The surface contains zeros

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Abstract
The critical line lies on a surface. And the critical line inherits the characteristics from this surface. Then, the location of the critical line can be determined.

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1 Introduction
Young [You10] gave Levinson’s result on the critical line but in a simpler way. Bredberg [Bre11] showed the very long zero-free interval in the critical line. Milinovich [Mil11] showed that the moments of \( \zeta \) occur between two bounds. Herichi and Lapidus [HL12] reformulated the Riemann hypothesis from the invertibility of spectral operator. Rezvyakova [Rez12] obtained that the Selberg’s result on the critical line holds for \( L \)-functions.

In this paper, we show the exact location of the critical line. Our paper is organized as follows. In §2, we deduce many results on the complex plane. In §3, the main results is given.

2 The geometric considerations
We consider the complex plane \( \mathbb{C} \). We also consider \( \mathbb{C} = \mathbb{R} \times \mathbb{R} \). The plane consists of the real axis \( X \) and the imaginary axis \( Y \). For the quadrants I, II, III and IV, we denote by \( K_1, K_2, K_3 \) and \( K_4 \) respectively. We define the set \( -E \) of \( E \), viz.,

\[
-E := \{ x : x = -a \text{ for some } a \in E \}.
\]

Proposition 2.1. \( K_1 = -K_3 \).
Proof. We know that \( K_1 = \{(x, y) : x, y > 0\} \) and \( K_3 = \{(-x, -y) : x, y > 0\} \). Then by definition, \(-K_3 = \{(x, y) : x, y > 0\}\). □

**Proposition 2.2.** \( K_2 = -K_1 \).

Proof. We know that \( K_2 = \{(-x, y) : x, y > 0\} \) and \( K_4 = \{(x, -y) : x, y > 0\} \). Then by definition, \(-K_4 = \{(-x, y) : x, y > 0\}\). □

**Theorem 2.3.** All quadrants are located.

Proof. The rectangular coordinate has four quadrants. In the quadrants I and III, \( x, y > 0 \) and \( x, y < 0 \). In the quadrants II and IV, \( x < 0, y > 0 \) and \( x > 0, y < 0 \). □

**Example 2.4.** A point \( \neq \) a hole.

Proof. A point is a singleton. And the singleton is not empty. However, a hole is a discontinuity on some set. This means that the set is free from an element. □

**Proposition 2.5.** Every point of convergence is invariant.

Proof. Let \( x_n \) converges to \( a \). We know that the number of \( x_n \) is \( > 1 \). However, \( x_n \neq a \) then \( a \) is not converging and the number of \( a = 1 \). □

**Example 2.6.** The origin \( 0 \) does not converge to everywhere.

Proof. Obviously, \( 0 \) is a point of convergence. □

**Lemma 2.7.** The point \( \infty \) converges to itself.

Proof. We know that, every large number \( p \) converges to \( \infty \). If we consider \( \infty \) as a large number then the statement follows. □

**Theorem 2.8.** Any line must have a mid-point.

Proof. Given a line \( \ell \). The line \( \ell \) has the length \( L \). And the half of \( L \) is \( L/2 \). Then a point on \( \ell \) of the distance \( L/2 \) be considered as a mid-point. □

**Theorem 2.9.** A point is constructible.

Proof. Let \{pt\} denote a singleton. Because we can construct the set \{pt\}, then we can construct a point. □

**Theorem 2.10.** The intersected points imply a line.

Proof. Given a point \( X \). Generate the other intersected point with respect to \( X \). Continue this process until we obtain a line. □
Theorem 2.11. There are no holes in $\mathbb{R}$.

Proof. A hole means a discontinuity in $\mathbb{R}$. Because $\mathbb{R}$ is the archimedean dense ordered field, then $\mathbb{R}$ is continuous in everywhere. \hfill $\square$

Theorem 2.12. The hole in $\mathbb{C} - \{0\}$ is unique.

Proof. It is clear that the origin is unique. Thereby, the hole in the origin is unique. \hfill $\square$

$H$ denotes the upper-half plane.

Example 2.13. The upper-half plane is unique.

