P vs NP: Solutions of NP Problems

Abstract

The simplest solution is usually the best solution---Albert Einstein

Best news. After over 30 years of debating, the debate is over. Yes, P is equal to NP.

For the first time, NP problems have been solved in this paper. Techniques and formulas were developed and used to solve these problems as well as produce simple equations to help programmers apply the techniques. The techniques and formulas are based on an extended Ashanti fairness wisdom as exemplified below.

If two people A and B are to divide items of different sizes which are arranged from the largest size to the smallest size, the procedure will be as follows. In the first round, A chooses the largest size, followed by B choosing the next largest size. In the second round, B chooses first, followed by A. In the third round, A chooses first, followed by B and the process continues up to the last item. To abbreviate the sequence in the above choices, one obtains the sequence "A, BB, AA, BB, AA". Let A and B divide the sum of the whole numbers, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 as equally as possible, by merely always choosing the largest number. Then A chooses 10, B chooses 9 and 8, followed by A choosing 7 and 6; followed by B choosing 5 and 4; followed by A choosing 3 an 2; and finally, B chooses 1. The sum of A’s choices is 10 + 7 + 6 + 3 + 2 = 28; and the sum of B’s choices is 9 + 8 + 5 + 4 + 1 = 27, with error, plus or minus 0.5. Observe the sequence "A, BB, AA, BB, AA". Observe also that the sequence is not "AB, AB, AB, AB, AB" as one might think. If one were to use AB, AB, AB, AB, AB, the sum for A would be 10 + 8 + 6 + 4 + 2 = 30 and the sum for B would be 9 + 7 + 5 + 3 + 1 = 25, with error, plus or minus 2.5.

The reason why the sequence is "A, BB, AA, BB, AA", and not "AB, AB, AB, AB, AB" is as follows. In the first round, when A chooses first, followed by B, A has the advantage of choosing the larger number and B has the disadvantage of choosing the smaller number. In the second round, if A were to choose first, A would have had two consecutive advantages, and therefore, in the second round, B will choose first to produce the sequence AB, B or ABB. In the third round, A chooses first, because B chose first in the second round. After three rounds, the sequence would be A, BB, AA. When this technique was applied to 100 items of different masses, by mere combinations, the total mass of A’s items was equal to the total mass of B’s items. Similarly, for 1000 items of different masses, the total mass of A’s items was equal to the total mass of B’s items.

By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. School secretaries, office assistants can learn and apply the techniques covered.

The author also confirmed the notion that a method that solves one of these problems can also solve other NP problems. Consider the following two problems which are modifications of suggested NP problems from the Wikipedia (Simple English) website.

Problem 1: A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Problem 2: A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

After solving Problem 1, one was able to solve Problem 2 by mere inspection of the solutions to Problem 1.

Since a method that solves a single NP problem can solve other NP problems, and six problems have been solved in this paper, all NP problems (well-posed problems) can be solved. If all NP problems can be solved, then all NP problems are P problems and therefore, P is equal to NP.

The CMI Millennium Prize requirements have been satisfied.
Solutions of NP Problems

The following sample problems will be solved and analyzed. They are based on the suggested sample problems from the Wikipedia (Simple English) website. Many Thanks to Wikipedia.

Basis of the method used in solving the NP problems: Ratios

Example 1  (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally.
14,13,12,11,10,9,8,7,6,5,4,3,2,1.

Example 2a  Consider the existence of dollar bills with denominations $100, $99, $98,...,$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, divide the total value of these dollar bills equally between A and B.

Example 2b  Consider the existence of dollar bills with denominations $100, $99, $98,...,$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question:
(a) If a computer costs $2,000, can A afford to buy this computer?
(b) If a computer costs $3,000, can A afford to buy this computer?

Example 3  Let one randomly delete some of the bills in Example 2a, a previous example, and divide the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining.

Example 4  A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Example 5  A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

Example 6  A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B.
Basis of the method used in solving the NP problems: Ratios
Method 2 below is the method used for the solutions of the NP problems.

Example 1: Divide $12 between A and B in the ratio 1:2

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = \( \frac{1}{1+2} = \frac{1}{3} \)
Fraction of the money B receives = \( \frac{2}{1+2} = \frac{2}{3} \)

Step 2: Amount A receives = \( \frac{1}{3} \times 12 = 4 \)
Amount B receives = \( \frac{2}{3} \times 12 = 8 \)

Therefore, A receives $4, and B receives $8

(Method 1 above is from the author's book entitled "Power of Ratios" by A. A. Frempong, and published by Yellowtextbooks.com.)

Method 2 (The process method)

The ratio 1:2 means whenever A receives $1, B receives $2.

