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Somewhat almost sg-continuous functions and Somewhat almost sg-open functions

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Abstract: In this paper we tried to introduce a new variety of continuous and open functions called Somewhat almost sg-continuous functions and Somewhat almost sg-open functions. Its basic properties are discussed.

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Keywords: Somewhat sg-continuous functions and Somewhat sg-open functions, Somewhat almost sg-continuous functions and Somewhat almost sg-open functions

1. Introduction:

b-open[1] sets are introduced by Andrijevic in 1996. K.R.Gentry[8] introduced somewhat continuous functions in the year 1971. V.K.Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T.Noiri and N.Rajesh[10] introduced somewhat b-continuous functions in the year 2011. Inspired with these developments we introduce in

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this paper somewhat almost *sg*-continuous functions, somewhat almost *sg*-open functions and study its basic properties and interrelation with other type of such functions available in the literature. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For $A\subset(X;\tau)$, $cl(A)$ and A^o denote the closure of A and the interior of A in X, respectively.

2. Preliminaries:

Definition 2.1: A subset *A* of X is said to be

(i) b-open[1] if $A\subset(cl\{A\})^o\cap cl\{A^o\}$.

(ii) sg-dense in X if there is no proper sg-closed set C in X such that $M\subset C\subset X$.

Definition 2.2: A function *f* is said to be

(i) somewhat continuous[8][resp: somewhat b-continuous[10]; somewhat sg-continuous[6]] if for $U\in\sigma$ and $f^{-1}(U)\neq\varphi$, there exists an open[resp: b-open; sg-open] set V in X such that $V\neq\varphi$ and $V\subset f^{-1}(U)$.

(ii) somewhat open[10][resp: somewhat b-open[8]; somewhat sg-open] provided that if $U\in\tau$ and $U\neq\varphi$, then there exists an open[resp: b-open; sg-open] set V in Y such that $V\neq\varphi$ and $V\subset f(U)$.

Definition 2.3: (X, τ) is said to be resolvable[7][b-resolvable[10]] if there exists a set A in (X, τ) such that both A and X - A are dense[b-dense] in (X, τ). Otherwise, (X, τ) is called irresolvable.

Definition 2.4: If X is a set and τ and σ are topologies on X, then τ is said to be equivalent[resp: sg- equivalent] to σ provided if $U\in\tau$ and $U\neq\varphi$, then there is an open[resp:sg-open] set V in *X* such that $V\neq\varphi$ and $V\subset U$ and if $U\in\sigma$ and $U\neq\varphi$, then there is an open[resp:sg-open] set V in (X, τ) such that $V\neq\varphi$ and $U\supset V$.

3. Somewhat almost sg-continuous function:

Definition 3.1: A function f is said to be somewhat almost sg-continuous if for $U \in RO(\sigma)$ and $f^{-1}(U) \neq \varphi$, there exists a $V \neq \varphi \in SGO(X)$ such that $V \subset f^{-1}(U)$.

Example 1: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, X\}$. The function $f:(X, \tau) \to (X, \sigma)$ defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ is somewhat almost sg-continuous but not somewhat continuous.

Example 2: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{b, c\}, X\}$, $\sigma = \{\varphi, \{b\}, \{a, c\}, X\}$ and $\eta = \{\varphi, \{a\}, X\}$. Then the identity functions $f:(X, \tau) \to (X, \sigma)$ and $g:(X, \sigma) \to (X; \eta)$ and $g \bullet f$ are somewhat almost sg-continuous.

In general composition of two somewhat almost sg-continuous functions is not somewhat almost sg-continuous. However, we have the following

Theorem 3.1: If f is somewhat almost sg-continuous and g is continuous[r-continuous], then $g \bullet f$ is somewhat almost sg-continuous.

Corollary 3.1: If f is somewhat almost sg-continuous and g is r-irresolute[r-continuous], then $g \bullet f$ is somewhat almost sg-continuous.

Theorem 3.2: For a surjective function f, the following statements are equivalent:

(i) f is somewhat almost sg-continuous.

