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ON ALMOST SEMIGENERALIZED α-CONTINUOUS FUNCTIONS

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Abstract. In this paper, we introduce and study the concept of almost $sg\alpha$-continuity in topological spaces.

Keywords: $sg\alpha$-open sets, almost $sg\alpha$-continuous functions.

1 INTRODUCTION AND PRELIMINARIES

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real Analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized closed sets. Recently, as generalization of closed sets, the notion of semi generalized closed sets were introduced and studied by Rajesh and Biljana [7]. A point $x \in X$ is called a θ-cluster point of A if $\mathrm{Cl}(V) \cap A \neq \phi$ for every open set V of X containing x. The set of all θ-cluster points of A is called the θ-closure of A and is denoted by $\mathrm{Cl}_\theta(A)$. If $A = \mathrm{Cl}_\theta(A)$, then A is said to be θ-closed. The complement of θ-closed set is said to be a θ-open set. The union of all θ-open sets contained in a subset A is called the θ-interior of A and is denoted by $\mathrm{Int}_\theta(A)$. It follows from [16] that the collection of θ-open sets in a topological space (X, τ) forms a topology τ_θ on X. For a subset A of a topological space (X, τ), we denote the closure of A and the interior of A by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be regular open [15] if $A = \mathrm{Int}(\mathrm{Cl}(A))$. A subset $A \subset X$ is said to be δ-open [16] if it is the union of regular open sets of X. The complement of a regular open (resp. δ-open) set is called regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure [16] of A and is denoted by $\mathrm{Cl}_\delta(A)$. A subset A of a topological space (X, τ) is said to be β-open [1] (resp. semiopen [4], preopen [5], α-open [6]) if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$(resp. $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$). The set of all regular open (resp. regular closed, δ-open, δ-closed, β-open, preopen, α-open)

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sets of (X, τ) is denoted by $RO(X)$ (resp. $RC(X)$, $\delta O(X)$, $\delta C(X)$, $\beta O(X)$, $PO(X)$, $\alpha O(X)$). The complement of an α-open set is called an α-closed set. The α-closure of a subset A of X, denoted by $\alpha\,\mathrm{Cl}(A)$ is defned to be the intersection of all α-closed sets of X containing A. A subset A of a space (X, τ) is called semigeneralized α-closed (briefly $sg\alpha$-closed) [7] if $\alpha\,\mathrm{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in (X, τ). The complement of a $sg\alpha$-closed set is said to be a $sg\alpha$-open set. The family of all $sg\alpha$-open subsets of a topological space (X, τ) forms a topology on X which is fi0ner than τ. The set of all $sg\alpha$-open sets of (X, τ) is denoted by $sg\alpha O(X)$. The set of all $sg\alpha$-open sets of (X, τ) containing a point $x \in X$ is denoted by $sg\alpha O(X, x)$. The intersection of all $sg\alpha$-closed sets containing S is called the $sg\alpha$-closure of S and is denoted by $sg\alpha\,\mathrm{Cl}(S)$. The $sg\alpha$-interior of S is defined by the union of all $sg\alpha$-open sets contained in S and is denoted by $sg\alpha\,\mathrm{Int}(S)$. In this paper, we introduce and study the concept of almost $sg\alpha$-continuity in topological spaces.

Definition 1.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:*

(1) *$sg\alpha$-continuous* [8] *if $f^{-1}(V)$ is $sg\alpha$-open in X for every open set V of Y;*

(2) *almost continuous* [14] *if $f^{-1}(V)$ is open in X for every regular open set V of X;*

(3) *R-map* [2] *if $f^{-1}(V)$ is regular open in X for every regular open set V of X;*

(4) *$sg\alpha$-irresolute* [8] *if $f^{-1}(V)$ is $sg\alpha$-open in X for every $sg\alpha$-open subset V of Y;*

(5) *faintly $sg\alpha$-continuous* [10] *if for each $x \in X$ and each θ-open set V of Y containing $f(x)$, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset V$;*

(6) *weakly $sg\alpha$-continuous* [9] *if for each point $x \in X$ if for each open subset V in Y containing $f(x)$, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset \mathrm{Cl}(V)$.*

Theorem 1.2. [10] *A function $f : (X, \tau) \to (Y, \sigma)$ is faintly $sg\alpha$-continuous if and only if $f^{-1}(V) \in sg\alpha C(X)$ for every θ-closed set V of Y.*

Definition 1.3. *A topological space (X, τ) is said to be:*

(1) *$sg\alpha$-T_1* [11] *(resp. r-T_1* [3]*) if for each pair of distinct points x and y of X, there exist $sg\alpha$-open (resp. regular open) sets and U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.*

(2) *$sg\alpha$-T_2* [11] *(resp. r-T_2* [3]*) if for each pair of distinct points x and y of X, there exist $sg\alpha$-open (resp. regular open) sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \varnothing$.*

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2 PROPERTIES OF ALMOST *sgα*-CONTINUOUS FUNCTIONS

Definition 2.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost sgα-continuous for each point $x \in X$ if for each open subset V of Y containing $f(x)$, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset \mathrm{Int}(\mathrm{Cl}(V))$.*

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Proposition 2.2. *Every almost sgα-continuious function is weakly sgα-continuous.* The following example shows that the converse of Proposition 2.2 is not true in general.

