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Numerical solution of fuzzy differential equations under generalized differentiability by Modified Euler method

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Abstract

In this paper, we interpret a fuzzy differential equation by using the strongly generalized differentiability concept. Utilizing the Generalized Characterization Theorem, we investigate the problem of finding a numerical approximation of solutions. The Modified Euler approximation method is implemented and its error analysis, which guarantees point-wise convergence, is given. The method applicability is illustrated by solving a linear first-order fuzzy differential equation.

Keywords: Fuzzy differential equations; Generalized differentiability; Generalized Characterization Theorem; Modified Euler method.

1 Introduction

The study of fuzzy differential equations (FDEs) forms a suitable setting for mathematical modeling of real-world problems in which uncertainties or vagueness pervade. There several approaches to the study of fuzzy differential equations [7,14,17]. The first and the most popular approach is using the Hukuhara differentiability for fuzzy number value functions. Under this setting, mainly the existence and uniqueness of the solution of a fuzzy differential equation are studied. This approach has a drawback: the solution becomes fuzzier as time goes by. Hence, the fuzzy solution behaves quite differently from the crisp solution. To alleviate the situation, Hullermeier interpreted FDEs as a family of differential inclusions. The main shortcoming of using differential inclusions is that we do not have a derivative of a fuzzy-number-valued function.

The strongly generalized differentiability was introduced in [4] and studied in [5,6,8]. This concept allows us to resolve the above-mentioned shortcoming. Indeed the strongly generalized derivative is defined for a larger class of fuzzy-number-valued functions than the Hukuhara derivative. Hence, we use this

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differentiability concept in the present paper. Under appropriate conditions, the fuzzy initial value problem (FIVP) considered under this interpretation has locally two solutions [5].

Numerical solution of an FDE is obtained now in a natural way, by extending the existing classical methods to the fuzzy case. Some numerical methods for FDEs under the Hukuhara differentiability concept such as the fuzzy Euler method, predictor- corrector method, Taylor method and Nystrom method are presented in [1,3,10,15]. The local existence of two solutions of an FDE under generalized differentiability implies that we present new numerical methods. In this paper, using strongly generalized differentiability, we generalize some numerical methods presented for solving FDEs. The original initial value problem is replaced by two parametric ordinary differential systems which are then solved numerically using classical algorithms.

After a preliminary section, we study fuzzy differential equations using the concept of generalized differentiability and present the generalized characterization theorem. In section 4, we propose numerical methods for solving FDEs. A scheme based on the classical modified Euler method is discussed and this is followed by a complete error analysis. Also, we present a numerical example to illustrate our method.

2 Preliminaries

In this section, we give some definitions and introduce the necessary notation which will be used throughout the paper. See for example [9]

Definition 2.1 *Let X be a nonempty set. A fuzzy set u in X is characterized by its membership function $u : X \to [0,1]$. Then $u(x)$ is interpreted as the degree of membership of a element x in the fuzzy set u for each $x \in X$.*

Let us denote by $\Re_F$ the class of fuzzy subsets of the real axis ($i.e. u : \Re \to [0,1]$) satisfying the following properties:

(i) u is normal, i.e., there exists $s_0 \in \Re$ such that $u(s_0) = 1$,

(ii) u is a convex fuzzy set (i.e. $u(ts + (1-t)r) \geq \min\ \{u(s), u(r)\}$, $\forall t \in [0,1], \quad s, r \in \Re$),

(iii)u is upper semicontinuous on $\Re$,

(iv) $\mathrm{cl}(s \in \Re | u(s) > 0\}$ is compact, where cl denotes the closure of a subset.

Then $\Re_F$ is called the space of fuzzy numbers. Obviously $\Re \subset \Re_F$. For $0 < \alpha \leq 1$ denote $[u]^\alpha = \{s \in \Re | u(s) \geq \alpha\}$ and $[u]^0 = cl\{s \in \Re | u(s) > 0\}$.

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Then from (i)-(iv) it follows that if u belongs to $\Re_F$ then the α-level set$[u]^\alpha$ is a non-empty compact interval for all $0 \leq \alpha \leq 1$.

The notation $[u]^\alpha = [\underline{u}^\alpha, \overline{u}^\alpha]$ denotes explicitly the α-level set of u. We refer to $\underline{u}$ and $\overline{u}$ as the lower and upper branches on u, respectively.

