International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 9 (September 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/

Structure of Chromatic Polynomials on Quasi - Total Graphs

R.V.N.SrinivasaRao[1], Dr.J.VenkateswaraRao[2],T.NageswaraRAo[3]

[1]Department of Mathematics, Guntur Engineering College, Guntur Dt., A.P, India.
[2]Department of Mathematics, Mekelle University Main Campus,Mekelle, Ethiopia.
[3]Department of Mathematics, St.Marys Women's Engineering College, Guntur Dt. India

Abstract. Total colouring is a combination of vertex and edge colouring. Quasi total coloring of a graph is an assignment of colours to vertices and edges such that distinct colours are assigned to adjacent vertices and edges or incident vertices and edges. This manuscript is going to count the possible number of assignments of different proper colourings of these quasi total graphs of a (p, q) connected graphs with $p \leq 3$ and $1 \leq q \leq 3$with given number of colours using deletion -contraction algorithm.

Keywords: Chromatic Polynomials, 1-quasi total graph, 2-quasi total graph.

1. **Introduction:** G.D.Birkhoff [1912] introduced the concepts of chromatic polynomials on maps. Afterward Whitney **[**1932**]** expanded the study of Chromatic Polynomials from maps to graphs. Behzad and vizing [1965] have posed independently a new concept of a graph colouring, called a total colouring .Further H.P.yap [1996] contributes on total colourings. R C. Read **[**1968**]** wrote a survey paper on chromatic polynomials, improved interest in chromatic polynomials of graphs. Lastly Brualadi (1999) contribute on chromatic polynomials. In recent times J.V.Rao and R.V.N.S.Rao [2012] contributes to 1-quasi and 2-quasi total graphs.

---

[1]CorrespondingAuthor: R.V.N.SrinivasaRao, mrvnrepalle@rediffmail.com

International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 9 (September 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/

2. Basic definitions and preliminaries:

2.1. Definition [West D.B.2003]: Chromatic polynomial of a graph G is a special function that describes the number of ways we can achieve a proper colouring with given λ colours .If G is a simple graph we write P(G, λ) as the number of ways we can achieve a proper colouring on the vertices of G with the given λ colours and $P_G(\lambda)$ or P(G, λ) is called the chromatic polynomial.

2.2. Lemma: The Chromatic Polynomial of a cycle graph of order n is $(\lambda-1)^n+(-1)^n(\lambda-1)$.

The proof of this lemma is obtained by using mathematical induction.

2.3. Definition [9]: Let G be a graph with vertex set V(G) and edge set E(G). The *1–quasitotal graph,* (denoted by $Q_1(G)$) of G is defined as follows: The vertex set of $Q_1(G)$, that is $V(Q_1(G)) = V(G) \cup E(G)$. Two vertices are adjacent if and only if they corresponding to two adjacent vertices of G or two adjacent edges in G.

2.4. Definition [9] Let G be a graph with vertex set V(G) and edge set E(G). The *2–quasitotal graph,* (denoted by $Q_2(G)$) of G is defined as follows: The vertex set of $Q_2(G)$, that is $V(Q_2(G)) = V(G) \cup E(G)$. Two vertices are adjacent if and only if they corresponding to two adjacent vertices of G or a vertex in G incident with an edge in G.

2.5. Deletion and Contraction Algorithm [Brualadi (1999)**]**: Brualadi provided a formal algorithm known as deletion and contraction algorithm for finding a chromatic polynomial.

2.6. Theorem [Brualadi (1999)]: Let G be a graph, and G-e and G/e, be the graphs obtained from G by deleting and contracting an edge e respectively .Then P(G, λ) =P(G-e, λ)- P(G/e, λ).

2.7.Theorem:[Erdos, R.J.Wilson,1977] Let G be a graph containing nonadjacent vertices u and v and let H be the graph obtained from G by contracting u and v. Then $P(G, \lambda) = P(G + uv, \lambda) + P(H, \lambda)$.

