A note on the harmonic series and the logarithm

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Abstract

A relationship between the harmonic series and the logarithm is presented. The formula \( H(n) - \log(n) \) for the Euler-Mascheroni constant is adopted accordingly.

\[
\gamma = \left( \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \right) - \left( \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n^2} \right)
\]

Figure 1: Illustration of \( H(n) \) and \( \log(n) \) as part of \( H(n+n^2) \)

Figure 2: Relationship between \( \log(n) \) and its approximation \( H(n+n^2) - H(n) \)

Mathematica Codes:

\[
\text{Limit}[2*\text{HarmonicNumber}[n] - \text{HarmonicNumber}[n + n^n], n \to \infty]
\]

\[
\text{Limit}[n*(\text{HarmonicNumber}[n + n^n] - \text{HarmonicNumber}[n] - \log[n]), n \to \infty]
\]

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