Conjectured Primality Test for Specific Class of $3 \cdot b^n - 1$

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August 16, 2014

Abstract: Conjectured polynomial time primality test for specific class of numbers of the form $3 \cdot b^n - 1$ is introduced.

Keywords: Primality test, Polynomial time, Prime numbers.

AMS Classification: 11A51.

1 Introduction

In 1969 Hans Riesel provided polynomial time primality test for numbers of the form $3 \cdot 2^n - 1$ with n > 2, see Theorem 5 in [1]. In this note I present polynomial time primality test for specific class of numbers of the form $3 \cdot b^n - 1$ that may be considered as generalization of the Riesel primality test for $3 \cdot 2^n - 1$.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4} \right)^m + \left(x + \sqrt{x^2 - 4} \right)^m \right)$, where m and x are nonnegative integers.

Conjecture 2.1. Let $N = 3 \cdot b^n - 1$ such that n > 2 and

$$\begin{cases} b \equiv 2 \pmod{10} \text{ with } n \equiv 0,3 \pmod{4} \\ b \equiv 4 \pmod{10} \text{ with } n \equiv 0,2 \pmod{4} \\ b \equiv 6 \pmod{10} \text{ with } n \equiv 0,1,2,3 \pmod{4} \\ b \equiv 8 \pmod{10} \text{ with } n \equiv 0,1 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{3b/2}(P_{b/2}(5778))$, then N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

References

[1] Riesel, Hans (1969), "Lucasian Criteria for the Primality of $k \cdot 2^n - 1$ ", *Mathematics of Computation* (AmericanMathematical Society), 23 (108): 869-875.