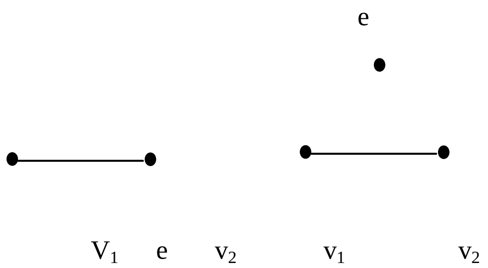
3. Chromatic Polynomials on quasi total graphs:

Now we have to find the chromatic polynomials of quasi total graphs of the (p, q)-connected graphs with $p \leq 3$ and $1 \leq q \leq 3$.

3.1. Lemma: The chromatic polynomial for 1-quasi total graph of a (p, q)-connected graph with $p \leq 3$, $1 \leq q \leq 3$ is $\lambda^{p+q} - a_1 \lambda^{p+q-1} + a_2 \lambda^{p+q-2}$- ----- $(-1)^{p+q} a_{n-2} \lambda^2$, where a_1 is the number of edges in 1-quasi total graph.

Proof: The proof of this lemma can be come apart in to two cases with (2, 1)-connected and (3, 3)-connected graphs.

Case.1: First we begin with (2,1) -connected graph. The following figure 1 represents a (2, 1) connected graph and its 1-quasi total graph



G Q$_1$(G)

Figure1: A graph G of order 2 size1 with its 1-quasi total graph

We have to find the chromatic polynomials for the quasi total graph using the deletion-contraction algorithm. The following figure 2 represents the deletion – contraction reduction of 1-quasi total graph. Hence $P(Q_1(G), \lambda)=N_3-N_2=\lambda^3-\lambda^2$,in general $\lambda^{p+q} +(-1)^{p+q} \lambda^{p+q-1}$.Hence the we get the chromatic polynomial of 1-quasi total graph as desired in this case.

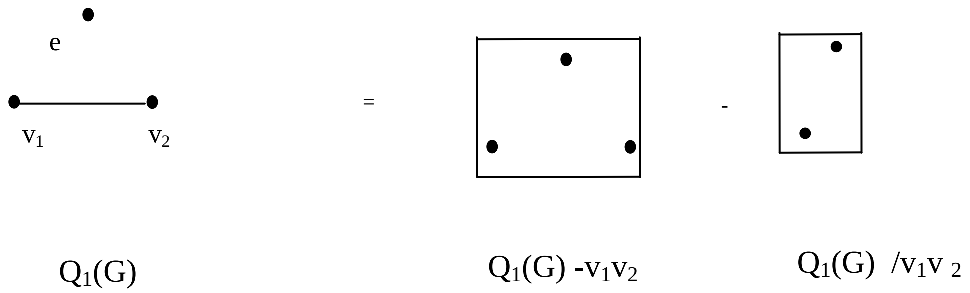


Figure2: Deletion – Contraction reduction of 1-quasi total graph

Case2: It represents the (3, 3)-connected graph and chromatic polynomial of its 1-quasi total graph .The following figure 3 represents (3, 3)-connected graph G with vertices, $V(G) = \{v_1, v_2, v_3\}$, edges $E(G) = \{e_1, e_2, e_3\}$ and its 1-quasi total graph $Q_1(G)$.

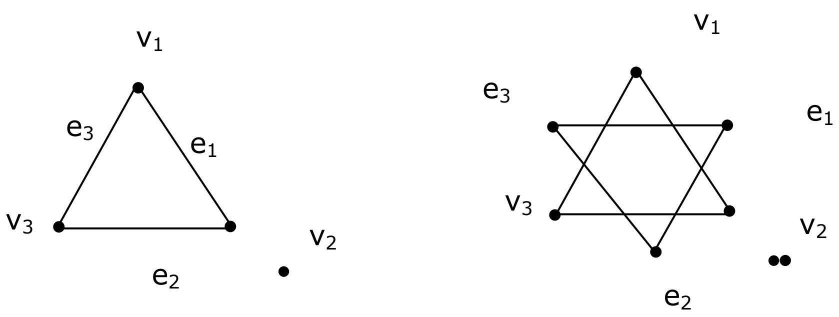


Figure 3: A (3, 3)-connected graph with its 1-quasi total graph

The deletion contraction algorithm for this1-quasitotal graph as observed in figure 4 , each step to the left represents a deletion, and the step on the right represents a contraction. After each step, if the graph is not reduced down to a null graph, the algorithm is repeated .At the end of the algorithm, only null graphs remains(here we eliminate numerous steps and write the final step). Since the chromatic polynomial of a null graph of order n is λ^n, the chromatic polynomial of 1-quasi total graph is

