Abstract:

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at www.valdostamuseum.com/hamsmith/www.tony5m17h.net/ and on viXra - list at vixra.org/author/frank_dodd_tony_smith_jr
Due to experimental observations and my learning new techniques over those 30 years my Physics Models have been in a state of evolving flux - for example, 30 years ago their basis was the Lie Algebra Spin(8), then to contain vectors and spinors it was F4, then to contain the geometry of bounded complex domains it was E6, then Real Clifford Algebras were used to describe evolution from a Void Empty Set ø, then Periodicity showed the importance of Cl(8) and tensor product Cl(8)xCl(8) = Cl(16), then E8 emerged from Cl(16) to give the structure of a realistic local E8 Lagrangian, then completion of the union of all tensor products of Cl(16) local structures produced a realistic Algebraic Quantum Field Theory (AQFT). Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate. Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient. The Cl(16)-E8 model must be consistent with experimental observations. As of now, given that most calculations are tree-level, the model is substantially so consistent. An interesting test over the 2015-2016 time frame will be whether or not the LHC sees two additional Higgs mass states with cross section about 20% of that of a full Standard Model Higgs.
Preface

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at http://www.valdostamuseum.com/hamsmith/
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Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate.

A lot of math is used in my Cl(16)-E8 model, some of which may be unfamiliar to many. My efforts to find a single volume for the math of Cl(16) - E8 Lagrangian - AQFT led me to my Princeton University Advanced Calculus text by H. K. Nickerson, D. C. Spencer, and N. E. Steenrod. However, it is over 50 years old, so I have added some Supplementary Material to produce a 21 MB pdf file on the web at http://www.valdostamuseum.com/hamsmith/NSS6313.pdf

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II. LINEAR TRANSFORMATIONS OF VECTOR SPACES
   Lie Groups and Symmetric Spaces
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IV. VECTOR PRODUCTS IN R3
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VI. VECTOR-VALUED FUNCTIONS OF A SCALAR
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IX. TENSOR PRODUCTS AND THE STANDARD ALGEBRAS
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X. TOPOLOGY AND ANALYSIS
XI. DIFFERENTIAL CALCULUS OF FORMS
XII. INTEGRAL CALCULUS OF FORMS
XIII. COMPLEX STRUCTURE
   Potential Theory, Green's Functions, Bergman Kernels, Schwinger Sources

Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient. My Cl(16)-E8 model must be, and is, consistent with experimental observations.
Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

**Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04**

Inflationary Gravitational Wave (IGW) tensor-to-scalar ratio $r = 7/28 = 0.25$

Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about $10^{\sim(-24)}$ cm.

<table>
<thead>
<tr>
<th>Particle/Force</th>
<th>Tree-Level</th>
<th>Higher-Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>e-neutrino</td>
<td>0</td>
<td>0 for $\nu_1$</td>
</tr>
<tr>
<td>mu-neutrino</td>
<td>0</td>
<td>$9 \times 10^{(-3)}$ eV for $\nu_2$</td>
</tr>
<tr>
<td>tau-neutrino</td>
<td>0</td>
<td>$5.4 \times 10^{(-2)}$ eV for $\nu_3$</td>
</tr>
<tr>
<td>electron</td>
<td>0.5110 MeV</td>
<td></td>
</tr>
<tr>
<td>down quark</td>
<td>312.8 MeV</td>
<td>charged pion = 139 MeV</td>
</tr>
<tr>
<td>up quark</td>
<td>312.8 MeV</td>
<td>proton = 938.25 MeV</td>
</tr>
<tr>
<td>muon</td>
<td>104.8 MeV</td>
<td>neutron - proton = 1.1 MeV</td>
</tr>
<tr>
<td>strange quark</td>
<td>625 MeV</td>
<td></td>
</tr>
<tr>
<td>charm quark</td>
<td>2090 MeV</td>
<td></td>
</tr>
<tr>
<td>tauon</td>
<td>1.88 GeV</td>
<td></td>
</tr>
<tr>
<td>beauty quark</td>
<td>5.63 GeV</td>
<td></td>
</tr>
<tr>
<td>truth quark (low state)</td>
<td>130 GeV</td>
<td>(middle state) 174 GeV</td>
</tr>
<tr>
<td>W+</td>
<td>80.326 GeV</td>
<td></td>
</tr>
<tr>
<td>W-</td>
<td>80.326 GeV</td>
<td></td>
</tr>
<tr>
<td>W0</td>
<td>98.379 GeV</td>
<td>$Z_0 = 91.862$ GeV</td>
</tr>
<tr>
<td>Mplanck</td>
<td>1.217x10^{19} GeV</td>
<td></td>
</tr>
<tr>
<td>Higgs VEV (assumed)</td>
<td>252.5 GeV</td>
<td></td>
</tr>
<tr>
<td>Higgs (low state)</td>
<td>126 GeV</td>
<td>(middle state) 182 GeV</td>
</tr>
<tr>
<td>(high state)</td>
<td>239 GeV</td>
<td></td>
</tr>
<tr>
<td>Gravity Gg (assumed)</td>
<td>1</td>
<td>($Gg)(M_{proton}^2 / M_{planck}^2)</td>
</tr>
<tr>
<td>EM fine structure</td>
<td>1/137.03608</td>
<td></td>
</tr>
<tr>
<td>Weak Gw</td>
<td>0.2535</td>
<td></td>
</tr>
<tr>
<td>Gw(M_{proton}^2 / (M_{W^+}^2 + M_{W^-}^2 + M_{Z^0}^2))</td>
<td>1.05 \times 10^{(-5)}</td>
<td></td>
</tr>
<tr>
<td>Color Force at 0.245 GeV</td>
<td>0.6286</td>
<td>0.106 at 91 GeV</td>
</tr>
</tbody>
</table>