Step 1: In the first round, A receives $1, and B receives $2.
After the first round, the amount of money remaining is $12 - ($1 + $2) = $9.
Step 2: In the second round, from this $9, A receives $1 and B receives $2.
After the second round, the amount of money remaining = $9 - ($1 + $2) = $6
Step 3: In the third round, A receives $1 and B receives $2.
The amount remaining = $6 - ($1 + $2) = $3
Step 4: In the fourth and final round, A receives $1 and B receives $2.
The amount remaining = $3 - ($1 + $2) = 0
Step 5: A's total = $1 + $1 + $1 + $1 = $4
B's total = $2 + $2 + $2 + $2 = $8

Example 2: Divide $12 between A and B in the ratio 1:1

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = \( \frac{1}{1+1} = \frac{1}{2} \)
Fraction of the money B receives = \( \frac{1}{1+1} = \frac{1}{2} \)

Step 2: Amount A receives = \( \frac{1}{2} \times 12 = 6 \)
Amount B receives = \( \frac{1}{2} \times 12 = 6 \)

Therefore, A receives $6, and B receives $6.

Method 2 (The process method)

The ratio 1:1 means whenever A receives $1, B receives $1.

Step 1: In the first round, A receives $1, and B receives $1.
After the first round, the amount of money remaining is $12 - ($1 + $1) = $10
Step 2: In the second round, from this $10, A receives $1 and B receives $1.
After the second round, the amount of money remaining = $10 - ($1 + $1) = $8
Step 3: In the third round, A receives $1, and B receives $1.
The amount remaining = $8 - ($1 + $1) = $6
Step 4: In the fourth round, A receives $1 and B receives $1
The amount remaining = $6 - ($1 + $1) = 4
Step 5: In the fifth round, A receives $1 and B receives $1.
The amount remaining = $4 - ($1 + $1) = 2
Step 6: In the sixth and final round, A receives $1 and B receives $1.
The amount remaining = $2 - ($1 + $1) = 0
Step 7: A's total = $1 + $1 + $1 + $1 + $1 + $1 = $6.
B's total = $1 + $1 + $1 + $1 + $1 + $1 = $6.
Case 1: Only two devisors A and B

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally.

\[14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1.\]

Solution
For communication purposes, one will call the numbers to be divided the "dividends"; and one will call A and B the "divisors". Let the sum of A's choices be \(Q_A\), and let the sum of B's choices be \(Q_B\).

Step 1: Check to ensure that the numbers are arranged in decreasing order.
One will apply the wisdom method of the introduction.
That is, one applies "A, BB, AA, BB, AA, BB, AA, B"

Method 1 Using braces
Step 2: A chooses the first element, 14
Step 3: B chooses the next two elements, 13 and 12.,
Step 4: A chooses the next two elements 11, and 10, and the alternating consecutive choices continue to the end.

\[
\begin{array}{ccccccccccccc}
14, & 13, & 12, & 11, & 10, & 9, & 8, & 7, & 6, & 5, & 4, & 3, & 2, & 1 \\
A, & B, & \overline{A}, & \overline{B}, & A, & B, & \overline{A}, & \overline{B}, & A, & B, & A, & B, & A, & B \\
\end{array}
\]  
(1)

Step 5: Add the choices for A and add the choices for B.
\[Q_A = 14 + 11 + 10 + 7 + 6 + 3 + 2 \]
\[= 53 \]
\[Q_B = 13 + 12 + 9 + 8 + 5 + 4 + 1 \]
\[= 52 \]
The sum for A = 53; and the sum for B = 52.

Method 2 (Tabular form)
Step 1: List the dividends as shown below

<table>
<thead>
<tr>
<th>14</th>
<th>13</th>
<th>12</th>
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<th>10</th>
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</thead>
</table>

Step 2: Write the devisors A, BB, AA, BB, AA, etc, above the numbers, This is the choosing step.

<table>
<thead>
<tr>
<th>14</th>
<th>13</th>
<th>12</th>
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</table>


Step 3: Collect and add the corresponding (dividends) choices

<table>
<thead>
<tr>
<th>(Q_A)</th>
<th>(Q_B)</th>
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<tbody>
<tr>
<td>14</td>
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<td>2</td>
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</tbody>
</table>

Total: 53  52
Mathematical formulas for choosing the elements

Let \( a_1 = 14, a_2 = 13, a_3 = 12, a_4 = 11, a_5 = 10, \)
\( a_6 = 9, a_7 = 8, a_8 = 7, a_9 = 6, a_{10} = 5, a_{11} = 4, a_{12} = 3, a_{13} = 2, a_{14} = 1 \)

By experimentation, one obtains the following formulas for A and B.