International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 9 (September 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/

$P(Q_1(G), \lambda) = \lambda^6 - 6\lambda^5 + 13\lambda^4 - 12\lambda^3 + 2\lambda^2$, in general $\lambda^{p+q} - a_1\lambda^{p+q-1} + \text{-----} (-1)^{p+q} a_4 \lambda^{p+q-1}$ where a_1 is number of edges in 1-quasi total graph .

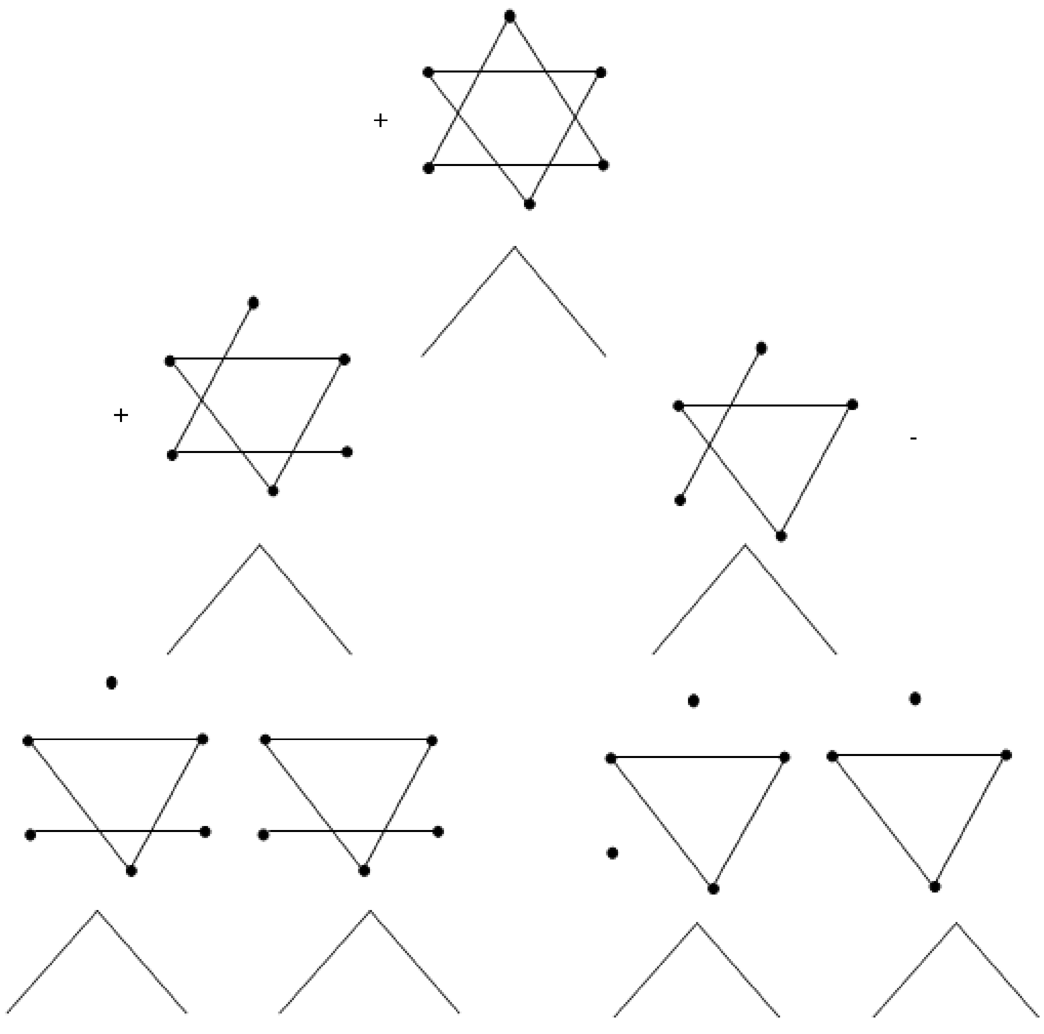


Figure4: Reduction of 1-Quasi total graph using Deletion-Contraction Algorithm.