Kobayashi-Maskawa parameters for W+ and W- processes are:

- $d$  
  - $u = 0.975$  
  - $c = -0.222 - 0.000161i$  
  - $t = 0.00698 - 0.003781i$  
- $s$  
  - $s = 0.222$  
  - $s = 0.974 - 0.0000365i$  
  - $t = -0.0418 - 0.000861i$  
- $b$  
  - $b = 0.00249 - 0.003881i$  
  - $b = 0.0423$  
  - $b = 0.999$

The phase angle $d_{13}$ is taken to be 1 radian.

The 3-state system of Higgs and Tquark masses is a property of the Cl(16)-E8 model that can be tested at the LHC 2015-2016 run by searching for Higgs middle and high mass states with cross section about 20% of that of a full SM Higgs.
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1. The First Grothendieck Universe is the Empty Set $\emptyset$ which grows by Clifford Iteration to $\text{Cl}(16)$ which contains $E_8$

$\emptyset = \text{Cl}(0) = 1$

$\emptyset \ (\emptyset) = \text{Cl}(1) = 2$

$\emptyset \ (\emptyset \ (\emptyset)) = \text{Cl}(2) = 4$

$\emptyset \ (\emptyset \ (\emptyset \ (\emptyset))) = \text{Cl}(4) = 16$

$\emptyset \ (\emptyset \ (\emptyset \ (\emptyset \ (\emptyset)))) = \text{Cl}(16) = 2^{16} = 65,536 = (64+64) \times (64+64)$

$\text{Cl}(16) \text{ BiVectors} = D_8 = 120 = 28 + 28 + 64$

$\text{Cl}(16) \text{ Spinors} = (64+64) + (64+64)$

$28 + 28 + 64 + 64 + 64 = E_8$
From \( \text{Cl}(1,3) = 16 \) to \( \text{Cl}((1,3)) = 65,536 \) with \( 16 \wedge 16 = 120 \)

(Color Scheme on this page for \( \text{Cl}(1,3) \) is not the same used for \( \text{Cl}(16) \) and \( \text{E}_8 \))

\[
\begin{array}{cccc}
1 & 4 & 6 & 4 \\
\wedge & & & \\
1 & 4 & 6 & 4
\end{array}
\]

\[
\begin{array}{cccc}
1 & 4 & = & 4 \\
4 & 6 & = & 24
\end{array}
\]

\[
\begin{array}{cccc}
1 & 4 & = & 4 \\
6 & 4 & = & 24
\end{array}
\]

\[
\begin{array}{cccc}
1 & 6 & = & 6 \\
1 & 1 & = & 1 \\
6 & 6 & = & 15 \\
6 & 1 & = & 6
\end{array}
\]

\[
\begin{array}{cccc}
4 & 4 & = & 6 \\
4 & 4 & = & 16 \\
4 & 4 & = & 6
\end{array}
\]

\[
\begin{array}{cccc}
4 & 1 & = & 4 \\
4 & 1 & = & 4
\end{array}
\]