\[
Q_A = a_1 + \sum_{n=2,4,6} a_{2n} + a_{2n+1} \quad (Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13})
\]

\[
Q_B = \sum_{n=1,3,5,7} a_{2n} + a_{2n+1} + a_{14} \quad (Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14})
\]

Apply the formulas to above (The above formulas are valid for only two divisors.)

\[
Q_A = a_1 + \sum_{n=2,4,6} a_{2n} + a_{2n+1} \\
= 14 + 11 + 10 + 7 + 6 + 3 + 2 \\
= 53
\]

\[
Q_B = \sum_{n=1,3,5,7} a_{2n} + a_{2n+1} + a_{14} \\
= 13 + 12 + 9 + 8 + 5 + 4 + 1 \\
= 52
\]

Note that the above formulas using the sigma notation are valid for only two divisors, A and B. For three divisors A, B, and C, different formulas would have to be derived, based on the solutions of the problem.
Example 2a  Consider the existence of dollar bills with denominations $100, $99, $98,...$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack.

Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Method 2a: Using the numerical values and braces

Apply, A, BB, AA, BB, AA. (as in Method 1 of Example 1)

Step 1: A chooses the first $100 bill. (Only a single item is removed).
Step 2: B chooses the next two bills, the $99 and $98 bills. (two items removed consecutively)
Step 3: A chooses the next two bills, the $97 and $96 bills, and the alternating removal continues to the end.

| 100 | 99 | 98 | 97 | 96 | 95 | 94 | 93 | 92 | 91 | 90 | 89 | 88 | 87 | 86 | 85 | 84 | 83 | 82 | 81 | 80 | 79 | 78 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

$Q_A = 100 + 97 + 96 + 93 + 92 + 91 + 90 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = 2525.$

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$Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54 + 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 = 2525.$

Conclusion: A receives $2525 and B receives $2525, Note the zero error for A and B.

Method 2b: Using tabular form

Step 1: Write the divisors A and B above the numbers (as done in Method 2 of Example 1)

| 100 | 99 | 98 | 97 | 96 | 95 | 94 | 93 | 92 | 91 | 90 | 89 | 88 | 87 | 86 | 85 | 84 | 83 | 82 | 81 | 80 | 79 | 78 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

| 80  | 79 | 78 | 77 | 76 | 75 | 74 | 73 | 72 | 71 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 | 62 | 61 | 60 | 59 | 58 | 57 | 56 | 55 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

| 60  | 59 | 58 | 57 | 56 | 55 | 54 | 53 | 52 | 51 | 50 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | 42 | 41 | 40 | 39 | 38 | 37 | 36 | 35 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

| 40  | 39 | 38 | 37 | 36 | 35 | 34 | 33 | 32 | 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

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Step 2: Collect and add the choices (dividends)

$Q_A = 100 + 97 + 96 + 93 + 92 + 91 + 90 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = 2525.$
\[ Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54 + 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 = 2525. \]

The above results are pleasantly astonishing. Of the 2^{100} possible ways to divide the above bills, the above technique and consequently the derived formulas divided the above mixture of bills into exactly two equal parts in value. Why has this technique been hiding for nearly 30 years? Note that the ratio \( Q_A : Q_B \) is 1:1.

Equations for above: \( Q_A = a_1 + \sum_{n=2,4,6,...}^{48} a_{2n} + a_{2n+1} + a_{100} \) and \( Q_B = \sum_{n=1,3,5,...}^{49} a_{2n} + a_{2n+1} \)

**Method 1b: Using term numbers and braces** Apply, A, BB, AA, BB, AA, ...

Using the term numbers and tabular form

| a_1 | a_2 | a_3 | a_4 | a_5 | a_6 | a_7 | a_8 | a_9 | a_{10} | a_{11} | a_{12} | a_{13} | a_{14} | a_{15} | a_{16} | a_{17} | a_{18} | a_19 | a_{20} |
| A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A | B | A |
| a_{21} | a_{22} | a_{23} | a_{24} | a_{25} | a_{26} | a_{27} | a_{28} | a_{29} | a_{30} | a_{31} | a_{32} | a_{33} | a_{34} | a_{35} | a_{36} | a_{37} | a_{38} | a_{39} | a_{40} |
| a_{41} | a_{42} | a_{43} | a_{44} | a_{45} | a_{46} | a_{47} | a_{48} | a_{49} | a_{50} | a_{51} | a_{52} | a_{53} | a_{54} | a_{55} | a_{56} | a_{57} | a_{58} | a_{59} | a_{60} |
| a_{61} | a_{62} | a_{63} | a_{64} | a_{65} | a_{66} | a_{67} | a_{68} | a_{69} | a_{70} | a_{71} | a_{72} | a_{73} | a_{74} | a_{75} | a_{76} | a_{77} | a_{78} | a_{79} | a_{80} |
| a_{81} | a_{82} | a_{83} | a_{84} | a_{85} | a_{86} | a_{87} | a_{88} | a_{89} | a_{90} | a_{91} | a_{92} | a_{93} | a_{94} | a_{95} | a_{96} | a_{97} | a_{98} | a_{99} | a_{100} |

Collect the terms for A and add them; and similarly collect the terms for B and add them.