$=N_6 - 6N_5 + 13N_4 - 11N_3 + 2N_2 = \lambda^6 - 6\lambda^5 + 13\lambda^4 - 12\lambda^3 + 2\lambda^2 = P(Q_1(G), \lambda)$.

Hence we get the chromatic polynomial of 1-quasi total graph as desired in this case also. Hence the chromatic polynomial for 1-quasi total graph of a (p, q)-connected graph is $\lambda^{p+q} - a_1\lambda^{p+q-1} + a_2\lambda^{p+q-2}$- ----- $(-1)^{p+q} a_{n-2}\lambda^2$, with $p \leq 3, 1 \leq q \leq 3$.

3.2.Lemma: The chromatic polynomial of 2-quasi total graph of a (p,q)-connected graph G is $P(Q_2(G),\lambda) = \lambda(\lambda-1)[(\lambda-2)^{p+q-2} + (\lambda-2)^{p+q-4}]$ if $p + q \geq 4$ and $\lambda(\lambda-1)[(\lambda-2)^{p+q-2}]$ if $p + q < 4$, where $p \leq 3, 1 \leq q \leq 3$.

Proof: The proof can befall spaced out in to two cases with $p + q < 4$ and $p + q \geq 4$ which give the following results 3.3 and 3.4respectively.

3.3. Result: The chromatic polynomial of a 2-quasi total graph of (2, 1)-connected graph is $\lambda(\lambda-1)[(\lambda-2)^{p+q-2}]$,in general.

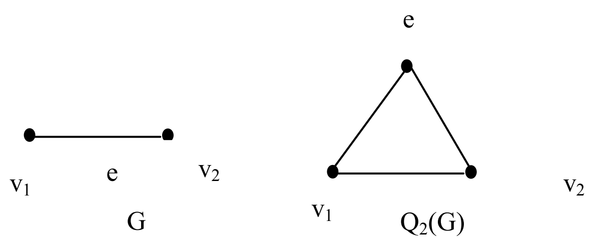


Figure5: A graph G of order 2 size 1 with its Total graph.

Since from the figure 5 the 2-quasi total graph $Q_2(G)$ is simple graph and a complete graph on three vertices ,since we know that the chromatic polynomial of a complete graph is on n vertices is $\lambda(\lambda-1)(\lambda-2)\ldots\ldots\ldots(\lambda-n+1)$.Hence the chromatic polynomial 2-quasi total graph $Q_2(G)$ is $P(Q_2(G), \lambda) = \lambda(\lambda-1)(\lambda-2) = \lambda(\lambda-1)(\lambda-2)^{p+q-2}$ .Hence $P(Q_2(G), \lambda) = \lambda^3 - 3\lambda^2 + 2\lambda = \lambda(\lambda-1)(\lambda-2)^{p+q-2}$,where $p + q < 4$.

3.4. Result: The chromatic polynomial of a 2-quasi total graph of a graph (3,3)-connected graph is $\lambda(\lambda-1)[(\lambda-2)^4+(\lambda-2)^2]$,in general $\lambda(\lambda-1)[(\lambda-2)^{p+q-2}+(\lambda-2)^{p+q-4}]$,where $p + q \geq 4$.

Proof: We have prove this result by consider the 2–quasi-total graph $Q_2(G)$of a (3,3)-connected graph G as in figure6.