Example 2.3. *Let $X = \{a,b,c\}$, $\tau = \{\varnothing, \{a\}, \{b,c\}\}$ and $\sigma = \{\varnothing, \{a\}, \{b\}, \{a,b\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is weakly sgα-continuous but not almost sgα-continuous.*

Theorem 2.4. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(1) *f is almost sgα-continuous;*
(2) $f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) \in sg\alpha O(X)$ *for every open set V of Y;*
(3) $f^{-1}(\mathrm{Cl}(\mathrm{Int}(V))) \in sg\alpha C(X)$ *for every closed set V of Y;*
(4) $f^{-1}(V) \in sg\alpha O(X)$ *for every $V \in RO(Y)$;*
(5) $f^{-1}(F) \in sg\alpha C(X)$ *for every $F \in RC(Y)$;*
(6) *for each $x \in X$ and each open set V of Y containing $f(x)$ there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset s\,\mathrm{Cl}(V)$;*
(7) $sg\alpha\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(F)))) \subset f^{-1}(F)$ *for every closed set F of Y;*
(8) $sg\alpha\,\mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl}(A))$ *for every $A \in \beta O(Y)$;*
(9) $sg\alpha\,\mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl}(A))$ *for every $A \in SO(Y)$;*
(10) $f^{-1}(V) \subset sg\alpha\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$ *for every open set $V \in PO(Y)$;*
(11) $f(sg\alpha\,\mathrm{Cl}(A)) \subset \mathrm{Cl}_\delta(f(A))$ *for every subset A of X;*
(12) $sg\alpha\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\delta(B))$ *for every subset B of Y;*
(13) $f^{-1}(F) \in sg\alpha C(X)$ *for every $F \in \delta C(Y)$;*
(14) $f^{-1}(V) \in sg\alpha O(X)$ *for every $V \in \delta O(Y)$.*

Proof. (4) $\Rightarrow$ (5): Let $F \in RC(Y)$. Then $Y \setminus F \in RO(Y)$. Take $x \in f^{-1}(Y \setminus F)$, then $f(x) \in Y \setminus F$ and since f is almost $sg\alpha$-continuous, there exists $W_x \in sg\alpha O(X, x)$ such that $x \in W_x$ and $f(W_x) \subset Y \setminus F$.

Then $x \in W_x \subset f^{-1}(Y \setminus F)$ so that $f^{-1}(Y \setminus F) = \Rightarrow \bigcup_{x \in f^{-1}(Y \setminus F)} W_x$. Since any union of $sg\alpha$-open sets is $sg\alpha$-open, $f^{-1}(Y \setminus F)$ is $sg\alpha$-open in X and hence $f^{-1}(F) \in sg\alpha C(X)$.

(5) $\Rightarrow$ (11): Let A be a subset of X. Since $\mathrm{Cl}_\delta(f(A))$ if δ-closed in Y, it is equal to $\cap$ $\{F_\alpha$: F_α is regular closed in Y, $\alpha \in \Lambda\}$, where Λ is an index set. From (5), we have $A \subset f^{-1}(\mathrm{Cl}_\delta(f(A))) = \cap\{f^{-1}(F_\alpha) : \alpha \in \Lambda\} \in sg\alpha C(X)$ and hence $sg\alpha\ \mathrm{Cl}(A) \subset f^{-1}(\mathrm{Cl}_\delta(f(A)))$. Therefore, we obtain $f(sg\alpha\ \mathrm{Cl}(A)) \subset \mathrm{Cl}_\delta(f(A))$.
(11) $\Rightarrow$ (12): Set $A = f^{-1}(B)$ in (11), then $f(sg\alpha\ \mathrm{Cl}(f^{-1}(B))) \subset \mathrm{Cl}_\delta(f(f^{-1}(B))) \subset \mathrm{Cl}_\delta(B)$ and hence $sg\alpha\ \mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\delta(B))$.
(12) $\Rightarrow$ (13): Let F be δ-closed set of Y, then $sg\alpha\ \mathrm{Cl}(f^{-1}(F)) \subset f^{-1}(F)$ so $f^{-1}(F) \in sg\alpha C(X)$.
(13) $\Rightarrow$ (14): Let V be δ-open set of Y, then $Y \setminus V$ is δ-closed set in Y.
This gives $f^{-1}(Y \setminus V) \in sg\alpha C(X)$ and hence $f^{-1}(V) \in sg\alpha O(X)$.
(14) $\Rightarrow$ (1): Let V be any regular open set of Y. Since V is δ-open in Y, $f^{-1}(V) \in sg\alpha O(X)$ and hence from $f(f^{-1}(V)) \subset V = \mathrm{Int}(\mathrm{Cl}(V))$. Then f is almost $sg\alpha$-continuous.