Proof. Given an upper-half plane $H$. Then the upper-half plane is unique because the quadrants $K_1$ and $K_2$ are unique. \hfill $\square$

Lemma 2.14. $\partial H = X$.

Proof. Obvious. \hfill $\square$

Theorem 2.15. There is no the indivisibles.

Proof. Given an interval $I$. Dividing $I$ by $n$ sub-interval (denote by $I/n$), we set $I_1 := I/n$. Choose a sub-interval in $I_1$. Dividing the sub-interval in $I_1$, by $n$ sub-interval again (denote by $I_1/n$), we set $I_2 := I_1/n = I/n^2$. Continue this process, hence we do not find the indivisibility of interval. \hfill $\square$

Corollary 2.16. The interval $(0, 1)$ is divisible at everywhere.

Definition 2.17. An infinitesimal square is called a block.

Theorem 2.18. The complex plane can be decomposed into the collection of blocks.

Proof. By definition, the collection of blocks can form a plane. \hfill $\square$

Corollary 2.19. All blocks in the complex plane are located.

Theorem 2.20. The universe is not empty.

Proof. $\emptyset^c = V$. \hfill $\square$

Theorem 2.21. The infinity $\infty$ does not occur in the polar coordinate.

Proof. In the polar coordinate, the line is cyclic. The point $\infty$ in the rectangular coordinate is replaced by 0 and $\pi/2$. \hfill $\square$
Example 2.22. The complex plane has no boundaries.

Proof. We know that the complex plane is open.

The set \((0, 1)\) is called the infinitesimal generator.

Proposition 2.23. \((0, 1) = (0, 1/n)\) for \(n \geq 1\).

Definition 2.24. A real number near zero in the right-side of zero is denoted by \(0^+_\text{near}\).

Proposition 2.25. For \(\Lambda \in (0, 1)\), \(0^+_\text{near} = \Lambda\).

The set is said to be periodic if and only if the set is of the form \(\pi Q\). 

A\((R)\) denotes the area of region \(R\).

Theorem 2.26. Given a closed region \(R\). Then the area \(A(R)\) is

\[
A(R) = (1 + \cdots + 1) + \left(\frac{1}{n_1} + \cdots + \frac{1}{n_k}\right),
\]

for some \(k \in \mathbb{Z}^+\).

Proof. A closed region must consists of the unit squares and the segments of a unit square. The area of a unit square = 1 and the area of the segment is the proper fraction \(1/n\).

Lemma 2.27. Let \(N\) be a positive integer and let \(Q\) be a square. Then 
\([-N, N] \times [-N, N] = 4N^2Q\).

Proof. In the quadrant \(K_1\), there is \(N^2\) squares. Nonetheless, there are four quadrants in the finite plane. The set \([-N, N] \times [-N, N]\) also forms the finite plane.

Proposition 2.28. Every lattice point is located.

Proof. It is clear.

Remark 2.29. Let \(e\) denote the set of unit squares. Then the set \(e\) in \(K_1\) is dense.

Proof. Given a unit square \(w\). Let \(|w|\) denote the area of \(w\). Then by the density, there exists \(w_2\) such that \(|w_1| < |w_2| < |w_3|\) for any \(w_1\) and \(w_3\).

Proposition 2.30. A line is a diameter of an unbounded circle.
Proof. Given an unbounded circle $C$. Because $C$ is a circle, then $C$ has a diameter. However, $C$ is unbounded and the diameter in $C$ can be prolonged indefinitely.

**Lemma 2.31.** Let $R$ be the radius of a circle, $c$ the chord length, $s$ the arc length and $d$ the height of the triangular portion. Then the area of intersection of two isoradius circles is

$$A = Rs - cd.$$

**Proof.** The area of the segment is

$$A_{\text{segment}} = \frac{1}{2}(Rs - cd).$$

And the intersection of two isoradius circles consists of two segments of each circle. However, the area of two segments is $2A_{\text{segment}} = Rs - cd$.

**Proposition 2.32.** Let $R$ denote the rectangle. Let $\Delta^\perp$ denote the right triangle. Then $\Delta^\perp = R/2$.