(ii) If C is regular closed in Y such that $f^{-1}(C) \neq X$, then there is a $D \neq \varphi \in SGC(X)$ such that $f^{-1}(C) \subset D$.

(iii)If M is a sg-dense subset of X, then $f(M)$ is a dense subset of Y.

Proof: (i) $\Rightarrow$(ii): Let $C \in RC(Y)$ such that $f^{-1}(C) \neq X$. Then $Y - C \in RO(Y)$ such that $f^{-1}(Y-C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists $V \neq \varphi \in SGO(X)$ and $V \subset f^{-1}(Y-C) = X - f^{-1}(C)$. Thus $X - V \supset f^{-1}(C)$ and $X - V = D$ is a proper sg-closed set in X.

(ii) ⇒(i): Let $U\in RO(\sigma)$ and $f^{-1}(U) \neq \varphi$ Then $Y\text{-}U\in RC(\sigma)$ and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists a proper $D\in SGC(X)$ such that $D\supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is sg-open and $X\text{-}D \neq \varphi$.

(ii) ⇒(iii): Let M be a sg-dense set in X. If $f(M)$ is not dense in Y. Then there exists a proper $C\in RC(Y)$ such that $f(M) \subset C\subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $D\in SGC(X)$ such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is sg-dense in X.

(iii) ⇒(ii): If (ii) is not true. there exists $C\in RC(Y)$ such that $f^{-1}(C) \neq X$ but there is no proper $D\in SGC(X)$ such that $f^{-1}(C) \subset D$. Thus $f^{-1}(C)$ is sg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ is dense in Y, which contradicts the choice of C.

Theorem 3.3: Let f be a function and $X = A\cup B$, where $A,B\in RO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat almost sg-continuous, then f is somewhat almost sg-continuous.

Proof: Let $U\in RO(\sigma)$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat almost sg-continuous, there exists $V\neq \varphi\in SGO(A)$ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since $V\in SGO(A)$ and $A\in RO(X)$, $V\in SGO(X)$. Thus f is somewhat almost sg-continuous. The proof of other cases are similar.

Theorem 3.4: Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat almost sg-continuous surjection and τ^* be a topology for X, which is sg-equivalent to τ. Then $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat almost sg-continuous.

Proof: Let $V\in RO(\sigma)$ such that $f^{-1}(V) \neq \varphi$. Since f is somewhat almost sg-continuous, there exists $U\neq \varphi\in SGO(X, \tau)$ such that $U\subset f^{-1}(V)$. But by hypothesis τ^* is sg-equivalent to τ. Therefore, there exists $U^*\neq \varphi\in SGO(X; \tau^*)$ such that $U^*\subset U$. But $U\subset f^{-1}(V)$. Then $U^*\subset f^{-1}(V)$; hence $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat almost sg-continuous.

Theorem 3.5: Let f:(X, τ)→ (Y, σ) be a somewhat almost sg-continuous surjection and σ* be a topology for Y, which is equivalent to σ. Then f:(X, τ)→ (Y, σ*) is somewhat almost sg-continuous.

Proof: Let V*∈RO(σ*) such that f^{-1}(V*) ≠ φ. Since σ* is equivalent to σ, there exists V≠ϕ∈RO(Y, σ) such that V⊂ V*. Now φ ≠ f^{-1}(V) ⊂ f^{-1}(V*). Since f is somewhat almost sg-continuous, there exists U≠ φ∈SGO(X, τ) such that U⊂f^{-1}(V). Then U⊂f^{-1}(V*); hence f:(X, τ)→ (Y, σ*) is somewhat almost sg-continuous.

4. Somewhat sg-irresolute function:

Definition 4.1: A function f is said to be somewhat sg-irresolute if for U∈SGO(σ) and f^{-1}(U) ≠ φ, there exists a non-empty sg-open set V in X such that V⊂f^{-1}(U).

Example3: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, {a, b}, X}. The function f:(X, τ)→ (X, σ) defined by f(a) = c, f(b) = a and f(c) = b is somewhat sg-irresolute but not somewhat-irresolute.