(5) $\Rightarrow$ (8): Let A be any β-open set in Y. Since $\mathrm{Cl}(A)$ is regular closed, $f^{-1}(\mathrm{Cl}(A))$ is δ-closed and $f^{-1}(A) \subset f^{-1}(\mathrm{Cl}(A))$. Hence, $sg\alpha\ \mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl}(A))$.

(8) $\Rightarrow$ (9): obvious.

(9) $\Rightarrow$ (10): Let V be a preopen set. Then we have $V \subset \mathrm{Int}(\mathrm{Cl}(V))$ and $\mathrm{Cl}(\mathrm{Int}(Y \setminus V)) \subset Y \setminus V$. Moreover, since the set $\mathrm{Cl}(\mathrm{Int}(Y \setminus V))$ is semiopen, it follows that $X \setminus sg\alpha\ \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))) = sg\alpha\ \mathrm{Cl}(X \setminus f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$
$= sg\alpha\ \mathrm{Cl}(f^{-1}(Y \setminus \mathrm{Int}(\mathrm{Cl}(V)))) = sg\alpha\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(Y \setminus V)))) \subset f^{-1}(\mathrm{Cl}(\mathrm{Int}(Y \setminus V))) \subset f^{-1}(Y \setminus V) \subset X \setminus f^{-1}(V)$. Hence, we obtain $f^{-1}(V) \subset sg\alpha\ \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$.

(10) $\Rightarrow$ (4): Let V be a regular open set. Since V is preopen, we get $f^{-1}(V) \subset sg\alpha\ \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))) = sg\alpha\ \mathrm{Int}(f^{-1}(V))$. Hence $f^{-1}(V) \in sg\alpha O(X)$.
The other implications are obvious.

Theorem 2.5. *A function $f : (X, sg\alpha(X)) \to (Y, \sigma)$ is almost sgα-continuous if and only if it is almost continuous.*

Proof. The proof is clear. ¤

Theorem 2.6. *The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(1) *f is upper almost sgα-continuous;*
(2) *$sg\alpha\ \mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(\mathrm{Cl}(V))$ for every $V \in \beta O(Y)$;*

(3) $sg\alpha\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(\mathrm{Cl}(V))$ *for every* $V \in SO(Y)$;

(4) $f^{-1}(V) \subset sg\alpha\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$ *for every* $V \in PO(Y)$.

Proof. (1) $\Rightarrow$ (2): Let V be any β-open set of Y. Since $\mathrm{Cl}(V) \in RC(Y)$, by Theorem 2.4 $f^{-1}(\mathrm{Cl}(V))$ is $sg\alpha$-closed in X and $f^{-1}(V) \subset f^{-1}(\mathrm{Cl}(V))$. Therefore, we obtain $sg\alpha\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(\mathrm{Cl}(V))$.
(2) $\Rightarrow$ (3): This is obvious since $SO(Y) \subset \beta O(Y)$.
(3) $\Rightarrow$ (1): Let $K \in RC(Y)$. Then $K \in SO(Y)$ and hence $sg\alpha\,\mathrm{Cl}(f^{-1}(K)) \subset f^{-1}(K)$. Therefore, $f^{-1}(K)$ is $sg\alpha$-closed in X and hence F is upper almost $sg\alpha$-continuous by Theorem 2.4.
(1) $\Rightarrow$ (4): Let V be arbitrary preopen set of Y. Since $\mathrm{Int}(\mathrm{Cl}(V)) \in RO(Y)$, by Theorem 2.4 we have $f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) \in sg\alpha O(X)$ and hence $f^{-1}(V) \subset f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) = sg\alpha\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$.
(4) $\Rightarrow$ (1): Let V be any regular open set of Y. Since $V \in PO(Y)$, we have $f^{-1}(V) \subset sg\alpha\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))) = sg\alpha\,\mathrm{Int}(f^{-1}(V))$ and hence $f^{-1}(V) \in sg\alpha O(X)$. It follows from Theorem 2.4 that f is upper almost $sg\alpha$-continuous.

