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Erratum on the paper notes on the commutativity of prime near-rings

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Abstract. In this paper, we correct some results of the article entitled "Notes on the commutativity of prime near-rings" published in the journal Miskolc Mathematical Notes, Vol. 12 (2011), no. 2.

Keywords: near-rings, $(\theta,\ \theta)$-derivation, generalized $(\theta,\ \theta)$-derivation.

1 Definitions and terminology

A right near-ring is a set N with two operation + and . such that $(N,+)$ is a group (not necessarily abelian) and $(N,\ .\ )$ is a semigroup satisfying the right distributive law $(x+y).z = x.z + y.z$ for all $x,\ y,\ z \in N$. Recalling that a near-ring N is called prime if for any $x,\ y \in N$, $xNy = \{0\}$ implies that $x = 0$ or $y = 0$. For $x,\ y \in N$ the symbol $[x,\ y]$ (resp. $x \circ y$) will denote $xy - yx$ (resp. $xy + yx$). $Z(N)$ is the multiplicative center of N. An additive mapping $d : N \to N$ is said to be a derivation if $d(xy) = xd(y) + d(x)y$ for all $x,\ y \in N$, or equivalently, as noted in [12], that $d(xy) = d(x)y + xd(y)$ for all $x,\ y \in N$. Recently, in [7], Bresar defined the following concept. An additive mapping $F : N \to N$ is called a generalized derivation if there exists a derivation $d : N \to N$ such that $F(xy) = F(x)y + xd(y)$ for all $x,\ y \in N$. Basic examples are derivations and generalized inner derivations (i.e., maps of type $x \to ax + xb$ for some $a,\ b \in N$). One may observe that the concept of generalized derivations includes the concept of derivations and of left multipliers (i.e., $F(xy) = F(x)y$ for all $x,\ y \in N$). Inspired by the definition of derivation (resp. generalized derivation), we define the notion of $(\theta,\ \phi)$-derivation (resp. generalized $(\theta,\ \phi)$-derivation) as follows: Let $\theta,\ \phi$ be two near-ring automorphisms of N. An additive mapping $d: N \to N$ is called a (θ, ϕ)-derivation if $d(xy) = \phi(x)d(y) + d(x)\theta(y)$ for all $x, y \in N$. An additive mapping $F: N \to N$ is called a generalized (θ, ϕ)-derivation if there is a (θ, ϕ)-derivation d such that $F(xy) = F(x)\theta(y) + \phi(x)d(y)$. It is noted that $d(xy) = d(x)\theta(y) + \phi(x)d(y)$ for all $x, y \in N$ in [9, Lemma 1].

2 The Main Results

In [9] the theorems 3, 4, 5 and 6 are not correct in general. Moreover the following theorems show the non existence of generalized (θ, θ)-derivation of N satisfying the theorems.

Theorem 1. Let N be a 2-torsion free 3-prime near-ring, then there is no generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that $F(x o y) = 0$ for all $x, y \in N$.

Proof. If there exists a generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that

$$F(x o y) = 0 \quad \text{for all } x, y \in N. \tag{1}$$

From the proof of [9, Theorem 3], we conclude that N is a commutative ring and using equation (1), we obtain

$$F(xy) = 0 \quad \text{for all } x, y \in N. \tag{2}$$

It follows

$$F(x)\theta(y) + \theta(x)d(y) = 0 \quad \text{for all } x, y \in N. \tag{3}$$

Replacing x by xz in (3) and using (2), we get

$$\theta(x)\theta(z)\theta(y) = 0 \quad \text{for all } x, y, z \in N.$$

Since θ is an automorphism of N, we have

$$\theta(x)N\theta(y) = \{0\} \quad \text{for all } x, y, z \in N. \tag{4}$$

By the primeness of N, we obtain that $d = 0$; a contradiction. This completes the proof of our theorem.

Theorem 2. Let N be a 2-torsion free 3-prime near-ring, then there is no generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that $F(x o y) = \pm\theta(x o y)$ for all $x, y \in N$.

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Proof. If there exists a generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that

$$F(x \circ y) = \pm\theta(x \circ y) \quad \text{for all } x, y \in N. \tag{5}$$

Using the proof of [9, Theorem 4], we conclude that N is a commutative ring and by (5), we arrive at

$$F(xy) = \pm\theta(xy) \quad \text{for all } x, y \in N \tag{6}$$

which implies that

$$F(x)\theta(y) + \theta(x)d(y) = \pm\theta(x)\theta(y) \quad \text{for all } x, y \in N. \tag{7}$$

Taking xz instead of x in (7) and using (6), we get

$$\theta(x)\theta(z)\theta(y) = 0 \quad \text{for all } x, y, z \in N.$$

Since θ is an automorphism of N, we have

$$\theta(x)N\theta(y) = \{0\} \quad \text{for all } x, y, z \in N. \tag{8}$$

By the primeness of N, we conclude that $d = 0$; a contradiction.

Theorem 3. Let N be a 2-torsion free 3-prime near-ring, then there is no generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that $F([x, y]) = \pm\theta(x \circ y)$ for all $x, y \in N$.

Proof. Suppose that there exists a generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that

$$F([x, y]) = \pm\theta(x \circ y) \quad \text{for all } x, y \in N. \tag{9}$$

According to the proof of [9, Theorem 5], N is a commutative ring and returning to (9), we get

$$\pm\theta(xy) = 0 \quad \text{for all } x, y \in N$$

it means that,

$$\pm\theta(x)\theta(y) = 0 \quad \text{for all } x, y \in N. \tag{10}$$

By (10) and the primeness of N, it is easy to verify that $x = 0$ for all $x \in N$; a contradiction.

Theorem 4. Let N be a 2-torsion free 3-prime near-ring, then there is no generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that $F(x \circ y) = \pm\theta([x, y])$ for all $x, y \in N$.

Proof. Let (F, d) be a generalized (θ, θ)-derivation (F, d) of N and $d \neq 0$ such that

$$F(\mathrm{xoy}) = \pm\theta([\mathrm{x}, \mathrm{y}]) \quad \text{for all} \quad \mathrm{x}, \mathrm{y} \in N. \tag{11}$$

Using the same proof of [9, Theorem 6], we find that N is a commutative ring and by (11), we arrive at

$$F(\mathrm{xy}) = 0 \quad \text{for all} \quad \mathrm{x}, \mathrm{y} \in N. \tag{12}$$

Since equation (12) is the same as (2), arguing as in the proof of Theorem 1 we conclude that $d = 0$; a contradiction.

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