Example 4: Let X = {a, b, c}, τ = {φ, {a}, {a, b}, X} and σ = {φ, {a}, {b, c}, X}. The function f defined by f(a) = c, f(b) = a and f(c) = b is not somewhat sg-irresolute.

Note 1: Every somewhat sg-irresolute function is slightly sg-irresolute.

Example 5: Let X = {a, b, c}, τ = {φ, {b, c}, X}, σ = {φ, {b}, {a, c}, X} and η = {φ, {a}, X}. Then the identity functions f:(X, τ)→ (X, σ) and g:(X, σ)→ (X; η) and g•f are somewhat sg-irresolute.

In general composition of two somewhat sg-irresolute functions is not somewhat sg-irresolute. However, we have the following

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Theorem 4.1: If f is somewhat sg-irresolute and g is irresolute, then $g \bullet f$ is somewhat sg-irresolute.

Theorem 4.2: For a surjective function f, the following statements are equivalent:

(i) f is somewhat sg-irresolute.

(ii) If $C \in SGC(Y)$ such that $f^{-1}(C) \neq X$, then there is a $D \neq \varphi \in SGC(X)$ such that $f^{-1}(C) \subset D$.

(iii)If M is a sg-dense subset of X, then $f(M)$ is a sg-dense subset of Y.

Proof: (i) ⇒(ii): Let $C \in SGC(Y)$ such that $f^{-1}(C) \neq X$. Then $Y\text{-}C \in SGO(Y)$ such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists $V \neq \varphi \in SGO(X)$ and $V \subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. This means $X\text{-}V \supset f^{-1}(C)$ and $X - V = D$ is proper sg-closed in X.

(ii) ⇒(i): Let $U \in SGO(\sigma)$ and $f^{-1}(U) \neq \varphi$ Then $Y\text{-}U \neq \varphi \in SGC(Y)$ and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists $D \neq \varphi \in SGC(X)$ such that $D \supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is sg-open and $X\text{-}D \neq \varphi$.

(ii) ⇒(iii): Let M be a sg-dense set in X. If $f(M)$ is not sg-dense in Y. Then there exists a proper $C \in SGC(Y)$ such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $D \in SGC(X)$ such that $M \subset f^{-1}(C) \subset D \subset X$. This is a contradiction to the fact that M is sg-dense in X.

(iii) ⇒(ii): Suppose (ii) is not true. there exists $C \in SGC(Y)$ such that $f^{-1}(C) \neq X$ but there is no proper $D \neq \varphi \in SGC(X)$ such that $f^{-1}(C) \subset D$. This means $f^{-1}(C)$ is sg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be sg-dense in Y, which is a contradiction to the choice of C.

Theorem 4.3: Let f be a function and $X = A \cup B$, where $A,B \in RO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat sg-irresolute, then f is somewhat sg-irresolute.

Proof: Let $U \in SGO(\sigma)$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. If $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat sg-irresolute, there exists $V \neq \varphi \in SGO(A)$ and $V \subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since V is sg-open in A and A is r-open in X, V is sg-open in X. Thus f is somewhat sg-irresolute.

The proof of other cases are similar.

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If f is the identity function and τ and σ are sg-equivalent. Then f and f^{-1} are somewhat sg-irresolute. Conversely, if the identity function f is somewhat sg-irresolute in both directions, then τ and σ are sg-equivalent.

Theorem 4.4: Let f:(X, τ)$\rightarrow$ (Y, σ) be a somewhat sg-irresolute surjection and τ^* be a topology for X, which is sg-equivalent to τ. Then f:(X, τ^*)$\rightarrow$ (Y, σ) is somewhat sg-irresolute.