28 \quad \text{D4 for Gravity} \\
+ \quad 28 \quad \text{D4 for Standard Model} \\
+ \quad 28 \quad \text{AntiSymmetric D4 rotations in 8-dim SpaceTime} \\
+ \quad 28 \quad 8x8 \quad \text{Symmetric Off-Diagonal} \\
+ \quad 8 \quad 8x8 \quad \text{Symmetric Diagonal for 4 + 4 Klauza-Klein M4 x CP2} \\
= 120
E8 structure gives a Fundamental Local Lagrangian

E8 Root Vectors = 112 + 64 + 64 = 24 + 24 + 64 + 64 + 64

Fundamental Local Lagrangian =

\[ \int \text{Gauge Gravity Standard Model} + \text{Fermion Particle-AntiParticle} \]

8-dim SpaceTime
where E8 structure of the Lagrangian Terms is given by:

\[
E8 / D8 = 64 + 64
\]

- 64 = 8 Components of 8 Fermion Particles (first generation)
- 64 = 8 Components of 8 Fermion AntiParticles (first generation)

\[
D8 / D4xD4 = 64
\]

- 64 = 8-dim SpaceTime Position and Momentum

( Triality Automorphisms: 64 = 64 = 64 )

\[
D4xD4 = 24 + 4 + 24 + 4
\]

- 24 + 4 = 28 = D4 for Gravity Gauge Bosons
- 24 + 4 = 28 = D4 for Standard Model Gauge Bosons

**Gauge Gravity Standard Model** term has total weight 28 x 1 = 28
- 16 generators for U(2,2) of Conformal Gravity
- 12 generators for SU(3) and U(2) Standard Model

\[
= 28 \text{ D4 Gauge Bosons each with 8-dim Lagrangian weight = 1}
\]

**Fermion Particle-AntiParticle** term also has total weight 8 x (7/2) = 28
- 8 Fermion Particle/Antiparticle types each with 8-dim Lagrangian weight = 7/2

Since Boson Weight 28 = Fermion Weight 28
the Cl(16)-E8 model has a Subtle SuperSymmetry and is UltraViolet Finite.

**The Cl(16)-E8 model** has 8-dim Lorentz structure satisfying Coleman-Mandula because its fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime.

( See pages 382-384 of Steven Weinberg's book "The Quantum Theory of Fields" Vol. III )

**The Cl(16)-E8 model is Chiral** because
- E8 contains Cl(16) half-spinors \((64+64)\) for a Fermion Generation
- but does not contain Cl(16) Fermion AntiGeneration half-spinors \((64+64)\).
- Fermion +half-spinor Particles with high enough velocity are seen as left-handed.
- Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

**The Cl(16)-E8 model obeys Spin-Statistics** because
- the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number \(2+1 = 3\) and Atiyah-Singer index \(-1/8\) which is not the net number of generations because
- CP2 has no spin structure but you can use a generalized spin structure

( Hawking and Pope (Phys. Lett. 73B (1978) 42-44) )
to get (for integral m) the generalized CP2 index \( n_R - n_L = (1/2) m (m+1) \)

Prior to Dimensional Reduction: \( m = 1 \), \( n_R - n_L = (1/2) \times 1 \times 2 = 1 \) for 1 generation
After Reduction to 4+4 Kaluza-Klein: \( m = 2 \), \( n_R - n_L = (1/2) \times 2 \times 3 = 1 \) for 3 generations
(second and third generations emerge as effective composites of the first)

Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...
what happens in CP2 … is a two-surface K which cannot be shrunk to zero.
Parallel propagation of tetrads around K produces a curve in SO(4)
which cannot be shrunk to zero … i.e. it correspond[s] to a rotation through 2 \pi ...
Thus one could not define spinors consistently over such a space … In ...
CP2 there is a covariant constant two-form which can be taken as the electromagnetic field ...
The index theorem then gives \( nR - nL = (1/2) m (m+1) \). This is always an integer
For an electromagnetic generalized spin structure [ \( U(1) \) on CP2 ]
the fermions would have to carry half the electric charge of any bosons.
This obviously does not correspond with the real universe.
However, one could replace the electromagnetic field by
a Yang-Mills field whose group \( G \) had a double covering \( G~ \).
The fermion field would have to occur in representations which changed sign
under the non-trivial element of the kernel of the projection … \( G\sim \to G \)
while
the bosons would have to occur in representations which did not change sign …".

