The axiomatic definition of infinitesimals

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Abstract

In this paper, we redefine the infinitesimals by using axiomatic method.

1 Introduction and results

There are several definitions of the infinitesimals:

1. Archimedes’ definition in the method of exhaustion.
2. Leibniz’s definition in differential calculus.
3. Cauchy’s definition in the theory of limit.
4. Robinson’s definition in non-standard analysis.

We try to propose the redefinition of infinitesimals in the following axiom:

Axiom 1. Let $e$ be the infinitesimal. Then:

1. $0 < e < 1$.
2. $|e - 0| \in \mathbb{R} \setminus \mathbb{Z}$.
3. $1/e < \infty$.

We hope the definition can help us to understand the behavior of infinitesimals.