Definition 2.7.

1. *A filterbase* Λ *is said to be sgα-convergent to a point* x *in* X *if for any* $U \in sg\alpha O(X, x)$, *there exists* $B \in \Lambda$ *such that* $B \subset U$.
2. *A filterbase* Λ *is said to be r-convergent to a point* x *in* X *if for any regular open set* U *of* X *containing* x, *there exists* $B \in \Lambda$ *such that* $B \subset U$.

Theorem 2.8. *If a function* $f: (X, \tau) \to (Y, \sigma)$ *is almost sgα-continuous, then for each point* $x \in X$ *and each filter base* Λ *in* X *sgα-converging to* x, *the filter base* $f(\Lambda)$ *is r-convergent to* $f(x)$.

Proof. Let $x \in X$ and Λ be any filter base in X $sg\alpha$-converging to x. Since f is $sg\alpha$-continuous, then for any open set V of (Y, σ) containing $f(x)$, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset V$. Since Λ is $sg\alpha$-converging to x, there exists $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and hence the filter base $f(\Lambda)$ is convergent to $f(x)$.

Definition 2.9. *A sequence* (x_n) *is said to be sgα-convergent to a point* x *if for every sgα-open set* V *containing* x, *there exists an index* n_0 *such that for* $n \geq n_0$, $x_n \in V$.

Theorem 2.10. *If a function* $f: (X, \tau) \to (Y, \sigma)$ *is almost sgα-continuous, then for each point* $x \in X$ *and each net* (x_n) *which is sgα-convergt to* x, *the net* $f((x_n))$ *is r-convergent to* $f(x)$.

Proof. The proof is similar to that of Theorem 2.8.

Theorem 2.11. *If an injective function $f:(X,\tau)\to(Y,\sigma)$ is almost sgα-continuous and (Y,σ) is r-T_1, then (X,τ) is sgα-T_1.*

Proof. Suppose that (Y,σ) is r-T_1. For any distict points x and y in X, there exist regular open sets V and W such that $f(x)\in V$, $f(y)\notin V$, $f(x)\notin W$ and $f(y)\in W$. Since f is almost $sg\alpha$-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $sg\alpha$-open subsets of (X,τ) such that $x\in f^{-1}(V)$, $y\notin f^{-1}(V)$, $x\notin f^{-1}(W)$ and $y\in f^{-1}(W)$. This shows that (X,τ) is $sg\alpha$-T_1.

Theorem 2.12. *If $f:(X,\tau)\to(Y,\sigma)$ is an almost sgα-continuous injective function and (Y,σ) is r-T_2, then (X,τ) is sgα-T_2.*

Proof. For any pair of distinct points x and y in X, there exist disjoint regular open sets U and V in Y such that $f(x)\in U$ and $f(y)\in V$. Since f is almost $sg\alpha$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $sg\alpha$-open sets in X containing x and y, respectively. Therefore, $f^{-1}(U)\cap f^{-1}(V)=\varnothing$ because $U\cap V=\varnothing$. This shows that (X,τ) is $sg\alpha$-T_2.

Theorem 2.13. *If $f:(X,\tau)\to(Y,\sigma)$ is an almost continuous function and $g:(X,\tau)\to(Y,\sigma)$ is an almost sgα-continuous function and Y is a r-T_2-space, then the set $E=\{x\in X: f(x)=g(x)\}$ is an sgα-closed set in (X,τ).*

Proof. If $x\in X\backslash E$, then it follows that $f(x)\neq g(x)$. Since Y is r-T_2, there exist disjoint regular open sets V and W of Y such that $f(x)\in V$ and $g(x)\in W$. Since f is almost continuous and g is almost $sg\alpha$-continuous, then $f^{-1}(V)$ is open and $g^{-1}(W)$ is $sg\alpha$-open in X with $x\in f^{-1}(V)$ and $x\in g^{-1}(W)$. Put $A=f^{-1}(V)\cap g^{-1}(W)$. Then A is $sg\alpha$-open in X. Therefore, $f(A)\cap g(A)=\varnothing$ and it follows that $x\notin sg\alpha\,\mathrm{Cl}(E)$. This shows that E is $sg\alpha$-closed in X.

Theorem 2.14. *The implications (i) ⇒ (ii) ⇒ (iii) ⇒ (iv) ⇒ (v) hold for the following properties of a function $f:(X,\tau)\to(Y,\sigma)$:*

(1) *f is sgα-continuous.*
(2) *$f^{-1}(\mathrm{Cl}_\delta(B))$ is sgα-closed in X for every subset B of Y.*
(3) *f is almost sgα-continuous.*
(4) *f is weakly sgα-continuous.*
(5) *f is faintly sgα-continuous.*

If, in addition, Y is regular, then the five properties are equivalent of one another.