**Theorem** \( Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33} + a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65} + a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} + a_{80} + a_{81} + a_{84} + a_{85} + a_{88} + a_{89} + a_{92} + a_{93} + a_{96} + a_{97} + a_{100} \)

**Theorem** \( Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{25} + a_{27} + a_{30} + a_{31} + a_{34} + a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} + a_{79} + a_{82} + a_{83} + a_{86} + a_{87} + a_{90} + a_{91} + a_{94} + a_{95} + a_{98} + a_{99} \)

Back to examples
Example 2b Consider the existence of dollar bills with denominations $100, $99, $98,...,$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs $2,000, can A afford to buy this computer?
(b) If a computer costs $3,000, can A afford to buy this computer?

Answers: (a) From the solution of Example 2a, A received $2,525, and therefore can afford to buy this computer. Yes. A can afford to buy this $2,000 computer.
(b) Since from the solution of Example 2a, A received $2,525, and the computer costs $3,000, A cannot afford to buy this $3,000 computer. No. A cannot afford to buy this $3,000 computer.

Example 3
Let one randomly delete some of the bills in Example 2a, a previous example, and divide as equally as possible the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining.

Solution: Using the numerical values and braces

\[
\begin{align*}
98,97,96,95,94,93,91,90,89,88,87,86,85,81,80,79,78,77,76, \\
A & \quad B \\
95,94,93,91,90,89,88,87,86,85,81,80,79,78,77,76, \\
A & \quad B \\
74,73,72,69,68,67,66,65,64,62,61,60,58,56,54,53,51,50, \\
A & \quad \quad B \\
47,46,45,43,41,40,37,36,35,34,32,31,30,29,26,25,24, \\
A & \quad B \\
23,22,21,20,19,18,16,14,13,12,11,10,9,8,7,5,4,2,1 \\
A & \quad B \\
\end{align*}
\]

For equality, interchange the 47 bill in \( Q_A \) and the 45 bill in \( Q_B \). Thus A gives $2 to B, resulting in equality of $1,935 each. Other bills can be interchanged.

Using term numbers

\[
\begin{align*}
Q_A &= a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33} \\
&\quad + a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65} \\
&\quad + a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} \\
Q_B &= a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34} \\
&\quad + a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66} \\
&\quad + a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} \\
\end{align*}
\]

Observe above that the last term for \( Q_A \) is \( a_{77} \) and the last term for \( Q_B \) is \( a_{78} \) (there are 78 terms)
Case 2: Three or more divisors

In the previous examples, for communication purposes, A and B were called the "divisors" and the numbers or terms to be divided were called "dividends". The concept of divisors A and B can be extended to three or more divisors such as A, B, C, or A, B, C, D, but in these cases, geometric figures will help keep track of the choices.

Geometric figures to keep track of the order and directions of the divisors
(For three or more divisors such as A, B, C; four divisors A, B, C,D)

For ABC:
Step 1: Go Clockwise ABC (In the first round, A chooses first and C chooses last))
Step 2: Begin with C and reverse the direction in Step 1 and go CBA.
   (Since C was at the largest disadvantage in the first round, by choosing last, C chooses
   chooses first in the second round followed by B)
Step 3: Begin with B and reverse previous direction (direction of C) and go clockwise BCA.
Step 4: Begin with A again, change previous direction (direction of B) and go counterclockwise ACB.

For ABCD:
Step 1: Go Clockwise ABCD, (first round)
Step 2: Begin with D and reverse the direction in Step 1 (direction of A) and go DCBA......
Step 3: Begin with C and reverse previous direction (direction of D) and go clockwise CDAB.
Step 4: Begin with B reverse previous direction (direction of C) and go counterclockwise BADC.
Step 5: Beginning again, reverse the direction but by coincidence go clockwise ABCD. (5th round)