For $u, v \in \Re_F$ and $\lambda \in \Re$, the sum $u+v$ and the product $\lambda \odot u$ are defined by $[u+v]^\alpha = [u]^\alpha + [v]^\alpha$, $[\lambda \odot u]^\alpha = \lambda[u]^\alpha$, $\forall \alpha \in [0,1]$, where $[u]^\alpha + [v]^\alpha$ means the usual addition of two intervals (subsets) of $\Re$ and $\lambda[u]^\alpha$ means the usual product between a scalar and a subset of $\Re$.

The metric structure is given by the Hausdroff distance

$$D : \Re_F \times \Re_F \to \Re_+ \cup \{0\},$$
$$D(u,v) = \sup_{\alpha\in[0,1]} \max\{|\underline{u}^\alpha - \underline{v}^\alpha|\,,\, |\overline{u}^\alpha - \overline{v}^\alpha|\}$$

$(\Re_F, D)$ is a complete metric space and the following properties are well known:

$$\begin{aligned} D(u+w, v+w) &= D(u,v), \forall u,v,w \in \Re_F, \\ D(k \odot u, k \odot v) &= |k|\, D(u,v),\ \forall k \in \Re, u, v \in \Re_F, \\ D(u+v, w+e) &\leq D(u,w) + D(v,e),\ \forall u,v,w,e \in \Re_F. \end{aligned}$$

Definition 2.2 *Let x, $y \in \Re_F$. If there exists $z \in \Re_F$ such that $x = y+z$, then z is called the H-difference of x, y and it is denoted $x \ominus y$.*

In this paper the sign "$\ominus$" always stands for the H- difference, and let us remark that $x \ominus y \neq x + (-1)y$. Usually we denote $x+(-1)y$ by $x-y$, while $x \ominus y$ stands for the H-difference. In what follows, we fix $I = (a,b)$, for a, b $\in \Re$.

Definition 2.3 *Let $F : I \to \Re_F$ be a fuzzy function. We say F is differentiable at $t_0 \in I$ if there exists an element $F'(t_0) \in \Re_F$ such that the limits $\lim_{h\to 0^+} \frac{F(t_0+h)\ominus F(t_0)}{h}$ and $\lim_{h\to 0^+} \frac{F(t_0)\ominus F(t_0-h)}{h}$, exist and are equal $F'(t_o)$. Here the limits are taken in the metric space $(\Re_F, D)$, since we have defined $h^{-1} \odot (F(t_0) \ominus F(t_0-h))$ and $h^{-1} \odot (F(t_0+h) \ominus F(t_0))$.*

The above definition is a straightforward generalization of the Hukuhara differentiability of a set-valued function. From proposition 4.2.8 in [9], it follows that a Hukuhara differentiable function has increasing length of support. Note that this definition of a derivative is very restrictive; for instance in [5], the authors showed that, if $F(t) = c \odot g(t)$, where c is a fuzzy number and $g : [a,b] \to \Re^+$ is a function with $g'(t) < 0$, then F is not differentiable. To avoid this difficulty, the authors of [5] introduced a more general definition of a derivative for a fuzzy-number-valued function. In this paper we consider the following definition [8]:

Definition 2.4 *Let $F : I \to \Re_F$. Fix $t_0 \in I$. We say F is differentiable at t_0, if there exists an element $F'(t_0) \in \Re_F$ such that*

(1) for all $h > 0$ sufficiently close to 0, there exist $F(t_0 + h) \ominus F(t_0)$, $F(t_0) \ominus F(t_0 - h)$ and the limits (in the metric D)

$$\lim_{h\to 0^+} \frac{F(t_0 + h) \ominus F(t_0)}{h} = \lim_{h\to 0^+} \frac{F(t_0) \ominus F(t_0 - h)}{h} = F\,'(t_0),$$

or

(2) for all $h > 0$ sufficiently close to 0, there exists $F(t_0 + h) \ominus F(t_0)$, $F(t_0) \ominus F(t_0 - h)$ and the limits (in the metric D)

$$\lim_{h\to 0^-} \frac{F(t_0 + h) \ominus F(t_0)}{h} = \lim_{h\to 0^-} \frac{F(t_0) \ominus F(t_0 - h)}{h} = F\,'(t_0).$$