For Cl(16)-E8 model gauge bosons are in the 28+28=56-dim D4 + D4 subalgebra of E8.
One D4 acts on the M4 part of M4 x CP2 through its \( SU(2,2) = Spin(2,4) \) Conformal Subalgebra to give MacDowell-Mansouri Gravity
The other \( D4 = SO(8) \) acts on the CP2 part of M4 x CP2 through its \( SU(4) \) subalgebra
that contains color \( SU(3) \). Electroweak \( SU(2) \times U(1) \) comes from \( CP2 = SU(3) / U(2) \).
This D4 coupling to the 8-dim fundamental fermion particles comes from
the way that 28-dim Spin(8) couples to 8-dim D4-half-spinors based on Triality.
This \( D4 = SO(8) \) is the Hawking-Pope \( G \) which has double covering \( G\sim = Spin(8) \).
The 8 fermion particles / antiparticles are D4 half-spinors represented within E8
by anti-commutators and so do change sign
while
the 28 gauge bosons are D4 adjoint represented within E8 by commutators
and so do not change sign.
The Octonionic structure of the 8-dim D4 half-spinors gives all the correct properties
(quantum numbers = electric charge, color charge, helicity).
This establishes what Hawking and Pope described as
"… the interesting possibility that there may be a connexion between
the topology of space-time and the spectrum of elementary particles …".

Further,
**E8 inherits from F4 the property whereby**
its Spinor Part need not be written as Commutators
but can also be written in terms of Fermionic AntiCommutators.

( vixra 1208.0145 )
2. The Second Grothendieck Universe is Hereditarily Finite Sets such as Discrete Clifford Algebras and Discrete Lattices.

Cl(16) x ... x Cl(16) where each Cl(16) contains E8 produces Emergent SpaceTime with consistently aligned E8 Lattice structure for all E8 Local 8-dim Octonionic SpaceTimes
H. S. M. Coxeter in his paper Regular and Semi-Regular Polytopes III (Math. Z. 200, 3-45, 1988) about the 240 units of an E8 Integral Domain said: "... the 16 + 16 + 16 octaves ±1, ±i, ±j, ±k, ±E, ±i, ±j, ±K, (±1 ± i ± j ± k)/2, (±E ± i ± j ± k)/2, and the 192 others derived from the last two expressions by the cyclic permutation (E, I, J, i, k, K, J), which preserves the integral domain ... the permutation (e I J i k K j), which is an automorphism of the whole ring of octaves (and of the finite [Fano] plane ...) transforms this particular integral domain into another one of R. H. Bruck's cyclic of seven such domains. ...". An 8th E8 Lattice (not a closed Integral Domain, Kirmse's mistake) can be taken to correspond the 1 Real Element of the Octonion Basis {1, i, j, k, E, I, J, K}.

There are 7 independent E8 Integral Domain Lattices corresponding to the 7 Octonion Imaginary Basis Elements {i, j, k, E, I, J, K}:

<table>
<thead>
<tr>
<th>Associative Triangle</th>
<th>Coassociative Square</th>
<th>Heptaverton</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>J---j</td>
<td>k J</td>
</tr>
<tr>
<td>i -- &gt; / \</td>
<td>I---i -- &gt;</td>
<td></td>
</tr>
<tr>
<td>E----i</td>
<td>K----k</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>k I</td>
</tr>
<tr>
<td>j -- &gt; / \</td>
<td>J---j -- &gt;</td>
<td></td>
</tr>
<tr>
<td>E----j</td>
<td>I----i</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>k i</td>
</tr>
<tr>
<td>k -- &gt; / \</td>
<td>K---k -- &gt;</td>
<td></td>
</tr>
<tr>
<td>E----k</td>
<td>J----j</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>I j</td>
</tr>
<tr>
<td>E -- &gt; / \</td>
<td>J---J -- &gt;</td>
<td></td>
</tr>
<tr>
<td>i---k</td>
<td>K---E</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>I j</td>
</tr>
<tr>
<td>I -- &gt; / \</td>
<td>J---j -- &gt;</td>
<td></td>
</tr>
<tr>
<td>i---k</td>
<td>K---E</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>I j</td>
</tr>
<tr>
<td>J -- &gt; / \</td>
<td>J---i -- &gt;</td>
<td></td>
</tr>
<tr>
<td>I---K</td>
<td>K---E</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>I j</td>
</tr>
<tr>
<td>K -- &gt; / \</td>
<td>J---i -- &gt;</td>
<td></td>
</tr>
<tr>
<td>I---K</td>
<td>j---E</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>I j</td>
</tr>
</tbody>
</table>
E8 Lattices

E8 Lattices are based on Octonions, which have 480 different multiplication products. E8 Lattices can be combined to form 24-dimensional Leech Lattices and 26-dimensional Bosonic String Theory, which describes E8 Physics when the strings are physically interpreted as World-Lines. A basic String Theory Cell has as its automorphism group the Monster Group whose order is $2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17.19.23.29.31.41.47.59.71 = \text{about } 8 \times 10^{53}$.