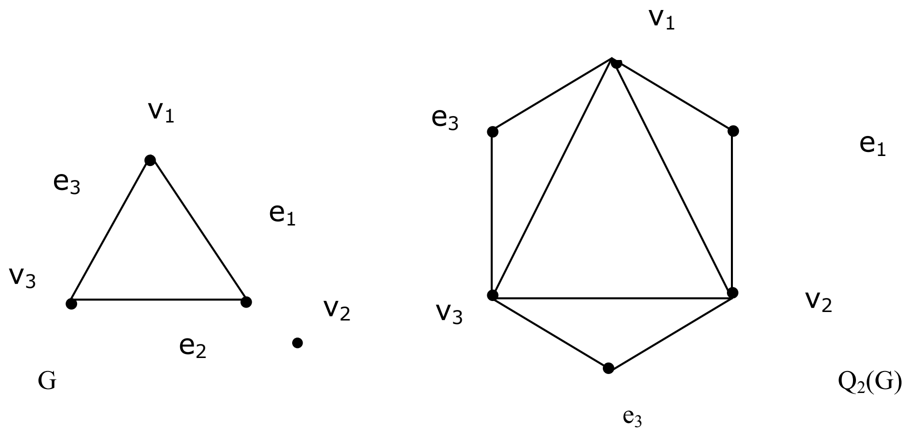
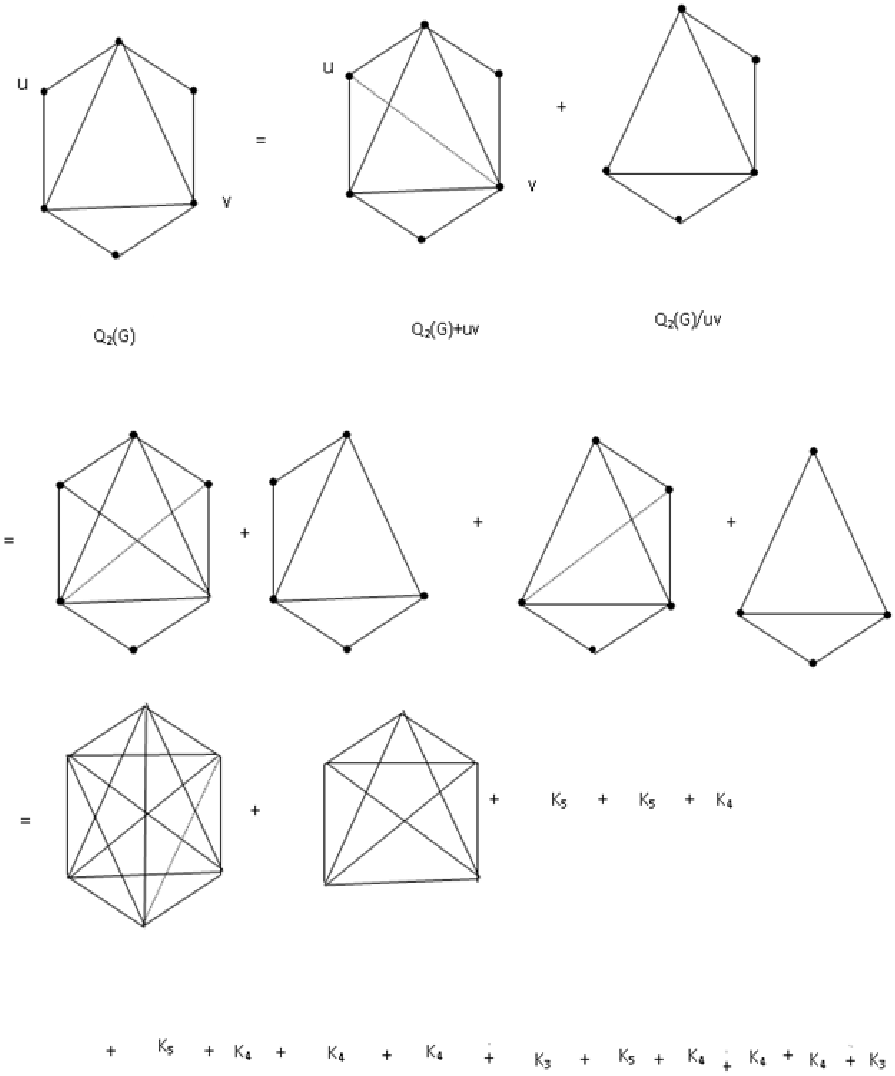