Proof: Let V$\in$SGO(σ) such that $f^{-1}(V) \neq \varphi$. Since f is somewhat sg-irresolute, there exists U$\neq$ $\varphi\in$SGO(X, τ) with U$\subset f^{-1}$(V). But for τ^* is sg-equivalent to τ. Therefore, there exists U*$\neq$ $\varphi\in$SGO(X; τ^*) such that U*$\subset$ U. But U$\subset f^{-1}$(V). Then U*$\subset f^{-1}$(V); hence f:(X, τ^*)$\rightarrow$ (Y, σ) is somewhat sg-irresolute.

Theorem 4.5: Let f:(X, τ)$\rightarrow$ (Y, σ) be a somewhat sg-irresolute surjection and σ^* be a topology for Y, which is equivalent to σ. Then f:(X, τ)$\rightarrow$ (Y, σ^*) is somewhat sg-irresolute.

Proof: Let V*$\in\sigma^*$ such that $f^{-1}(V^*) \neq \varphi$. Since σ^* is equivalent to σ, there exists V$\neq$ $\varphi\in$(Y, σ) such that V$\subset$ V*. Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat sg-irresolute, there exists U$\neq$ $\varphi\in$SGO(X, τ) such that U$\subset f^{-1}$(V). Then U$\subset f^{-1}$(V*); hence f:(X, τ)$\rightarrow$ (Y, σ^*) is somewhat sg-irresolute.

5. Somewhat almost sg-open function:

Definition 5.1: A function f is said to be somewhat almost sg-open provided that if U$\in$RO(τ) and U$\neq$ φ, then there exists a V$\neq$ $\varphi\in$SGO(Y) such that V$\subset f$(U).

Example 6: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function f: defined by f(a) = a, f(b) = c and f(c) = b is somewhat almost sg-open, somewhat sg-open and somewhat open.

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Example 7: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, X}. The function *f*: defined by *f*(a) = a, *f*(b) = c and *f*(c) = b is not somewhat almost sg-open.

Theorem 5.1: Let *f* be an r-open function and *g* somewhat almost sg-open. Then *g*• *f* is somewhat almost sg-open.

Theorem 5.2: For a bijective function *f*, the following are equivalent:

(i) *f* is somewhat almost sg-open.

(ii) If C is regular closed in X, such that *f*(C) ≠ Y, then there is a D ≠ $\varphi \in$SGC(Y) and D$\supset$ *f*(C).

Proof: (i) $\Rightarrow$(ii): Let C$\in$RC(X) such that *f*(C) ≠ Y. Then X-C≠ $\varphi\in$RO(X). Since *f* is somewhat almost sg-open, there exists V≠ $\varphi\in$SGO(Y) such that V$\subset$ *f*(X-C). Put D = Y-V. Clearly D≠ $\varphi\in$SGC(Y). If D = Y, then V = φ, which is a contradiction. Since V$\subset$ *f*(X-C), D = Y-V $\supset$ (Y- *f*(X-C)) = *f*(C).

(ii) $\Rightarrow$(i): Let U≠ $\varphi\in$RO(X). Then C = X-U$\in$RC(X) and *f*(X-U) = *f*(C) = Y- *f*(U) implies *f*(C) ≠ Y. Then by (ii), there is D ≠ $\varphi\in$SGC(Y) and *f*(C) $\subset$ D. Clearly V = Y-D≠ $\varphi\in$SGO(Y). Also, V = Y-D $\subset$ Y- *f*(C) = Y- *f*(X-U) = *f*(U).

Theorem 5.3: The following statements are equivalent:

(i) *f* is somewhat almost sg-open.

(ii)If A is a sg-dense subset of Y, then f^{-1}(A) is a dense subset of X.

Proof: (i) $\Rightarrow$(ii): Let A be a sg-dense set in Y. If f^{-1}(A) is not dense in X, then there exists B$\in$RC(X) such that f^{-1}(A) $\subset$ B$\subset$ X. Since *f* is somewhat almost sg-open and X-B$\in$RO(X), there exists C ≠ $\varphi\in$SGO(Y) such that C$\subset$ *f*(X-B). Therefore, C$\subset$ *f*(X-B) $\subset$ $f(f^{-1}$(Y-A)) $\subset$ Y-A. That is, A$\subset$ Y-C $\subset$ Y. Now, Y-C is a sg-closed set and A$\subset$ Y-C $\subset$ Y. This implies that A is not a sg-dense set in Y, which is a contradiction. Therefore, f^{-1}(A) is a dense set in X.