Proof. (1) ⇒ (2): Since $\mathrm{Cl}_\delta(B)$ is closed in Y for every subset B of Y, by Theorem 2.4, $f^{-1}(\mathrm{Cl}_\delta(B))$ is $sg\alpha$-closed in X.

(2) $\Rightarrow$ (3): For any subset B of Y, f^{-1} ($\mathrm{Cl}_\delta(B)$) is $sg\alpha$-closed in X and hence we have $sg\alpha$ Cl(f^{-1} (B)) $\subset$ $sg\alpha$ Cl(f^{-1} ($\mathrm{Cl}_\delta(B)$) = f^{-1} ($\mathrm{Cl}_\delta(B)$). It follows from Theorem 2.4 that f is almost $sg\alpha$-continuous.

(3) $\Rightarrow$ (4): This is obvious.

(4) $\Rightarrow$ (5): Let F be any θ-closed set of Y. It follows from Theorem 1.2 that $sg\alpha$ Cl(f^{-1} (F)) $\subset$ f^{-1} (Cl_θ (F)) = f^{-1} (F). Therefore, f^{-1} (F) is $sg\alpha$-closed in X and hence f is faintly $sg\alpha$-continuous.

Suppose that Y is regular. We prove that (5) $\Rightarrow$ (1). Let V be any open set of Y. Since Y is regular, V is θ-open in Y. By the faint $sg\alpha$-continuity of f, f^{-1} (V) is $sg\alpha$-open in X. Therefore, f is $sg\alpha$- continuous. ¤

Definition 2.15. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $sg\alpha$-preopen if $f(U) \in PO(Y)$ for every $sg\alpha$-open set U of X.*

Theorem 2.16. *If a function $f : (X, \tau) \to (Y, \sigma)$ is $sg\alpha$-preopen and weakly $sg\alpha$-continuous, then f is almost $sg\alpha$-continuous.*

Proof. Let $x \in X$ and let V be an open set of Y containing $f(x)$. Since f is weakly $sg\alpha$-continuous, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset \mathrm{Cl}(V)$. Since f is $sg\alpha$-preopen, $f(U) \subset \mathrm{Int}(\mathrm{Cl}(f(U))) \subset \mathrm{Int}(\mathrm{Cl}(V))$ and hence f is almost $sg\alpha$-continuous.

Theorem 2.17. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $g : X \to X \times Y$ the graph function defned by $g(x) = (x, f(x))$ for every $x \in X$. Then g is almost $sg\alpha$-continuous if and only if f is almost $sg\alpha$-continuous.*

Proof. Let x be any point of X and V any regular open set of Y containing $f(x)$. Then we have $g(x) = (x, f(x)) \in X \times V$ is regular open in $X \times Y$. Since g is almost $sg\alpha$-continuous, there exists $U \in sg\alpha O(X, x)$ such that $g(U) \subset X \times V$. Therefore, we obtain $f(U) \subset V$; hence f is almost $sg\alpha$-continuous. Conversely, let $x \in X$ and W be a regular open set of $X \times Y$ containing $g(x)$. There exist a regular open set U_1 in X and a regular open set V in Y such that $U_1 \times V \subset W$. Since f is almost $sg\alpha$-continuous, there exist $U_2 \in sg\alpha O(X, x)$ such that $f(U_2) \subset V$. Put $U = U_1 \cap U_2$, then we obtain $x \in U \in sg\alpha O(X)$ and $g(U) \subset U \times V \subset W$. This shows that g is almost $sg\alpha$-continuous.

Theorem 2.18. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be functions. Then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ is almost $sg\alpha$-continuous if f and g satisfy one of the following conditions:*

(1) *f is almost $sg\alpha$-continuous and g is R-map.*

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(2) *f is sgα-irresolute and g is almost sgα-continuous.*
(3) *f is sgα-continuous and g is almost continuous.*

Proof. The proof is clear.

Theorem 2.19. *If a function $f : X \to \prod Y_\alpha$ is almost sgα-continuous then $p_\alpha \circ f : (X, \tau) \to (Y, \sigma)_\alpha$ is almost sgα-continuous for each $\alpha \in I$, where p_α is the projection of $\prod Y_\alpha$ onto Y_α.*

Proof. Let V_α be any regular open set of Y_α. Since p_α is continuous open, it is an R-map and hence $p_\alpha^{-1}(V_\alpha)$ is regular open in Y_α, then $f^{-1}\left(p_\alpha^{-1}(V_\alpha)\right) = (p_\alpha \circ f)^{-1}(V_\alpha) \in sg\alpha O(X)$. This shows that $p_\alpha \circ f$ is almost $sg\alpha$- continuous for each $\alpha \in I$