For five divisors A, B, C, D, E

ABCDE, EDCBA, DEABC, CBAED, BCDEA
Step 1: Go Clockwise ABCDE
Step 2: Begin with E and reverse the direction in Step 1 and go EDCBA
Step 3: Begin with D and reverse previous direction and go clockwise DEABC
Step 4: Begin with C reverse previous direction and go counterclockwise CBAED
Step 5: Begin with B reverse previous direction and go clockwise BCDEA.
Step 6: Beginning again with A, reverse the direction but by coincidence, go clockwise ABCDE.
Example 4: A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Step 1: Arrange the items in decreasing order of their masses. Let the mass of the first item (largest) be 100 units, and let the masses of the rest of the items be respectively, 99, 98, 97, and so on down to smallest item of mass 1 unit. Let the 10 boxes be labeled A, B, C, D, E, F, G, H, J, and K. The ten boxes are to divide the 100 items. Imitate Example 2 but with 10 divisors.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>99</td>
<td>98</td>
<td>97</td>
<td>96</td>
<td>95</td>
<td>94</td>
<td>93</td>
<td>92</td>
<td>91</td>
</tr>
</tbody>
</table>

Step 2: Collect the choices for A, B, C, D, E, F, G, H, J, K.

<table>
<thead>
<tr>
<th>(Q_A)</th>
<th>(Q_B)</th>
<th>(Q_C)</th>
<th>(Q_D)</th>
<th>(Q_E)</th>
<th>(Q_F)</th>
<th>(Q_G)</th>
<th>(Q_H)</th>
<th>(Q_J)</th>
<th>(Q_K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>99</td>
<td>98</td>
<td>97</td>
<td>96</td>
<td>95</td>
<td>94</td>
<td>93</td>
<td>92</td>
<td>91</td>
</tr>
</tbody>
</table>

Guide 1: ABCDEFGHJK Guide 6: FEDCBAKJHG
Guide 2: KJHGFEDCBA Guide 7: EFGHKJABCD
Guide 3 JKABCDEFGH Guide 8: DBCAKJHGF
Guide 5: GHJKABCDFE Guide 10: BAKJHGFEDC

Imitate Example 2 but with 10 divisors.
Condition for sufficiency:
The 10 boxes would be sufficient to carry all the 100 items to the market if the mass of the contents of each box is equal to or less than 560 units. Since mass of the contents in each box is 505 units, which is less than 560 units, each box satisfies this sufficiency condition. Therefore, the 10 boxes would be sufficient to carry the 100 items to the market.


Method 4b Using the term numbers

| A | B | C | D | E | F | G | H | J | K | K | J | H | G | F | E | D | C | B | A |
| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ | $a_{17}$ | $a_{18}$ | $a_{19}$ | $a_{20}$ |

| J | K | A | B | C | D | E | F | G | H | H | G | F | E | D | C | B | A | K | J |
| $a_{21}$ | $a_{22}$ | $a_{23}$ | $a_{24}$ | $a_{25}$ | $a_{26}$ | $a_{27}$ | $a_{28}$ | $a_{29}$ | $a_{30}$ | $a_{31}$ | $a_{32}$ | $a_{33}$ | $a_{34}$ | $a_{35}$ | $a_{36}$ | $a_{37}$ | $a_{38}$ | $a_{39}$ | $a_{40}$ |

| G | H | J | K | A | B | C | D | E | F | E | D | C | B | A | K | J | H | G |
| $a_{41}$ | $a_{42}$ | $a_{43}$ | $a_{44}$ | $a_{45}$ | $a_{46}$ | $a_{47}$ | $a_{48}$ | $a_{49}$ | $a_{50}$ | $a_{51}$ | $a_{52}$ | $a_{53}$ | $a_{54}$ | $a_{55}$ | $a_{56}$ | $a_{57}$ | $a_{58}$ | $a_{59}$ | $a_{60}$ |

| E | F | G | H | J | K | A | B | C | D | C | B | A | K | J | H | G | F | E |
| $a_{61}$ | $a_{62}$ | $a_{63}$ | $a_{64}$ | $a_{65}$ | $a_{66}$ | $a_{67}$ | $a_{68}$ | $a_{69}$ | $a_{70}$ | $a_{71}$ | $a_{72}$ | $a_{73}$ | $a_{74}$ | $a_{75}$ | $a_{76}$ | $a_{77}$ | $a_{78}$ | $a_{79}$ | $a_{80}$ |

| C | D | E | F | G | H | J | K | A | B | B | A | K | J | H | G | F | E | D | C |
| $a_{81}$ | $a_{82}$ | $a_{83}$ | $a_{84}$ | $a_{85}$ | $a_{86}$ | $a_{87}$ | $a_{88}$ | $a_{89}$ | $a_{90}$ | $a_{91}$ | $a_{92}$ | $a_{93}$ | $a_{94}$ | $a_{95}$ | $a_{96}$ | $a_{97}$ | $a_{98}$ | $a_{99}$ | $a_{100}$ |

Collect the terms for $A, B, C, D, E, F, G, H, J, K$:

- $Q_A = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_B = a_2 + a_1 + a_4 + a_3 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_C = a_3 + a_1 + a_5 + a_4 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_D = a_4 + a_1 + a_6 + a_5 + a_8 + a_7 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_E = a_5 + a_1 + a_7 + a_6 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_F = a_6 + a_1 + a_8 + a_7 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_G = a_7 + a_1 + a_9 + a_8 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_H = a_8 + a_1 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_J = a_9 + a_1 + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$
- $Q_K = a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}$

Sub-Conclusion

The fairness wisdom method has performed perfectly.