(ii) $\Rightarrow$(i): If $A\neq \varphi \in RO(X)$. We want to show that $sg(f(A))^o \neq \varphi$. Suppose $sg(f(A))^o = \varphi$. Then, $sgcl\{(f(A))\} = Y$. Then by (ii), $f^{-1}(Y - f(A))$ is dense in X. But $f^{-1}(Y- f(A)) \subset X-A$. Now, $X-A\in RC(X)$. Therefore, $f^{-1}(Y- f(A)) \subset X-A$ gives $X = cl\{(f^{-1}(Y- f(A)))\} \subset X-A$. Thus $A = \varphi$, which contradicts $A \neq \varphi$. Therefore, $sg(f(A))^o\neq \varphi$. Hence f is somewhat almost sg-open.

Theorem 5.4: Let f be somewhat almost sg-open and A be any r-open subset of X. Then $f_{/A}$ is somewhat almost sg-open.

Proof: Let $U\neq \varphi\in RO(\tau_{/A})$. Since $U\in RO(A)$ and $A\in RO(X)$, $U\in RO(X)$ and since f is somewhat almost sg-open function, there exists $V\in SGO(Y)$, such that $V\subset f(U)$. Thus $f_{/A}$ is a somewhat almost sg-open function.

Theorem 5.5: Let f be a function and $X = A\cup B$, where $A,B\in RO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat almost sg-open, then f is somewhat almost sg-open.

Proof: Let $U \neq \varphi\in RO(X)$. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since $U\in RO(X)$, $U\in RO(A)$ and $U\in RO(B)$.

Case (i): If $A\cap U \neq \varphi\in RO(A)$. Since $f_{/A}$ is somewhat almost sg-open, there exists $V\in SGO(Y)$ such that $V\subset f(U\cap A) \subset f(U)$, which implies f is somewhat almost sg-open.

Case (ii): If $B\cap U \neq \varphi\in RO(B)$. Since $f_{/B}$ is somewhat almost sg-open, there exists $V\in SGO(Y)$ such that $V\subset f(U\cap B) \subset f(U)$, which implies f is somewhat almost sg-open.

Case (iii): If both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Then by case (i) and (ii) f is somewhat almost sg-open.

Remark 1: Two topologies τ and σ for X are said to be sg-equivalent if and only if the identity function f: $(X, \tau)\rightarrow (Y, \sigma)$ is somewhat almost sg-open in both directions.

Theorem 5.6: If f:$(X, \tau)\rightarrow(Y, \sigma)$ is somewhat almost open. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is equivalent to τ and σ^* is sg-equivalent to σ. Then f: $(X; \tau^*)\rightarrow (Y; \sigma^*)$ is somewhat almost sg-open.

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6. Somewhat M-sg-open function:

Definition 6.1: A function f is said to be somewhat M-sg-open provided that if $U \in SGO(\tau)$ and $U \neq \varphi$, then there exists a $V \neq \varphi \in SGO(Y)$ such that $V \subset f(U)$.

Example 8: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, X\}$. The function f defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat M-sg-open, somewhat sg-open and somewhat open.

Example 9: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{a, b\}, X\}$. The function f defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is not somewhat M-sg-open.

Theorem 6.1: Let f be an r-open function and g somewhat M-sg-open. Then $g \bullet f$ is somewhat M-sg-open.

Theorem 6.2: For a bijective function f, the following are equivalent:
(i) f is somewhat M-sg-open.
(ii) If $C \in SGC(X)$, such that $f(C) \neq Y$, then there is a $D \in SGC(Y)$ such that $D \neq Y$ and $D \supset f(C)$.