Definition 2.20. *A topological space (X, τ) is said to be:*

(1) *almost regular* [13] *if for any regular closed set F of X and any point $x \in X \backslash F$ there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$.*

(2) *semi-regular* [15] *if for any open set U of X and each point $x \in U$ there exists a regular open set V of X such that $x \in V \subset U$.*

Theorem 2.21. *If $f : (X, \tau) \to (Y, \sigma)$ is a weakly sgα-continuous function and Y is almost regular, then f is almost sgα-continuous.*

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the almost regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset \mathrm{Cl}(G) \subset \mathrm{Int}(\mathrm{Cl}(V))$ [[13], Theorem 2.2]. Since f is weakly $sg\alpha$-continuous, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset \mathrm{Cl}(G) \subset \mathrm{Int}(\mathrm{Cl}(V))$. Therefore, f is almost $sg\alpha$-continuous.

Theorem 2.22. *If $f : (X, \tau) \to (Y, \sigma)$ is an almost sgα-continuous function and Y is semi-regular, then f is sgα-continuous.*

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost $sg\alpha$-continuous, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset \mathrm{Int}(\mathrm{Cl}(G)) = G \subset V$ and hence f is $sg\alpha$-continuous.

Definition 2.23. [8] *An sgα-frontier of a subset A of (X, τ), denoted by $sg\alpha Fr(A)$, is defned by $sg\alpha Fr(A) = sg\alpha\, \mathrm{Cl}(A) \cap sg\alpha\, \mathrm{Cl}(X \backslash A)$.*

Theorem 2.24. *The set of all points $x \in X$ in which a function $f : (X, \tau) \to (Y, \sigma)$ is not almost sgα-continuous is identical with the union of sgα-frontier of the inverse images of regular open sets containing $f(x)$.*

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Proof. Suppose that f is not almost $sg\alpha$-continuous at $x \in X$. Then there exists a regular open set V of Y containing $f(x)$ such that $U \cap (X \setminus f^{-1}(V)) \neq \varnothing$; for every $U \in sg\alpha O(X, x)$. Therefore, we have $x \in sg\alpha\ \mathrm{Cl}(X \setminus f^{-1}(V)) = X \setminus sg\alpha\ \mathrm{Int}(f^{-1}(V))$ and $x \in f^{-1}(V)$. Thus, we obtain $x \in sg\alpha Fr(f^{-1}(U))$. Conversely, suppose that f is almost $sg\alpha$-continuous at $x \in X$ and let V be a regular open set of Y containing $f(x)$. Then there exists $U \in sg\alpha O(X, x)$ such that $U \subset f^{-1}(V)$. That is $x \in sg\alpha Int\left(f^{-1}(V)\right)$. Therefore, $x \in X \setminus sg\alpha Fr\left(f^{-1}(V)\right)$.

Definition 2.25. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be com-plementary almost sgα-continuous if for each regular open set V of Y, $f^{-1}(Fr(V))$ is sgα-closed in X, where $Fr(V)$ denotes the frontier of V.*

Theorem 2.26. *If $f : (X, \tau) \to (Y, \sigma)$ is weakly sgα-continuous and complementary almost sgα-continuous, then f is almost sgα-continuous.*

Proof. Let $x \in X$ and V be a regular open set of Y containing $f(x)$. Then $f(x) \in Y \setminus Fr(V)$ and hence $x \in X \setminus f^{-1}(Fr(V))$. Since f is weakly $sg\alpha$-continuous there exists $G \in sg\alpha O(X, x)$ such that $f(G) \subset \mathrm{Cl}(V)$. Put $U = G \cap (X \setminus f^{-1}(Fr(V)))$. Then $U \in sg\alpha O(X, x)$ and $f(U) \subset f(G) \cap (Y \setminus Fr(V)) \subset \mathrm{Cl}(V) \cap (Y \setminus Fr(V)) = V$ this shows that f is almost $sg\alpha$-continuous. ¤

Theorem 2.27. *If $f : (X, \tau) \to (Y, \sigma)$ is almost sgα-continuous, $g : (X, \tau) \to (Y, \sigma)$ is weakly sgα-continuous and Y is Hausdorff, then the set $\{x \in X : f(x) = g(x)\}$ is sgα-closed in (X, τ).*

