Lucasian Primality Criterion for Specific Class of $k \cdot b^n - 1$

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Abstract: Conjectured polynomial time primality test for specific class of numbers of the form $k \cdot b^n - 1$ is introduced.

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1 Introduction

In 1969 Hans Riesel provided polynomial time primality test for numbers of the form $k \cdot 2^n - 1$ with n > 2, k odd and $k < 2^n$ see Theorem 5 in [1]. In this note I present polynomial time primality test for specific class of numbers of the form $k \cdot b^n - 1$.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4} \right)^m + \left(x + \sqrt{x^2 - 4} \right)^m \right)$, where m and x are nonnegative integers .

Conjecture 2.1. Let $N = k \cdot b^n - 1$ such that n > 2 , $k < b^n$ and

$$\begin{cases} k \equiv 21 \pmod{30} \ with \ b \equiv 2 \pmod{10} \ and \ n \equiv 2, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \ with \ b \equiv 4 \pmod{10} \ and \ n \equiv 1, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \ with \ b \equiv 8 \pmod{10} \ and \ n \equiv 1, 2 \pmod{4} \end{cases}$$
 Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(3))$, thus N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

References

[1] Riesel, Hans (1969), "Lucasian Criteria for the Primality of $k \cdot 2^n - 1$ ", Mathematics of Computation (American Mathematical Society), 23 (108): 869-875.