Homology Classes of Generalised Triangulations Made up of a Small Number of Simplexes

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Abstract

By means of a computer, all the possible homogeneous compact generalised triangulations made up of a small number of 3-simplexes (from 1 to 3) have been classified in homology classes. The analysis shows that, with a small number of simplexes, it is already possible to build quite a large number of separate topological spaces.

Key Words: topology, generalised triangulation, homology.

1 Introduction

Given a number T of 3-simplexes, having n (with $n = 4 \cdot T$) faces, it is possible to build several compact topological spaces by identifying the n faces in couples. There are:

$$(n-1) \cdot (n-3) \cdot \ldots \cdot 1 \tag{1}$$

different combinations possible for identifying faces. For each of the above combination there are $\frac{n}{2}$ couple of identified faces and, for each couple, we have 6 different orientations for the identification.

In total, all the possible compact generalised triangulations that is possible to build up with a number T of 3-simplexes are:

$$(n-1) \cdot (n-3) \cdot \ldots \cdot 1 \cdot 6^{\frac{n}{2}} \tag{2}$$

where the above equation takes already into account the most obvious symmetries.

For example, with 1 simplex is possible to build 108 different compact generalised triangulations, with 2 simplexes is possible to build 136080 compact generalised triangulations and so on. Note that some of the combinations lead to non feasible triangulations meaning that, given the instruction for identifying faces, it is simply not possible to build a space that makes sense.

By means of a computer we have evaluated the homology groups of all the possible generalised triangulations made up of T simplexes (with T going from 1 to 3) and we have classified them is classes of triangulations having the same homology groups.

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The homology groups have been evaluated from the boundary maps ∂_i, expressed as matrices D_i of integers, in the usual way:
- the rank of the homology group H_i has been evaluated as:

$$\#Ker(\partial_i) - \#Im(\partial_{i+1}) = \#C_i - Rank(D_i) - Rank(D_{i+1}) \tag{3}$$

where # means space dimension, C_i is the i-chain of the generalised triangulation under study and, in our representation, $\#C_i$ is the number of columns of the matrix D_i.
- the torsion part of the group H_i has been read across from the matrix D_{i+1} expressed in Smith Normal Form.

The results are reported below.

Type of Triang.	Num. of Triang.	Note
Non feasible triangulations	69	
Path connected triangulations	39	in 3 homology classes
Non path connected triangulations	N/A	
Total	108	

Table 1 : Triangulations made of 1 simplex

Type of Triang.	Num. of Triang.	Note
Non feasible triangulations	98991	
Path connected triangulations	35568	in 13 homology classes
Non path connected triangulations	1521	in 6 homology classes
Total	136080	

Table 2 : Triangulations made of 2 simplexes

Type of Triang.	Num. of Triang.	Note
Non feasible triangulations	366924249	
Path connected triangulations	113844096	in 43 homology classes
Non path connected triangulations	4220775	in 47 homology classes
Total	484989120	

Table 3 : Triangulations made of 3 simplexes

2 Spaces Made of 1 Simplex - Sum up Table

The following table sums up the homology classes of all path connected compact homogeneous generalised triangulations made up of only 1 simplex.

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
1	$\mathbb{Z}$	0	0	$\mathbb{Z}$	0	27
2	$\mathbb{Z}$	$\mathbb{Z}_4$	0	$\mathbb{Z}$	0	6
3	$\mathbb{Z}$	$\mathbb{Z}_5$	0	$\mathbb{Z}$	0	6

Table 4 : Homology classes (spaces made of 1 simplex)

3 Spaces Made of 2 Simplexes - Sum up Table

The following table sums up the homology classes of of all path connected compact homogeneous generalised triangulations made up of 2 simplexes.

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
1	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}^2$	0	12096
2	$\mathbb{Z}$	0	0	$\mathbb{Z}^2$	-1	10080
3	$\mathbb{Z}$	0	0	$\mathbb{Z}$	0	5880
4	$\mathbb{Z}$	$\mathbb{Z}_3$	0	$\mathbb{Z}$	0	2496
5	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}$	1	1296
6	$\mathbb{Z}$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	1224
7	$\mathbb{Z}$	0	$\mathbb{Z}_2$	0	1	576
8	$\mathbb{Z}$	$\mathbb{Z}$	0	$\mathbb{Z}$	-1	576
9	$\mathbb{Z}$	$\mathbb{Z}_7$	0	$\mathbb{Z}$	0	576
10	$\mathbb{Z}$	$\mathbb{Z}_5$	0	$\mathbb{Z}$	0	288
11	$\mathbb{Z}$	$\mathbb{Z}_8$	0	$\mathbb{Z}$	0	288
12	$\mathbb{Z}$	$\mathbb{Z}_5$	0	$\mathbb{Z}^2$	-1	144
13	$\mathbb{Z}$	$\mathbb{Z}_2 + \mathbb{Z}_2$	0	$\mathbb{Z}$	0	48