**Proof.** The rectangle $R$ can be broken into two right triangles $\Delta^\perp$.

**Theorem 2.33.** A rectangle is the model of universe.

**Proof.** Suppose the rectangle $V$ is the universe. Then there is any set $X$ in $V$ and its complement $X^c$.

Let $\alpha$ denote the area of $C$.

**Proposition 2.34.** $A(H) = \alpha/2$.

**Proof.** We know that $H$ is the half of the complex plane $C$.

**Lemma 2.35.** Any area of any region in the upper-half plane is always positive.

**Proof.** It is clear that the upper-half plane is above the $x$-axis.

**Theorem 2.36.** $A(C) = 4S^2$.

**Proof.** Observe the first quadrant $K_1$. Consider a unit square and $K_1$ is decomposed into many unit squares. We know that the area of unit square $= 1$. Because of the side of $K_1$ is infinite, then we define the side $S := 1 + 1 + 1 + \cdots$. Then $A(K_1) = S^2$. And the complex plane $C$ consists of four quadrants.
3 Critical lines

Let $\ell_{\text{crit}}$ denote the critical line. Let $Z$ denote the critical strip. And we denote the line $\sigma = 1$ by $1_{\text{line}}$. Let $r^*$ denote the zero-free region.

$G$ denotes the right-half plane. A trivial zero of the Riemann zeta-function is called a zerox. And a non-trivial zero of the Riemann zeta-function is called a zeron.

Let $V$ denote the number of vertices. Indeed, in a rectangle, $V = 4$.

**Proposition 3.1.** For $V = 4$, $1/2 = 2/V$.

**Example 3.2.** The zerons lie on two quadrants.

*Proof.* The zerons lie on the right-half plane $G$. And $G$ consists of two quadrants $K_1$ and $K_4$. □

We assume $\ell_{\text{crit}}$ as a ray and $[0, 1]$ as a lens.

**Theorem 3.3.** The zero-free region $r^*$ cannot be continued into the region $[0; 0.4]$.

*Proof.* Obvious. Because for $0 \leq \sigma \leq 0.4$, $\sigma \not\approx 1$. □

**Lemma 3.4.** The Riemann hypothesis holds in $\mathbb{C} - \{\pm n, \pm n\}$.

*Proof.* Given the set $\mathbb{C} - \{\pm n, \pm n\}$. This means $\mathbb{C}$ without the lattice points. The lattice points are the ordered pair of integers. If the lattice points are deleted then the remainder is the ordered pair of non-integer real numbers. □

**Theorem 3.5.** $A(Z) = 0$.

*Proof.* The critical strip $Z$ consists of two regions $Z_1$ and $Z_2$. $Z_1$ is the positive height critical strip and $Z_2$ is the negative height critical strip. Let $A(Z_1) = F$, then $A(Z_2) = -F$. Hence, $A(Z_1) + A(Z_2) = 0$. □

**Lemma 3.6.** $(1/2, 0)$ is a center of critical strip.

*Proof.* The critical strip $Z$ can be represented as a rectangle. This rectangle has a center. We know that $[0, 1]$ and $\ell_{\text{crit}}$ are the symmetry axes. The intersection of two symmetry axes is the center. And the intersection lies on $(1/2, 0)$. □

**Lemma 3.7.** $1_{\text{line}}$ is a boundary of the zero-free region $r^*$.
Proof. The zero-free region $r^*$ is bounded left and unbounded right. The bound is $I_{\text{line}}$. □


Proof. Obvious. □

Lemma 3.9. The critical strip $Z$ occurs at the quadrants $K_1$ and $K_4$.

Proof. The critical strip $Z$ lies on the right-half plane $G$. And the right-half plane $G$ consists of the quadrants $K_1$ and $K_4$. □

Theorem 3.10. $r^*$ is free from the critical line.

Proof. There are no zerons in $r^*$. Then this implies that there is no the critical line in $r^*$. □

Theorem 3.11. The critical line is located.

Proof. The critical strip consists of two regions. The regions are contiguous. We color each region with two different colors, say red and blue. Because we want one color no more dominant that the other color, then the color red and blue obtain the balanced regions. Hence, there is a bound between the red region and blue region. Consequently, this bound is a critical line. □

References