Proof: (i) $\Rightarrow$(ii): Let $C \in SGC(X)$ such that $f(C) \neq Y$. Then $X\text{-}C \neq \varphi \in SGO(X)$. Since f is somewhat M-sg-open, there exists $V \neq \varphi \in SGO(Y)$ such that $V \subset f(X\text{-}C)$. Put $D = Y\text{-}V$. Clearly $D \neq \varphi \in SGC(Y)$. If $D = Y$, then $V = \varphi$, which is a contradiction. Since $V \subset f(X\text{-}C)$, $D = Y\text{-}V \supset (Y\text{-} f(X\text{-}C)) = f(C)$.
(ii) $\Rightarrow$(i): Let $U \neq \varphi \in RO(X)$. Then $C = X\text{-}U \in SGC(X)$ and $f(X\text{-}U) = f(C) = Y\text{-} f(U)$ implies $f(C) \neq Y$. Then by (ii), there is $D \in SGC(Y)$ such that $D \neq Y$ and $f(C) \subset D$. Clearly $V = Y\text{-}D \neq \varphi \in SGO(Y)$. Also, $V = Y\text{-}D \subset Y\text{-} f(C) = Y\text{-} f(X\text{-}U) = f(U)$.

Theorem 6.3: The following statements are equivalent:

(i) f is somewhat M-sg-open.

(ii)If A is a sg-dense subset of Y, then $f^{-1}(A)$ is a sg-dense subset of X.

Proof: (i) ⇒(ii): Let A be a sg-dense set in Y. If $f^{-1}(A)$ is not sg-dense in X, then there exists $B \in SGC(X)$ in X such that $f^{-1}(A) \subset B \subset X$. Since f is somewhat M-sg-open and X-B is sg-open, there exists a $C \neq \varphi \in SGO(Y)$ such that $C \subset f(X-B)$. Therefore, $C \subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y-A$. That is, $A \subset Y-C \subset Y$. Now, Y-C is a sg-closed set and $A \subset Y-C \subset Y$. This implies that A is not a sg-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a sg-dense set in X.

(ii) ⇒(i): Let $A \neq \varphi \in SGO(X)$. We want to show that $sg(f(A))^o \neq \varphi$. Suppose $sg(f(A))^o = \varphi$. Then, $sgcl\{(f(A))\} = Y$. Then by (ii), $f^{-1}(Y - f(A))$ is sg-dense in X. But $f^{-1}(Y- f(A)) \subset X-A$. Now, $X-A \in SGC(X)$. Therefore, $f^{-1}(Y- f(A)) \subset X-A$ gives $X = cl\{(f^{-1}(Y- f(A)))\} \subset X-A$. Thus $A = \varphi$, which contradicts $A \neq \varphi$. Therefore, $sg(f(A))^o \neq \varphi$. Hence f is somewhat M-sg-open.

Theorem 6.4: If f is somewhat M-sg-open and $A \in RO(X)$. Then $f_{/A}$ is somewhat M-sg-open.

Proof: Let $U \neq \varphi \in SGO(\tau_{/A})$ and $A \in RO(X)$. Since f is somewhat M-sg-open, there exists $V \in SGO(Y)$, such that $V \subset f(U)$. Thus $f_{/A}$ is a somewhat M-sg-open.

Theorem 6.5: Let f be a function and $X = A \cup B$, where $A,B \in SGO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat M-sg-open, then f is somewhat M-sg-open.

Proof: Same as Theorem 5.5.

.

Remark 2: Two topologies τ and σ for X are said to be sg-equivalent if and only if the identity function f: $(X, \tau) \rightarrow (Y, \sigma)$ is somewhat M-sg-open in both directions.

Theorem 6.6: If $f:(X, \tau) \rightarrow (Y, \sigma)$ is somewhat M-open. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is equivalent to τ and σ^* is sg-equivalent to σ. Then f: $(X; \tau^*) \rightarrow (Y; \sigma^*)$ is somewhat M-sg-open.

CONCLUSION: In this paper we defined Somewhat-sg-continuous functions, studied its properties and their interrelations with other types of Somewhat-continuous functions.

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