For more about the Leech Lattice and the Monster and E8 Physics, see viXra 1210.0072 and 1108.0027.

E8 Root systems and lattices are discussed by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":

"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomalous basis $\{ 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 \}$ labeled by the projective line $\text{PL}(7) = \{ oo \} u F7$ ...

The E8 root system embeds in this algebra ... take the 240 roots to be ...

112 octonions $ +/- it +/- iu$ for any distinct $t,u$ ...

128 octonions $ (1/2)( +/- 1 +/- i0 +/- ... +/- i6 )$ ...

Denote by $L$ the lattice spanned by these 240 octonions ...

Let $s = (1/2)( - 1 + i0 + ... + i6 )$ so $s$ is in $L$ ... write $R$ for $Lbar$ ...

$(1/2) \cdot ( 1 + i0 ) \cdot L = (1/2) \cdot R \cdot ( 1 + i0 )$ is closed under multiplication ... Denote this ... by $A$ ...

Writing $B = (1/2) \cdot ( 1 + i0 ) \cdot A \cdot ( 1 + i0 )$ ... from ... Moufang laws ... we have $L \cdot R = 2 \cdot B$ , and ... $B \cdot L = L$ and $R \cdot B = R$ ...

[also] ... $2 \cdot B = Lbar$

... the roots of $B$ are [
16 octonions ]... $ +/- it$ for $t$ in $\text{PL}(7)$ ...
... together with ...
[112 octonions ]... $(1/2)( +/- 1 +/- it +/- i(t+1) +/- i(t+3) )$ ...
... and ...
[112 octonions ]... $(1/2)( +/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6) )$ ...
... for $t$ in $F7$

... $B$ is not closed under multiplication ... Kirmse's mistake ...
...[ but ]... as Coxeter ... pointed out ...
... there are seven non-associative rings $At = (1/2) \cdot ( 1 + it ) \cdot B \cdot ( 1 + it )$ , obtained from $B$ by swapping 1 with it ... for $t$ in $F7$

... $LR = 2B$ and $BL = L$ ...

[which]... appear[s] not to have been noticed before ... some work ... by Geoffrey Dixon ...
"
Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\} for the Octonion basis elements that Robert A. Wilson denotes by \{1=ioo, i0, i1, i2, i3, i4, i5 , i6\} and I sometimes denote by \{1,i,j,k,E,I,J,K\}: "...

\[
\Xi_0 = \{\pm e_a\}, \\
\Xi_2 = \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a, b, c, d \text{ distinct}, e_a(e_b(e_ce_d)) = \pm 1\}, \\
\Xi_{\text{even}} = \Xi_0 \cup \Xi_2, \\
\Xi_{\text{even}}^8 = \text{span}\{\Xi_{\text{even}}\}, \\
\Xi_1 = \{(\pm e_a \pm e_b)/\sqrt{2} : a, b \text{ distinct}\}, \\
\Xi_3 = \{\sum_{a=0}^7 \pm e_a)/\sqrt{8} : \text{even number of } 's\}, \\
\Xi_{\text{odd}} = \Xi_1 \cup \Xi_3, \\
\Xi_{\text{odd}}^8 = \text{span}\{\Xi_{\text{odd}}\}
\]

(spans over integers)

\Xi_{\text{even}} \text{ has } 16+224 = 240 \text{ elements ... } \Xi_{\text{odd}} \text{ has } 112+128 = 240 \text{ elements ... }

\Xi_{\text{even}}^8 \text{ does not close with respect to our given octonion multiplication} 
...[but]...
the set \Xi_{\text{even}}[0-a], \text{ derived from } \Xi_{\text{even}} \text{ by replacing each occurrence of } e_0 \text{ ... with } e_a, 
and vice versa, \text{ is multiplicatively closed. }...".