Proof. Let $A = \{x \in X : f(x) = g(x)\}$ and $x \in X \setminus A$. Then $f(x) \neq g(x)$. Since (Y, σ) is Hausdorff, there exist open sets V and W of Y such that $f(x) \in V$, $g(x) \in W$ and $V \cap W = \varnothing$, hence $\mathrm{Int}(\mathrm{Cl}(V)) \cap \mathrm{Cl}(W) = \varnothing$. Since f is almost $sg\alpha$-continuous, there exists $G \in sg\alpha O(X, x)$ such that $f(G) \subset \mathrm{Int}(\mathrm{Cl}(V))$. Since g is weakly $sg\alpha$-continuous, there exists $H \in sg\alpha O(X)$ such that $g(H) \subset \mathrm{Cl}(W)$. Now put $U = G \cap H$, then $U \in sg\alpha O(X, x)$ and $f(U) \cap g(U) \subset \mathrm{Int}(\mathrm{Cl}(V)) \cap \mathrm{Cl}(W) = \varnothing$. Therefore, we obtain $U \cap A = \varnothing$ and hence A is $sg\alpha$-closed in X.

Theorem 2.28. *Suppose that the product of two sgα-open sets is sgα-open. If $f_1 : (X_1, \tau) \to (Y, \sigma)$ is weakly sgα-continuous, $f_2 : (X_2, \tau) \to (Y, \sigma)$ is almost sgα-continuous and (Y, σ) is Hausdorff, then the set $\{(x_1, x_2) \in X_1 \times X_2 : f_1(x_1) = f_2(x_2)\}$ is sgα-closed in $X_1 \times X_2$.*

Proof. Let $A = \{(x_1, x_2) \in X_1 \times X_2 : f(x_1) = f(x_2)\}$. If $(x_1, x_2) \in (X_1 \times X_2) \setminus A$, then we have $f(x_1) \neq f(x_2)$. Since (Y, σ) is Hausdorff, there exist disjoint open sets V_1 and V_2 in Y such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $\mathrm{Cl}(V_1) \cap \mathrm{Int}(\mathrm{Cl}(V_2)) = \varnothing$. Since f_1 (resp. f_2) is

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weakly $sg\alpha$-continuous (resp. almost $sg\alpha$-continuous), there exists $U_1 \in sg\alpha O(X_1, x_1)$ such that $f(U_1) \subset \mathrm{Cl}(V_1)$ (resp. $U_2 \in sg\alpha O(X_2, x_2)$ such that $f(sg\alpha\ \mathrm{Cl}(U_1)) \subset (\mathrm{Int}(\mathrm{Cl}(V_2))))$. Therefore, we obtain $(x_1, x_2) \in U_1 \times U_2 \subset X_1 \times X_2 \setminus A$. Therefore, $(X_1 \times X_2)\setminus A$ is $sg\alpha$-open and hence A is $sg\alpha$-closed in $X_1 \times X_2$

Proof. Let S be any δ-closed set of $X \times Y$ and $x \in sg\alpha\ \mathrm{Cl}(p_X (S \cap G(g))$. Let U be any open set of X containing x and V any open set of Y containing $g(x)$. Since g is almost $sg\alpha$-continuous, we have $x \in g^{-1}(V) \subset sg\alpha\ \mathrm{Int}(g^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$ and $U \cap sg\alpha\ \mathrm{Int}(g^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))) \in sg\alpha O(X, x)$. Since $x \in sg\alpha\ \mathrm{Cl}(p_X (S \cap G(g))$, $(U \cap sg\alpha\ \mathrm{Int}(g^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))) \cap p_X (S \cap G(g))$ contains some point u of X. This implies that $(u, g(u)) \in S$ and $g(u) \in \mathrm{Int}(\mathrm{Cl}(V))$. Thus, we have $\varnothing \neq (U \times \mathrm{Int}(\mathrm{Cl}(V)) \cap S \subset \mathrm{Int}(\mathrm{Cl}(U \times V)) \cap S$ and hence $(x, g(x)) \in \mathrm{Cl}_\delta(S)$. Since S is δ-closed, $(x, g(x)) \in p_X (S \cap G(g))$ and $x \in p_X (S \cap G(g))$. Then $p_X (S \cap G(g))$ is $sg\alpha$-closed.

Corollary 2.30. *If $f : (X, \tau) \to (Y, \sigma)$ has a δ-closed graph and $g : (X, \tau) \to (Y, \sigma)$ is almost $sg\alpha$-continuous, then the set $\{x \in X : f(x) = g(x)\}$ is $sg\alpha$-closed in X.*

Proof. Since $G(f)$ is δ-closed and $p_X (G(f) \cap G(g)) = \{x \in X : f(x) = g(x)\}$ it follows from Theorem 2.29 that $\{x \in X : f(x) = g(x)\}$ is
$sg\alpha$-closed in X. ¤

