Five conjectures on a diophantine equation involving two primes and a square of prime

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I make five conjectures about the primes \( r, t \) and the square of prime \( p^2 \), which appears as solutions in the diophantine equation \( 120*n*q*r + 1 = p^2 \), where \( n \) is non-null positive integer.

Conjecture 1:

For any \( n \) non-null positive integer there exist \( q, r \) primes such that \( 120*n*q*r + 1 = p^2 \), where \( p \) is prime or a power of prime.

Conjecture 2:

For any \( q \) odd prime there exist \( n \) non-null positive integer and \( r \) prime such that \( 120*n*q*r + 1 = p^2 \), where \( p \) is prime or a power of prime.

Conjecture 3:

For any \( q, r \) odd primes there exist \( n \) non-null positive integer such that \( 120*n*q*r + 1 = p^2 \), where \( p \) is prime or a power of prime.

Conjecture 4:

For any \( n \) non-null positive integer and any \( q \) prime there exist \( r \) prime such that \( 120*n*q*r + 1 = p^2 \), where \( p \) is prime or a power of prime.

Examples:

: For \([n, q] = [1, 5]\) there exist \( r = 17 \) such that \( p = 101 \) prime; also \( r = 37 \) such that \( p = 149 \) prime;
: For \([n, q] = [1, 7]\) there exist \( r = 23 \) such that \( p = 139 \) prime; also \( r = 53 \) such that \( p = 211 \) prime;
: For \([n, q] = [1, 11]\) there exist \( r = 13 \) such that \( p = 131 \) prime; also \( r = 83 \) such that \( p = 331 \) prime;
: For \([n, q] = [2, 5]\) there exist \( r = 19 \) such that \( p = 151 \) prime;
For \([n, q] = [2, 7]\) there exist \(r = 3\) such that \(p = 71\) prime; also \(r = 17\) such that \(p = 169\) square of prime;

For \([n, q] = [2, 11]\) there exist \(r = 3\) such that \(p = 89\) prime;

For \([n, q] = [3, 7]\) there exist \(r = 13\) such that \(p = 181\) prime;

For \([n, q] = [3, 11]\) there exist \(r = 3\) such that \(p = 109\) prime;

For \([n, q] = [4, 5]\) there exist \(r = 67\) such that \(p = 401\) prime;

For \([n, q] = [4, 7]\) there exist \(r = 17\) such that \(p = 239\) prime;

For \([n, q] = [4, 11]\) there exist \(r = 11\) such that \(p = 241\) prime.

**Conjecture 5:**

For any \(n\) non-null positive integer there exist \(q\) prime such that \(120*n*q^2 + 1 = p^2\), where \(p\) is prime or a power of prime.

Note, for instance, the case from the examples below: \(480*11^2 + 1 = 241^2\).