Figure 6: A graph G with its 2-quasi total graph $Q_2(G)$

Since by its definition the 2-quasi total graph contains more vertices and edges .Hence it is difficult to find the chromatic polynomial by analysis of the structure of the graphs. Hence we use deletion- contraction algorithm to find its chromatic polynomial. We use an equivalent form of deletion- contraction algorithm as proved in theorem 2.7.That is if G be a graph containing nonadjacent vertices u and v and let H be the graph obtained from G by contracting u and v. Then $P(G, \lambda) = P(G + uv, \lambda) + P(H, \lambda)$ .In this process we add new edges between non adjacent vertices until the given graph translate to a complete graph .Since we know that the chromatic polynomial of a complete graph on n vertices is with given λ colours is $(\lambda)(\lambda-1)(\lambda-2)\ldots\ldots(\lambda-n+1)$,we get the chromatic polynomial of Total graph T(G).

International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 9 (September 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/



$= K_6+6K_5+8K_4+2K_3$

Figure.7: Reduction of 2-quasi total graph

This can be completely explained in the figure7. (To avoid the graphical confusion we eliminate numerous steps). Hence the chromatic polynomial of 2-quasi total graph of a graph G is $P(Q_2,(G), \lambda)= \lambda^6-9\lambda^5+33\lambda^4-61\lambda^3+56\lambda^2-20\lambda = \lambda(\lambda-1)[(\lambda-2)^4+(\lambda-2)^2]$,in general $\lambda(\lambda-1)[(\lambda-2)^{p+q-2}+(\lambda-2)^{p+q-4}]$,where $p + q \geq 4$ as desired .

3.5. Result: The chromatic polynomial for 1-quasi total graph is free from the constant term and first degree term.

Proof: As we observed from the above results, in each result the chromatic polynomial of 1-quasi total graph is in the form of $P(Q_1(G), \lambda)=\lambda^{p+q}-a_1\lambda^{p+q-1}+a_2\lambda^{p+q-2}$- -----$(-)^{p+q-1}\lambda^2$,where

a_1 is the number of edges in 1-quasi total graph. That is the chromatic polynomial for 1-quasi total graph was ends with λ^2 terms only. Hence it does not contain a constant term and first degree term.

Conclusion: The manuscript was established the chromatic polynomials of 1-quasi and 2-quasi total graphs of (p, q) -connected graphs with $p \leq 3$ and $1 \leq q \leq 3$, using deletion –contraction algorithm.

References:

[1] G.D.Birkhoff (1912) A determinant formula for the number of ways of coloring a map. Ann. Math. **14** 42-46.

[2] H.P.Yap, total colourings of a graph .Lecture notes in Mathematics, Springer-verlag,1623,Berlin(1996).

[3] H.Whitney (1932), The colorings of graphs. Ann. Math. 33 688-718

[4] J.Venkateswara Rao, R.V.N.Sinivasa Rao(2012) ,A unique approach on upper bounds for the chromatic number of total graphs, Asian journal of Applied sciences 5(4) 240-246.

[5] P.Erdos and R.J.Wilson, *on the chromatic index of almost all graphs*. J.Combin. Theory Ser.B 23 (1977) 255-257.

[6] R.A.Brualadi1 (1999), Introductory Combinatorics, 3rd ed.Prentice–Hall, Upper saddle River, New Jersey.

[7] R.C.Read (1968), An introduction to chromatic polynomials.J.Combin.Theory 4 52- 71.

[8] Robin J.Willson (1996), Introduction to graph theory, by Addison Wesley Publishing Company.

[9] R.V.N.Sinivasa Rao , J.Venkateswara Rao, Contribution to 1-Quasi total graphs , Asian journal of Mathematics and statistics 2012.(Published online) http://scialert.net/onlinefirst.php?issn=1994-5418.

[10] R.V.N.Srinivasarao, J.Venkateswarao and D.Srinivasulu, A discussion on bounds for 1- quasi Total colourings, International journal of Mathematical Archive-3(6), 2012, 2314-2320.

[11]. R.V.N.Srinivasarao, J.Venkateswarao , Characterizations of 2-quasi total coloring graphs, International journal of Mathematical Archive. 3(7), 2012, 2739-2744.

[12]West.D.B (2003), Introduction to Graph Theory, 2nd Edition, Prentice-Hall India edition.