Remark 2.5 *([5]). This definition agrees with the one introduced in [5]. Indeed, if F is differentiable in the senses (1) and (2) simultaneously, then for $h > 0$ sufficiently small, we have $F(t_0+h) = F(t_0)+u_1$, $F(t_0) = F(t_0-h)+u_2$, $F(t_0) = F(t_0+h)+v_1$ and $F(t_0) = F(t_0+h)+v_2$, with u_1, u_2, v_1, $v_2 \in \Re_F$. Thus, $F(t_0) = F(t_0) + (u_2 + v_1)$, i.e., $u_2 + v_1 = X_{\{0\}}$, which implies two possibilities: $u_2 = v_1 = X_{\{0\}}$ if $F'(t_0) = X_{\{0\}}$; or $u_2 = X_{\{a\}} = -v_1$, with $a \in \Re$, if $F'(t_0) \in \Re$. Therefore if there exists $F'(t_0)$ in the first form (second form) with $F'(t_0) \notin \Re$, then $F'(t_0)$ does not exist in the second form (first form, respectively).*

Remark 2.6 *In the previous definition, case(1) corresponds to the H-derivative, so this differentiability concept is a generalization of the H-derivative.*

Remark 2.7 *In [5], the authors consider four cases for derivatives. Here we only consider the two first cases of Definition 5 in [5]. In the other cases, the derivative is trivial because it is reduced to a crisp element (more precisely, $F' \in \Re$; for details see Theorem 7 in [5]).*

Definition 2.8 *Let $F : I \to \Re_F$. we say F is (1)-differentiable on I if F is differentiable in the sense (1)of Definition 2.4 and its derivative is denoted D_1F, and similarly for (2)-differentiability we have D_2F.*

The principal properties of defined derivatives are well known and can be found in [5,8]. In this paper, we make use of the following Theorem [8].

Theorem 2.9 *Let $F : I \to \Re_F$ and put $[F(t)]^\alpha = [f_\alpha(t),\ g_\alpha(t)]$ for each $\alpha \in [0, 1]$.*

(i) If F is (1)-differentiable then f_α and g_α are differentiable functions and $[D_1F(t)]^\alpha = [f'_\alpha(t), g'_\alpha(t)]$.

(ii) If F is (2)-differentiable then f_α and g_α are differentiable functions and we have $[D_2F(t)]^\alpha = [g'_\alpha(t), f'_\alpha(t)]$.

proof. see [8]

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3 Generalized characterization theorem for FDEs under generalized differentiability

Let us consider the fuzzy differential equations with initial value condition

$$x'(t) = f(t, x), \quad x(0) = x_0, \tag{1}$$

where $f : I \times \Re_F \to \Re_F$ is a continuous fuzzy mapping and x_0 is a fuzzy number. The interval I may be $[0, A]$ for some $A > 0$ or $I = [0, \infty)$.

Theorem 3.1 *Let $f : I \times \Re_F \to \Re_F$ be a continuous fuzzy function such that there exists $k > 0$ such that $D(f(t, x), f(t, z)) \leq kD(x, z)$, $\forall t \in I$, x, $z \in \Re_F$. Then problem (3) has two solutions (one (1)- differentiable and the other one (2)-differentiable on I.*

Proof. see [12].

Let $y : I \to \Re_F$ be a fuzzy function such that $D_1 y$ or $D_2 y$ exists. If y and $D_1 y$ satisfy problem (3), we say y is a (1)-solution of problem (3). Similarly, if y and $D_2 y$ satisfy problem (3), we say y is a (2)-solution of problem (3).

Then Theorem 2.9 shows us a way to translate the FIVP(3) into a system of ODEs. Let $[x(t)]^\alpha = [\underline{x}_\alpha(t), \overline{x}_\alpha(t)]$. If $x(t)$ is (3)-differentiable then $[D_1 x(t)]^\alpha = [\underline{x}'_\alpha(t), \overline{x}'_\alpha(t)]$,and (3)-translates into the following system of ODEs:

$$\begin{cases} \underline{x}'(t) = \underline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha) = F(t, \underline{x}, \overline{x}), & \underline{x}(0) = \underline{x}_0, \\ \\ \overline{x}'(t) = \overline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha) = G(t, \underline{x}, \overline{x}), & \overline{x}(0) = \overline{x}_0. \end{cases} \tag{2}$$