Observe above in Step 2 that the totals for $Q_A, Q_B, Q_C, Q_D, Q_E, Q_F, Q_G, Q_H, Q_J, Q_K$ are all the same. The technique applied picked combinations to produce these equal totals.

Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order, a task a computer performs very fast.

In the next example, Example 5, one will confirm the notion that a method that solves one of the NP problems can be used to solve other similar problems. One will use the results of the above example Example 4b to do the next problem.
Example 5: A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

Step 1: Final Exam Schedule 8AM – 6PM

Let the course numbers be $a_1, a_2, a_3, \ldots, a_{100}$ Using the result of Example 4b

Step 2: Collect the choices for A, B, C, D, E, F, G, H, J, K

Final Exam Schedule: 8 AM-6 PM

The final exam for every course has been scheduled. However, if a student takes for example, Course $a_1$ and course $a_{20}$, because the duration for the final exams for these two courses is 8-9 AM, the student cannot take the final exams for these two courses simultaneously. Therefore, it is not possible to prepare a schedule to allow every student to take the final exams for all registered courses on the same day. However, below is what is possible.
In order for every student to take the final exam for all courses registered for, ten days would be needed as shown below, where the course numbers are \(a_1, a_2, a_3, \ldots, a_{100}\).

<table>
<thead>
<tr>
<th>DAY</th>
<th>(Q_A)</th>
<th>(Q_B)</th>
<th>(Q_C)</th>
<th>(Q_D)</th>
<th>(Q_E)</th>
<th>(Q_F)</th>
<th>(Q_G)</th>
<th>(Q_H)</th>
<th>(Q_I)</th>
<th>(Q_J)</th>
<th>(Q_K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a_1)</td>
<td>(a_2)</td>
<td>(a_3)</td>
<td>(a_4)</td>
<td>(a_5)</td>
<td>(a_6)</td>
<td>(a_7)</td>
<td>(a_8)</td>
<td>(a_9)</td>
<td>(a_{10})</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(a_{20})</td>
<td>(a_{19})</td>
<td>(a_{18})</td>
<td>(a_{17})</td>
<td>(a_{16})</td>
<td>(a_{15})</td>
<td>(a_{14})</td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>(a_{23})</td>
<td>(a_{24})</td>
<td>(a_{25})</td>
<td>(a_{26})</td>
<td>(a_{27})</td>
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<td>(a_{29})</td>
<td>(a_{30})</td>
<td>(a_{21})</td>
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<td></td>
</tr>
<tr>
<td>4</td>
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<td>(a_{37})</td>
<td>(a_{36})</td>
<td>(a_{35})</td>
<td>(a_{34})</td>
<td>(a_{33})</td>
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<tr>
<td>5</td>
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<td>(a_{46})</td>
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<td>(a_{48})</td>
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</tr>
<tr>
<td>6</td>
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<td>(a_{55})</td>
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</tr>
<tr>
<td>7</td>
<td>(a_{67})</td>
<td>(a_{68})</td>
<td>(a_{69})</td>
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<td>(a_{72})</td>
<td>(a_{71})</td>
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<td>9</td>
<td>(a_{89})</td>
<td>(a_{90})</td>
<td>(a_{81})</td>
<td>(a_{82})</td>
<td>(a_{83})</td>
<td>(a_{84})</td>
<td>(a_{85})</td>
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</tr>
<tr>
<td>10</td>
<td>(a_{92})</td>
<td>(a_{91})</td>
<td>(a_{100})</td>
<td>(a_{99})</td>
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<td>(a_{96})</td>
<td>(a_{95})</td>
<td>(a_{94})</td>
<td>(a_{93})</td>
<td></td>
</tr>
</tbody>
</table>

Observe how one used the results of the previous example (Example 4b) to solve the above problem, Example 5.
In the next example, one will cover an example involving 1000 items, which will be similar to Example 2a.
Example 6  A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. Review Example 2a before proceeding.

