A conjecture on the squares of primes of the form $6k + 1$

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Abstract. In this paper I make a conjecture on the squares of primes of the form $6k + 1$, conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

Conjecture:

For any square of a prime $p$ of the form $p = 6k + 1$ is true at least one of the following six statements:

1. $p^2$ can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
2. $p^2$ can be deconcatenated into a semiprime $3^n q$ and a number congruent to 1 modulo 6;
3. $p^2$ can be deconcatenated into a number $n$ such that $n + 1$ is prime or power of prime and the digit 1;
4. $p^2$ can be deconcatenated into a number $n$ such that $n + 1$ is prime or power of prime and the digit 9;
5. $p^2$ can be deconcatenated into a number of the form $49 + 120k$ and a number congruent to 0 modulo 6;
6. $p^2$ can be deconcatenated into a number of the form $121 + 24k$ and a number congruent to 0 modulo 6.

Examples for case (1):

- for $67^2 = 4489$ we got 89 prime and $44 \equiv 2 \pmod{6}$;
- for $73^2 = 5329$ we got 29 prime and $53 \equiv 5 \pmod{6}$;
- for $79^2 = 6241$ we got 41 prime and $62 \equiv 2 \pmod{6}$;
- for $109^2 = 11881$ we got 881 prime and $11 \equiv 2 \pmod{6}$;
- for $163^2 = 26569$ we got 569 prime and $26 \equiv 2 \pmod{6}$;
- for $181^2 = 32761$ we got 761 prime and $32 \equiv 5 \pmod{6}$;
- for $199^2 = 39601$ we got 601 prime and $39 \equiv 3 \pmod{6}$.

Examples for case (2):

- for $13^2 = 169$ we got $69 = 3*23$ and $1 \equiv 1 \pmod{6}$;
- for $37^2 = 1369$ we got $369 = 3^2*41$ and $1 \equiv 1 \pmod{6}$;
- for $43^2 = 1849$ we got $849 = 3*283$ and $1 \equiv 1 \pmod{6}$;
- for $61^2 = 3721$ we got $21 = 3*7$ and $37 \equiv 1 \pmod{6}$;
- for $127^2 = 16129$ we got $6129 = 3^3*227$ and $1 \equiv 1 \pmod{6}$;
- for $193^2 = 37249$ we got $249 = 3*83$ and $37 \equiv 1 \pmod{6}$.
Examples for case (3):

: for $19^2 = 361$ we got $36 + 1 = 37$ prime;
: for $31^2 = 961$ we got $96 + 1 = 97$ prime;
: for $79^2 = 6241$ we got $624 + 1 = 625$ power of prime.
: for $139^2 = 19321$ we got $1932 + 1 = 1933$ prime;
: for $151^2 = 22801$ we got $2280 + 1 = 2281$ prime.

Examples for case (4):

: for $7^2 = 49$ we got $4 + 1 = 5$ prime;
: for $97^2 = 9409$ we got $940 + 1 = 941$ prime;
: for $103^2 = 10609$ we got $1060 + 1 = 1061$ prime.

Examples for case (5):

: for $157^2 = 24649$ we got $649 = 49 + 120*5$ and $24 \equiv 0 \pmod{6}$.

Examples for case (6):

: for $79^2 = 6241$ we got $241 = 121 + 24*5$ and $6 \equiv 0 \pmod{6}$.

Note:

This conjecture is verified up to $p = 199$.

Note:

I mention that this conjecture and the one from my previous paper “A conjecture on the squares of primes of the form $6k - 1$” were made with the title of jocandi causa. It is not relevant if they are not true if they raise interesting questions about squares of primes.