A set of Poulet numbers and generalizations of the twin primes and de Polignac’s conjectures inspired by this

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Abstract. In this paper I show a set of Poulet numbers, each one of them having the same interesting relation between its prime factors, and I make four conjectures, one about the infinity of this set, one about the infinity of a certain type of duplets respectively triplets respectively quadruplets and so on of primes and finally two generalizations, of the twin primes conjecture respectively of de Polignac’s conjecture.

Conjecture 1:

There exist an infinity of Poulet numbers of the form \(n^2 + 120n\), where \(n\) is prime or a composite positive integer.

Note:

In the first case, obviously \(n\) is a prime factor of such a Poulet number and the product of the other prime factors is equal to \(n + 120\); for instance, the number 1729 is a part of this set of Poulet numbers because \(1729 = 7 \times 13 \times 19\) can be written as \(13^2 + 13 \times 120\) and implicitly \(7 \times 19 = 13 + 120\). First few such Poulet numbers are:

\[
\begin{align*}
1729 &= 7 \times 13 \times 19 = 13^2 + 13 \times 120; \\
4681 &= 31 \times 151 = 31^2 + 31 \times 120; \\
6601 &= 7 \times 23 \times 41 = 41^2 + 41 \times 120.
\end{align*}
\]

Note:

In the second case, obviously \(n\) is a product of few prime factors of such a Poulet number and the product of the other prime factors is equal to \(n + 120\). Such a Poulet number is 75361 = \(11 \times 13 \times 17 \times 31 = 221^2 + 221 \times 120\) and implicitly \(11 \times 31 = 13 \times 17 + 120\).
Comment:

Interesting (outside the topic of this paper) that there are other pairs of twin primes of the form \([m = 10k + 1, n = 10k + 3]\) for which exist primes \([p, q]\) such that \(m*p = n*q + 120\); for instance, \(41*227 = 43*241 + 120\).

Conjecture 2:

There exist an infinity of duplets of primes \([p, q]\) such that \(p - q = 120\); there also exist an infinity of triplets of primes \([p_1, p_2, q]\) such that \(p_1*p_2 - q = 120\); there also exist an infinity of quadruplets of primes \([p_1, p_2, p_3, q]\) such that \(p_1*p_2*p_3 - q = 120\); generally, for any non-null positive integer \(i\) there exist \(i\) primes \(p_1, p_2, ..., p_i\) and a prime \(q\) such that \(p_1*p_2*...*p_i - q = 120\).

Examples:

: \(151 - 31 = 120\);
: \(7*19 - 13 = 120\);
: \(7*17*37 - 4283 = 120\).

Conjecture 3:

(generalization of the twin primes conjecture)

For any positive integer \(i\) there exist \(i\) primes \(p_1, p_2, ..., p_i\) and a prime \(q\) such that \(p_1*p_2*...*p_i - q = 2\).

Conjecture 4:

(generalization of de Polignac’s conjecture)

For any \(n\) even positive integer and for any \(i\) non-null positive integer there exist \(i\) primes \(p_1, p_2, ..., p_i\) and a prime \(q\) such that \(p_1*p_2*...*p_i - q = n\).