| 200| 199| 198| 197| 196| 195| 194| 193| 192| 191| 190| 189| 188| 187| 186| 185| 184| 183| 182| 181|   |
| 180| 179| 178| 177| 176| 175| 174| 173| 172| 171| 170| 169| 168| 167| 166| 165| 164| 163| 162| 161|   |
| 160| 159| 158| 157| 156| 155| 154| 153| 152| 151| 150| 149| 148| 147| 146| 145| 144| 143| 142| 141|   |
| 140| 139| 138| 137| 136| 135| 134| 133| 132| 131| 130| 129| 128| 127| 126| 125| 124| 123| 122| 121|   |
| 120| 119| 118| 117| 116| 115| 114| 113| 112| 111| 110| 109| 108| 107| 106| 105| 104| 103| 102| 101|   |
| 100| 99 | 98 | 97 | 96 | 95 | 94 | 93 | 92 | 91 | 90 | 89 | 88 | 87 | 86 | 85 | 84 | 83 | 82 | 81 |   |
| 80 | 79 | 78 | 77 | 76 | 75 | 74 | 73 | 72 | 71 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 | 62 | 61 |   |
| 60 | 59 | 58 | 57 | 56 | 55 | 54 | 53 | 52 | 51 | 50 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | 42 | 41 |   |
| 40 | 39 | 38 | 37 | 36 | 35 | 34 | 33 | 32 | 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 |   |
| 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 |  9 |  8 |  7 |  6 |  5 |  4 |  3 |  2 |  1 |
Concrete masses for Pile A

Step 2: Collect and add the Choices (dividends):

\[ Q_{A1} = 1000 + 997 + 996 + 993 + 992 + 989 + 988 + 985 + 984 + 981 + 980 + 977 + 976 + 973 + 972 + 969 + 968 + 965 + 964 + 961 + 960 + 957 + 956 + 953 + 952 + 949 + 948 + 945 + 944 + 941 + 940 + 937 + 936 + 933 + 932 + 929 + 928 + 925 + 924 + 921 + 920 + 917 + 916 + 913 + 912 + 909 + 908 + 905 + 904 + 901 = 47,525 \]

\[ Q_A = 900 + 897 + 896 + 893 + 892 + 889 + 888 + 885 + 884 + 881 + 880 + 877 + 876 + 873 + 872 + 869 + 868 + 865 + 864 + 861 + 860 + 857 + 856 + 853 + 852 + 849 + 848 + 845 + 844 + 841 + 840 + 837 + 836 + 833 + 832 + 829 + 828 + 825 + 824 + 821 + 820 + 817 + 816 + 813 + 812 + 809 + 808 + 805 + 804 + 801 = 42,525 \]

\[ Q_A = 800 + 797 + 796 + 793 + 792 + 789 + 788 + 785 + 784 + 781 + 780 + 777 + 776 + 773 + 772 + 769 + 768 + 765 + 764 + 761 + 760 + 757 + 756 + 753 + 752 + 749 + 748 + 745 + 744 + 741 + 740 + 737 + 736 + 733 + 732 + 729 + 728 + 725 + 724 + 721 + 720 + 717 + 716 + 713 + 712 + 709 + 708 + 705 + 704 + 701 = 37,525 \]

\[ Q_A = 700 + 697 + 696 + 693 + 692 + 689 + 688 + 685 + 684 + 681 + 680 + 677 + 676 + 673 + 672 + 669 + 668 + 665 + 664 + 661 + 660 + 657 + 656 + 653 + 652 + 649 + 648 + 645 + 644 + 641 + 640 + 637 + 636 + 633 + 632 + 629 + 628 + 625 + 624 + 621 + 620 + 617 + 616 + 613 + 612 + 609 + 608 + 605 + 604 + 601 = 32,525 \]

\[ Q_A = 600 + 597 + 596 + 593 + 592 + 589 + 588 + 585 + 584 + 581 + 580 + 577 + 576 + 573 + 572 + 569 + 568 + 565 + 564 + 561 + 560 + 557 + 556 + 553 + 552 + 549 + 548 + 545 + 544 + 541 + 540 + 537 + 536 + 533 + 532 + 529 + 528 + 525 + 524 + 521 + 520 + 517 + 516 + 513 + 512 + 509 + 508 + 505 + 504 + 501 = 27,525 \]

\[ Q_A = 500 + 497 + 496 + 493 + 492 + 489 + 488 + 485 + 484 + 481 + 480 + 477 + 476 + 473 + 472 + 469 + 468 + 465 + 464 + 461 + 460 + 457 + 456 + 453 + 452 + 449 + 448 + 445 + 444 + 441 + 440 + 437 + 436 + 433 + 432 + 429 + 428 + 425 + 424 + 421 + 420 + 417 + 416 + 413 + 412 + 409 + 408 + 405 + 404 + 401 = 22,525 \]

