{\varepsilon \varepsilon}{\mu_0 \mu}} \] (30)

This is the current induced by the flow of energy in the wire, permeated by the flow of energy \( S_o \). This energy flow in the air is determined as
\[ S_o = E_o H_o = E_o^2 \sqrt{\frac{\varepsilon \varepsilon}{\mu_0}}, \] (31)

where \( E_o, H_o \) – are the intensity of external field where the wire is located. As the flow density does not change at its transition from the air into the wire, so from (29, 31) we find:
\[ E_o^2 \sqrt{\frac{\varepsilon \varepsilon}{\mu_0}} = E^2 \sqrt{\frac{\varepsilon \varepsilon}{\mu_0 \mu}}, \] (32)

or
\[ E = E_o \frac{\sqrt{\mu}}{\varepsilon}, \] (33)

Combining (30) and (33), we get:
\[ j_o = \frac{2E_o}{\sqrt{d\rho}} \sqrt{\frac{\varepsilon \varepsilon}{\mu_0}}. \] (34)

Let us find the emf generated in the wire:
\[ e_o = j_o \rho. \] (35)

Combining (35) and (34), we find:
\[ e_o = 2E_o \sqrt{\frac{\rho}{d}} \sqrt{\frac{\varepsilon \varepsilon}{\mu_0}} \] (36)

Given that \( \sqrt{\frac{\varepsilon \varepsilon}{\mu_0}} = 19.4 \), we get
\[ e_o \approx 40E_o \sqrt{\frac{\rho}{d}}. \] (37)

Thus, in the wire located in the flow (31), emf (37) is generated.
References