Also, if $x(t)$ is (2)-differentiable then $[D_2 x(t)]^\alpha = [\overline{x}'_\alpha(t), \underline{x}'_\alpha(t)]$, and (1.1) translates into the following system of ODEs:

$$\begin{cases} \underline{x}'(t) = \overline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha) = G(t, \underline{x}, \overline{x}), & \underline{x}(0) = \underline{x}_0, \\ \\ \overline{x}'(t) = \underline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha) = F(t, \underline{x}, \overline{x}), & \overline{x}(0) = \overline{x}_0, \end{cases} \tag{3}$$

where $[f(t, x)]^\alpha = [\underline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha), \overline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha)]$. Then, the authors of [8] state that if we ensure that the solution $[\underline{x}_\alpha(t), \overline{x}_\alpha(t)]$ of the system (4) are valid level sets of a fuzzy number valued function and if $[\underline{x}'_\alpha(t), \overline{x}'_\alpha(t)]$ are valid level sets of a fuzzy valued function, then by the stacking Theorem [14], it is possible to construct the (1)-solution of FIVP (3). Also, for the (2)-solution, we can proceed in a similar way.

The characterization theorem [5] states that a fuzzy differential equation is equivalent to a system of ordinary differential equations under certain conditions. The next result extends Bede's characterization theorem to fuzzy differential equations under generalized differentiability.

Theorem 3.2 *Let us consider the FIVP (3) where $f : I \times \Re_F \to \Re_F$ is such that*

(i) $[f(t,x)]^\alpha = [\underline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha), \overline{f}_\alpha(t, \underline{x}_\alpha, \overline{x}_\alpha)]$;

(ii) $\underline{f}_\alpha$ *and* $\overline{f}_\alpha$ *are equicontinuous;*

(iii) there exists $L > 0$ *such that*
$$\left|\underline{f}_\alpha(t, x_1, y_1) - \underline{f}_\alpha(t, x_2, y_2)\right| \le L \max\{|x_1 - x_2|, |y_1 - y_2|\}, \quad \forall\alpha \in [0,1],$$
$$\left|\overline{f}_\alpha(t, x_1, y_1) - \overline{f}_\alpha(t, x_2, y_2)\right| \le L \max\{|x_1 - x_2|, |y_1 - y_2|\}, \quad \forall\alpha \in [0,1].$$

Then , for (1)-differentiability, the FIVP (3) and the system of ODEs (4)are equivalent and in (2)-differentiability, the FIVP (3) and the system of ODEs (5) are equivalent.

Proof. In the paper [5] , the authors proved for (1)- differentiability. The result for (2)- differentiability is obtained analogously by using Theorem 2.9 .

4 Numerical solution of FDE by generalized characterization theorem

In this section we present numerical methods for solving (3) by the generalized characterization theorem. Here we assume the existence of two solutions for (3) based on Theorem 3.1 .

Lemma 4.1 *([5]). The fuzzy differential equation (3), where $f : I \times \Re_F \to \Re_F$ is supposed to be continuous, is equivalent to one of the integral equations:*

$$\begin{aligned} x(t) &= x_0 + \int_0^t f(s, x(s))ds, \quad \forall t \in I, \quad \textit{or} \\ x_0 &= x(t) + (-1) \odot \int_0^t f(s, x(s))ds, \quad \forall t \in I, \end{aligned}$$

depending on the strongly differentiability considered, (1)-differentiability or (2)-differentiability, respectively. Here the equivalence between two equations means that any solution of an equation is a solution too for the other one.

Remark 4.2 *([5]). Under appropriate conditions, the fuzzy initial value problem (3) considered under generalized differentiability has locally two solutions, and the successive iterations*

$$x(0) = x_0, \quad x_{n+1}(t) = x_0 + \int_0^t f(s, x_n(s))ds, \quad \textit{and}$$
$$x(0) = x_0, \quad x_{n+1}(t) = x_0 \ominus (-1) \odot \int_0^t f(s, x_n(s))ds,$$

converge to the (1)- solution and the (2)- solution, respectively.

In the interval $I = [0, A]$ we consider a set of discrete equally spaced grid points $0 = t_0 < t_1 < t_2 < ... < t_N = A$ at which two exact solutions $[Y_1(t)]^\alpha = [\underline{Y}_1(t,\alpha), \overline{Y}_1(t,\alpha)]$ and $[Y_2(t)]^\alpha = [\underline{Y}_2(t,\alpha), \overline{Y}_2(t,\alpha)]$ are approximated by some $[y_1(t)]^\alpha = [\underline{y}_1(t,\alpha), \overline{y}_1(t,\alpha)]$ and $[y_2(t)]^\alpha = [\underline{y}_2(t,\alpha), \overline{y}_2(t,\alpha)]$, respectively. The grid points at which the solutions are calculated are $t_n = t_0 + nh, h = A/N$. The exact and approximate solutions at t_n, $0 \leq n \leq N$ are denoted by $Y_{1_n}(\alpha)$, $Y_{2_n}(\alpha)$, $y_{1_n}(\alpha)$, and $y_{2_n}(\alpha)$ respectively.