\[ Q_A = 400 + 397 + 396 + 393 + 392 + 389 + 388 + 385 + 384 + 381 + 380 + 377 + 376 + 373 + 372 + 369 + 368 + 365 + 364 + 361 + 360 + 357 + 356 + 353 + 352 + 349 + 348 + 345 + 344 + 341 + 340 + 337 + 336 + 333 + 332 + 329 + 328 + 325 + 324 + 321 + 320 + 317 + 316 + 313 + 312 + 309 + 308 + 305 + 304 + 301 = 17,525 \]

\[ Q_A = 300 + 297 + 296 + 293 + 292 + 289 + 288 + 285 + 284 + 281 + 280 + 277 + 276 + 273 + 272 + 269 + 268 + 265 + 264 + 261 + 260 + 257 + 256 + 253 + 252 + 249 + 248 + 245 + 244 + 241 + 240 + 237 + 236 + 233 + 232 + 229 + 228 + 225 + 224 + 221 + 220 + 217 + 216 + 213 + 212 + 209 + 208 + 205 + 204 + 201 = 12,525 \]

\[ Q_A = 200 + 197 + 196 + 193 + 192 + 189 + 188 + 185 + 184 + 181 + 180 + 177 + 176 + 173 + 172 + 169 + 168 + 165 + 164 + 161 + 160 + 157 + 156 + 153 + 152 + 149 + 148 + 145 + 144 + 141 + 140 + 137 + 136 + 133 + 132 + 129 + 128 + 125 + 124 + 121 + 120 + 117 + 116 + 113 + 112 + 109 + 108 + 105 + 104 + 101 = 7,525 \]

\[ Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = 2,525 \]

Total for \( Q_A = 250,250 \) units
Concrete masses for Pile B

\[ Q_B = \text{sum of all masses for } Pile B \]

\[ Q_B = 250,250 \text{ units} \]
Overall Conclusion

An extended Ashanti fairness wisdom technique was applied to solve six NP problems. In the first step, one did not think about mathematical models to apply, but rather thought about the process involved, and this led to the development of the fairness wisdom technique which was applied to a set of 100 items of different values or masses. Two people A and B were able to divide the items equally by merely choosing in turns from a set of ordered items. The total value or mass of A’s items was found to be equal total value or mass of B’s items, and these results are combinations of the items of different values or masses. This problem was followed by similar problems in which in addition to dividing the masses, certain conditions were to be satisfied before definite answers could be given to the problems. It is very pleasing that such a simple technique can produce desired combinations. Even though the solutions are combinations, no knowledge of combinatorial mathematics was involved or required. In fact, if one was asked to name the mathematics involved, the answer would be arithmetic. Thus, high school and middle school graduates could be taught the technique involved. From the solutions, formulas or simple equations were produced to help programmers apply the techniques. Confirmed was the notion that a method that solves one of these problems can also solve other NP problems.

Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order, a task a computer performs very fast. The technique was also applied to 1000 items, and the results were perfect, just like the results for the 100 items.

For 100 items, out of the possible $2^{100}$ combinations, the technique produced the desired combinations, and similarly, for 1000 items, out of $2^{1000}$ possible combinations, the technique produced the desired combinations. Therefore, the technique covered does not care whether there are $2^{100}$ or $2^{1000}$ possibilities. The desired and correct combinations are always produced. This technique can divide a set of items of different lengths, masses, volumes, value, or sizes into equal parts by combinations only.

There are social consequences of the method and principles used to divide the set of items into equal totals. The results can be applied by government agencies in the distribution of goods and services. Management personnel should be aware of the principles involved in the above technique. From the elementary school, through high school, and perhaps college, students should be taught the principles in the above wisdom technique, since throughout life, one is going to encounter situations in which two or more people are asked to choose in turns, from items of different values or sizes, and in this case, the sequence by which the choices are made matters; one may be either a participant or one may be in charge of the distribution process.

By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. School secretaries and office assistants can learn and apply the techniques covered.

Finally, if a method can solve one NP problem, that method can also solve other NP problems. Since six NP problems have been solved in this paper, the formerly NP problems are now P problems, and therefore, it is concluded that P is equal to NP.

Perhaps, one may make the following statements.

1. NP plus human ability equals P.
2. NP plus human inability is not equal to P.
3. NP minus human inability equals P.

References:

For paper edition of the above paper, see Appendix 6 of the book entitled "Power of Ratios" by A. A. Frempong, published by Yellowtextbooks.com. After solving the NP problems and reviewing the solutions, the author realized that a ratio process had been applied in solving the NP solutions, and in the beginning, did not consider including the solutions of NP problems in the "Power of Ratios" book which contains also the author's previous solutions of the Navier-Stokes equations plus solutions of the magnetohydrodynamic equations, (viXra.org). The "Power of Ratios" book covers definition of ratio and applications of ratios in mathematics, science, pharmacology, engineering, economics and business fields.