Theorem 2.31. *If for each pair of distinct x_1 and x_2 in a topological space (X, τ) there exists a function f on (X, τ) into a Hausdorff space (Y, σ) such that $f(x_1) \neq f(x_2)$, f is weakly $sg\alpha$-continuous at x_1 and f is almost $sg\alpha$-continuous at x_2, then X is $sg\alpha$-T_2.*

Proof. Since (Y, σ) is Hausdorff, for each pair of distinct point x_1 and x_2 there exist disjoint open sets V_1 and V_2 of Y containing $f(x_1)$ and $f(x_2)$, respectively, hence $\mathrm{Cl}(V_1) \cap \mathrm{Int}(\mathrm{Cl}(V_2)) = \varnothing$. Since f is weakly $sg\alpha$-continuous at x_1, there exists $U_1 \in sg\alpha O(X, x_1)$ such that $f(U_1) \subset \mathrm{Cl}(V_1)$. Since f is almost $sg\alpha$-continuous at x_2, there exists $U_2 \in sg\alpha O(X, x_2)$ such that $f(U_2) \subset \mathrm{Int}(\mathrm{Cl}(V_2))$. Therefore, we obtain $U_1 \cap U_2 = \varnothing$. This shows that (X, τ) is $sg\alpha$-T_2.

Definition 2.32. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to have an $sg\alpha$-strongly closed graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exists an $sg\alpha$-open subset U of X and an open subset V of Y such that $(U \times \mathrm{Cl}(V)) \cap G(f) = \varnothing$.*

Lemma 2.33. *A function $f : (X, \tau) \to (Y, \sigma)$ has $sg\alpha$-strongly closed graph $G(f)$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$ there exists an $sg\alpha$-open set U and an open set V containing x and y, respectively such that $f(U) \cap \mathrm{Cl}(V) = \varnothing$.*

Theorem 2.34. *If $f : (X, \tau) \to (Y, \sigma)$ is an almost sgα-continuous function and (Y, σ) is Hausdorff, then f has an sgα-strongly closed graph.*

Proof. Let $(x, y) \in (X \times Y)$ such that $y \neq f(x)$. Since (Y, σ) is Hausdorff, there exist open sets V and W of Y containing $f(x)$ and y, respectively, such that $V \cap W = \varnothing$. Then $f(x) \in Y \setminus \mathrm{Cl}(W)$ and $Y \setminus \mathrm{Cl}(W)$ is regular open in Y. There exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset Y \setminus \mathrm{Cl}(W)$ and hence $f(U) \cap \mathrm{Cl}(W) = \varnothing$. Therefore, by Lemma 2.33 f has an $sg\alpha$-strongly closed graph. ¤

Corollary 2.35. *If $f : (X, \tau) \to (Y, \sigma)$ is an sgα-continuous function and (Y, σ) is Hausdorff, then f has an sgα-strongly closed graph.*

References

1. M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud, β-open sets and β-continuous functions, *Bull. Fac. Sci. Assiut Univ. A*, 12(1983), 77-90.
2. D. Carnahan, Some properties related to compactness in topological spaces, *Ph. D. Thesis*, Univ. Arkansas (1973).
3. E. Ekici, Generalization of perfectly continuous, Regular set-connected and clopen functions, *Acta. Math. Hungar.*, 107(3)(2005), 193-206.
4. N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.
5. A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc. Egypt*, 53(1982), 47-53.
6. O. Njstad, *On some classes of nearly open sets*, Pacific J. Math. 15(1965), 961-970.
7. N. Rajesh and K. Biljana, Semigeneralized α-closed sets, *Antarctica J. Math.*, 6(1) 2009, 1-12.
8. N. Rajesh, G. Shanmugam and K. Biljana, Semigeneralized α-continuous functions (submitted).
9. N.Rajesh and G. Shanmugam, Weakly semigeneralized α-continuous functions (submitted).
10. N.Rajesh and G. Shanmugam, On faintly semigeneralized α-continuous functions (submitted).
11. N. Rajesh, G. Shanmugam and K. Biljana, Characterization of $sg\alpha$-T_0, $sg\alpha$-T_1 and $sg\alpha$-T_2 topological spaces (submitted).
12. N.Rajesh and G. Shanmugam and K. Biljana, Slightly semigeneralized α-continuous functions (submitted).
13. M. K. Singal and S. P. Arya, On almost regular spaces, *Glasnik Mat.*, 4 (24) (1969), 89-99.

14. M. K. Singal and A. R. Singal, Almost-continuous mappings, *Yokohama Math. J.*, 16(1968), 63-73.
15. M. Stone, Applications of the theory of boolean rings to general topology, *Trans. Amer. Math. Soc.*, 41(1937), 374-381.
16. N. V. Veli·cko, *H*-closed topological spaces, *Amer. Math. Soc. Transl.(2)*, 78(1968), 103-118.