The generalized Modified Euler method based on the first-order approximation of $\underline{Y}'_1(t,\alpha)$, $\overline{Y}'_1(t,\alpha)$, and $\underline{Y}'_2(t,\alpha)$, $\overline{Y}'_2(t,\alpha)$ are equations (2)and (3) is obtained as follows:

$$\begin{cases} \underline{y_1}_{n+1}(\alpha) = \underline{y_1}_n(\alpha) + hF\Big[t_n + \frac{h}{2},\ \underline{y_1}_n(\alpha) + \frac{h}{2}F[t_n, \underline{y_1}_n(\alpha), \overline{y_1}_n(\alpha)], \\ \qquad\qquad \overline{y_1}_n(\alpha) + \frac{h}{2}F[t_n, \underline{y_1}_n(\alpha), \overline{y_1}_n(\alpha)]\Big], \\ \overline{y_1}_{n+1}(\alpha) = \overline{y_1}_n(\alpha) + hG\Big[t_n + \frac{h}{2},\ \underline{y_1}_n(\alpha) + \frac{h}{2}G[t_n, \underline{y_1}_n(\alpha), \overline{y_1}_n(\alpha)], \\ \qquad\qquad \overline{y_1}_n(\alpha) + \frac{h}{2}G[t_n, \underline{y_1}_n(\alpha), \overline{y_1}_n(\alpha)]\Big], \\ \underline{y_1}_0(\alpha) = \underline{y_0}(\alpha), \\ \overline{y_1}_0(\alpha) = \overline{y_0}(\alpha), \end{cases} \tag{4}$$

$$\begin{cases} \underline{y_2}_{n+1}(\alpha) = \underline{y_2}_n(\alpha) + hG\Big[t_n + \frac{h}{2},\ \underline{y_2}_n(\alpha) + \frac{h}{2}G[t_n, \underline{y_2}_n(\alpha), \overline{y_2}_n(\alpha)], \\ \qquad\qquad \overline{y_2}_n(\alpha) + \frac{h}{2}G[t_n, \underline{y_2}_n(\alpha), \overline{y_2}_n(\alpha)]\Big], \\ \overline{y_2}_{n+1}(\alpha) = \overline{y_2}_n(\alpha) + hF\Big[t_n + \frac{h}{2},\ \underline{y_2}_n(\alpha) + \frac{h}{2}F[t_n, \underline{y_2}_n(\alpha), \overline{y_2}_n(\alpha)], \\ \qquad\qquad \overline{y_2}_n(\alpha) + \frac{h}{2}F[t_n, \underline{y_2}_n(\alpha), \overline{y_2}_n(\alpha)]\Big], \\ \underline{y_2}_0(\alpha) = \underline{y_0}(\alpha), \\ \overline{y_2}_0(\alpha) = \overline{y_0}(\alpha), \end{cases} \tag{5}$$

where y_0 is an initial value. Our next result determines the pointwise convergences of the generalized Modified Euler approximates to the exact solutions. Let $F(t, u, v)$ and $G(t, u, v)$ be the functions F and G of equations (2) and (3), where u and v are constants and u$\leq$ v. The domain where F and G are defined is therefore

$K = \{(t, u, v)\ |0 \leq t \leq A,\ -\infty < v < \infty,\ -\infty < u \leq v\}$.

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Theorem 4.3 *Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C^1(K)$ and let the partial derivatives of F, G be bounded over K. Then, for arbitrary fixed $\alpha : 0 \leq \alpha \leq 1$, the generalized Modified Euler approximates of equations (4) and (5) converge to the exact solutions $Y_1(t;\alpha)$, $Y_2(t;\alpha)$ uniformly in t.*

Proof. If we consider (1)-differentiability, then convergence of equation (4) is obtained from Theorem 5.3 in [11]. In the same way, if we consider(2)-differentiability then analogously to the demonstration of Theorem 5.3 in [11], we can prove the convergence of equation (5).