Table 5 : Homology classes (spaces made of 2 simplexes)

4 Spaces Made of 3 Simplexes - Sum up Table

The following table sums up the homology classes of of all path connected compact homogeneous generalised triangulations made up of 3 simplexes.

n	H_0	H_1	H_2	H_3	χ	Num. of Triang.
1	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}^3$	-1	32514048
2	$\mathbb{Z}$	0	0	$\mathbb{Z}^2$	-1	24178176
3	$\mathbb{Z}$	0	0	$\mathbb{Z}^3$	-2	21202560
4	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}^2$	0	12109824
5	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}$	1	5059584
6	$\mathbb{Z}$	0	0	$\mathbb{Z}$	0	4942080
7	$\mathbb{Z}$	0	$\mathbb{Z}_2$	0	1	1928448
8	$\mathbb{Z}$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	1679616
9	$\mathbb{Z}$	0	$\mathbb{Z}+\mathbb{Z}_2$	0	2	1202688
10	$\mathbb{Z}$	0	$\mathbb{Z}^2$	$\mathbb{Z}^2$	1	1161216
11	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}^2$	0	1119744
12	$\mathbb{Z}$	$\mathbb{Z}_3$	0	$\mathbb{Z}^2$	-1	705024
13	$\mathbb{Z}$	$\mathbb{Z}_3$	0	$\mathbb{Z}$	0	587520
14	$\mathbb{Z}$	0	$\mathbb{Z}^2$	$\mathbb{Z}$	2	580608
15	$\mathbb{Z}$	$\mathbb{Z}_2$	0	$\mathbb{Z}^2$	-1	539136
16	$\mathbb{Z}$	$\mathbb{Z}_4$	0	$\mathbb{Z}^2$	-1	539136
17	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	456192
18	$\mathbb{Z}$	$\mathbb{Z}_5$	0	$\mathbb{Z}^2$	-1	456192
19	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	1	393984
20	$\mathbb{Z}$	0	$\mathbb{Z}^2$	$\mathbb{Z}^3$	0	331776
21	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	1	331776
22	$\mathbb{Z}$	0	$\mathbb{Z}+\mathbb{Z}_2$	$\mathbb{Z}$	1	248832
23	$\mathbb{Z}$	$\mathbb{Z}_5$	0	$\mathbb{Z}$	0	200448
24	$\mathbb{Z}$	$\mathbb{Z}_4$	0	$\mathbb{Z}^3$	-2	179712
25	$\mathbb{Z}$	$\mathbb{Z}_5$	0	$\mathbb{Z}^3$	-2	179712
26	$\mathbb{Z}$	$\mathbb{Z}_4$	0	$\mathbb{Z}$	0	138240
27	$\mathbb{Z}$	$\mathbb{Z}_3$	$\mathbb{Z}$	$\mathbb{Z}$	1	124416
28	$\mathbb{Z}$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	0	1	124416
29	$\mathbb{Z}$	$\mathbb{Z}$	0	$\mathbb{Z}$	-1	82944
30	$\mathbb{Z}$	$\mathbb{Z}$	0	$\mathbb{Z}^2$	-2	82944
31	$\mathbb{Z}$	$\mathbb{Z}_7$	0	$\mathbb{Z}$	0	82944
32	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$	0	0	41472
33	$\mathbb{Z}$	$\mathbb{Z}_{11}$	0	$\mathbb{Z}$	0	41472
34	$\mathbb{Z}$	$\mathbb{Z}_6$	0	$\mathbb{Z}$	0	41472
35	$\mathbb{Z}$	$\mathbb{Z}_6$	$\mathbb{Z}_2$	0	1	41472
36	$\mathbb{Z}$	$\mathbb{Z}_8$	0	$\mathbb{Z}$	0	41472
37	$\mathbb{Z}$	$\mathbb{Z}_9$	0	$\mathbb{Z}$	0	41472
38	$\mathbb{Z}$	$\mathbb{Z}+\mathbb{Z}_2$	$\mathbb{Z}_2$	0	0	41472
39	$\mathbb{Z}$	$\mathbb{Z}_{10}$	0	$\mathbb{Z}$	0	20736
40	$\mathbb{Z}$	$\mathbb{Z}_{12}$	0	$\mathbb{Z}$	0	20736
41	$\mathbb{Z}$	$\mathbb{Z}_{13}$	0	$\mathbb{Z}$	0	20736
42	$\mathbb{Z}$	$\mathbb{Z}_2+\mathbb{Z}_2$	0	$\mathbb{Z}$	0	13824
43	$\mathbb{Z}$	$\mathbb{Z}_2+\mathbb{Z}_2$	$\mathbb{Z}_2$	0	1	13824

Table 6 : Homology classes (spaces made of 3 simplexes)