Remark 4.4 *By Theorem 3.1 we observe that the solution of the fuzzy differential equations is not unique. This may seem a deficiency of the method. However, this disadvantage can be converted into an advantage since we may sometimes choose between two solutions, so for example we can study the real system and choose the solution which better reflects the behavior of the system and then consider that solution in all similar cases. This advantage is shown by the following simple modeling Example [6].*

5 Example

Let us consider the equation

$$x'(t) = -\lambda \odot x(t), \qquad x(0) = x_0. \tag{6}$$

Let $\lambda = 1$, $I = [0,1]$ and $x_0 = [\alpha - 1, 1 - \alpha]$.
By using the formulation (2) we get the exact solution

$$x(t,\alpha) = [(\alpha - 1)e^t, (1-\alpha)e^t],$$

that is a (1)-differentiable solution of the problem (1).
Using the formulation (3),

$$x(t,\alpha) = [(\alpha - 1)e^{-t}, (1-\alpha)e^{-t}],$$

is a (2)-differentiable solution of the problem (1).
To get the generalized Modified Euler approximation we divide I into N =10 equally spaced subintervals and calculate

$$\begin{cases} \underline{y_1}^{\alpha}_{n+1} = \underline{y_1}^{\alpha}_{n}\left(1 + \dfrac{h^2}{2}\right) - h\ \overline{y_1}^{\alpha}_{n}, \\ \overline{y_1}^{\alpha}_{n+1} = \overline{y_1}^{\alpha}_{n}\left(1 + \dfrac{h^2}{2}\right) - h\ \underline{y_1}^{\alpha}_{n}, \\ \underline{y_1}^{\alpha}_{0} = \underline{x}_0, \\ \overline{y_1}^{\alpha}_{0} = \overline{x}_0, \end{cases} \tag{7}$$

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for finding the (1)-solution and compute

$$\begin{cases} \underline{y2}^{\alpha}_{n+1} = \underline{y2}^{\alpha}_{n}\Big(1-h\Big) + \dfrac{h^2}{2}\ \overline{y2}^{\alpha}_{n}, \\ \\ \overline{y2}^{\alpha}_{n+1} = \overline{y2}^{\alpha}_{n}\Big(1-h\Big) + \dfrac{h^2}{2}\ \underline{y2}^{\alpha}_{n}, \\ \quad \underline{y2}^{\alpha}_{0} = \underline{x}_0, \\ \quad \overline{y2}^{\alpha}_{0} = \overline{x}_0, \end{cases} \tag{8}$$

for finding the (2)-solution.

A comparison between the exact the and the approximate solutions at t = 1 is shown in the following figures (1) and (2).

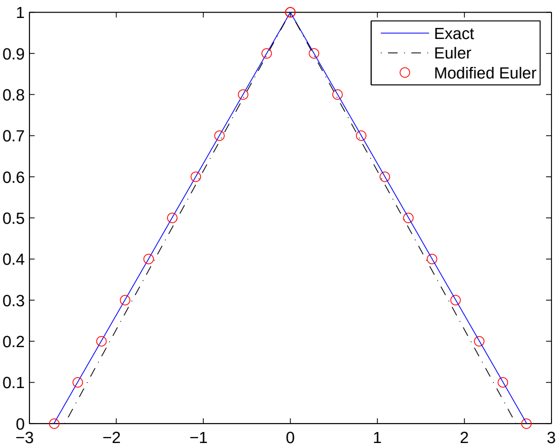

Figure 1: (1)-solution for h=0.1

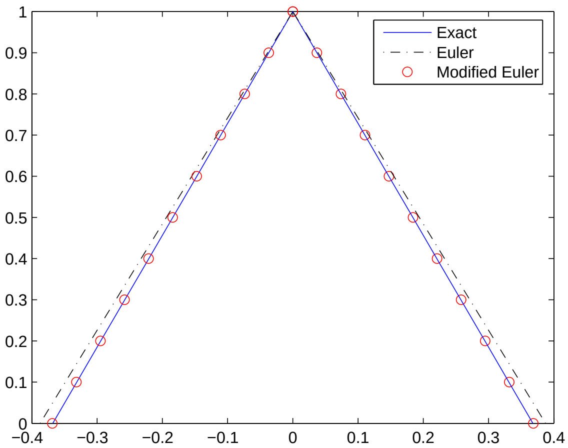

Figure 2: (2)-solution for h=0.1

6 Conclusion

In this paper we presented the solution of fuzzy differentiable equations under generalized differentiability by using generalized characterization theorem, we translate the fuzzy differential equations into two systems of ordinary differential equations and then solve numerically by Modified Euler method. From figures (1) and (2) we see that our proposed modified Euler method gives better solution than Euler method which was studied by Nieto et all.[16].

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