Geoffrey Dixon's \Xi_{\text{even}} \text{ corresponds to Wilson's } B \text{ which I denote as } 1E8.

Geoffrey Dixon's \Xi_{\text{even}}[0-a] \text{ correspond to Wilson's seven } \text{At} 
which I denote as \{iE8, jE8, kE8, EE8, IE8, JE8, KE8\}.

Geoffrey Dixon's \Xi_{\text{odd}} \text{ corresponds to Wilson's } L.

My view is that \textbf{the E8 domains } 1E8 = \Xi_{\text{even}} = B \text{ is fundamental} 
because 
E8 domains \{iE8, jE8, kE8, EE8, IE8, JE8, KE\} = \Xi_{\text{even}}[0-a] \text{ are derived from } 1E8 
and L and L s are also derived from } 1E8 = \Xi_{\text{even}} = B.
Using the notation \(\{1, i, j, k, E, I, J, K\}\) for Octonion basis
notice that in the Cl(16)-E8 model introduction of Quaternionic substructure
to produce (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime
requires breaking Octonionic light-cone elements
\((\pm 1 \pm i \pm j \pm k \pm E \pm I \pm J \pm K)/2\)
into Quaternionic 4-term forms like \((\pm A \pm B \pm C \pm D)/2\).

To do that, consider that there are \((8!4) = 70\) ways to choose 4-term subsets
of the 8 Octonion basis element terms. Using all of them produces
224 4-term subsets in each of the 7 Octonion Imaginary E8 lattices
\(iE8, jE8, kE8, EE8, IE8, JE8, KE8\) each of which also has 16 1-term first-shell vertices.

56 of the 70 4-term subsets appear as 8 in each of the 7 Octonion Imaginary E8 lattices.

The other 70-56 = 14 4-term subsets occur in sets of 3 among \(7 \times 6 = 42\) 4-term subsets
as indicated in the following detailed list of the 7 Octonion Imaginary E8 lattices:

**EE8:**

112 of D8 Root Vectors
16 appear in all 7 of \(iE8, jE8, kE8, EE8, IE8, JE8, KE8\)
\(\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K\)
96 appear in 3 of \(iE8, jE8, kE8, EE8, IE8, JE8, KE8\)
\((\pm 1 \pm k \pm E \pm j)/2 (\pm i \pm t \pm I \pm J)/2 \ kE8 , \ EE8 , \ KE8\)
\((\pm 1 \pm J \pm i \pm E)/2 (\pm I \pm k \pm t \pm i)/2 \ jE8 , \ EE8 , \ JE8\)
\((\pm 1 \pm E \pm I \pm i)/2 (\pm k \pm t \pm J \pm j)/2 \ iE8 , \ EE8 , \ IE8\)

128 of D8 half-spinors appear only in EE8
\((\pm 1 \pm I \pm J \pm K)/2 (\pm E \pm i \pm j \pm k)/2\)
\((\pm 1 \pm k \pm i \pm J)/2 (\pm t \pm I \pm K \pm E)/2\)
\((\pm 1 \pm i \pm K \pm j)/2 (\pm k \pm J \pm E \pm I)/2\)
\((\pm 1 \pm J \pm k \pm I)/2 (\pm I \pm E \pm i \pm K)/2\)
\textbf{iE8:}

112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±i ±E)/2 (±j ±k ±I ±K)/2 iE8 , EE8 , IE8
(±1 ±i ±I)/2 (±j ±k ±E ±J)/2 iE8 , JE8 , KE8
(±1 ±i ±J)/2 (±J ±K ±E ±I)/2 iE8 , jE8 , kE8

128 of D8 half-spinors appear only in iE8
(±1 ±i ±E ±I ±J ±K)/2
(±1 ±J ±E ±k ±I ±I)/2
(±1 ±K ±j ±J ±i ±I)/2

\textbf{jE8:}

112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±i ±E ±I ±J ±K)/2 iE8 , jE8 , kE8
(±1 ±i ±I ±K ±j ±J)/2 jE8 , IE8 , KE8
(±1 ±i ±I ±K ±j ±J)/2 jE8 , EE8 , JE8

128 of D8 half-spinors appear only in jE8
(±1 ±i ±E ±I ±J ±K)/2
(±1 ±i ±I ±J ±k ±K)/2
(±1 ±i ±J ±k ±I ±J)/2

\textbf{kE8:}

112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±J ±k ±I ±J ±K)/2 kE8 , IE8 , JE8
(±1 ±j ±i ±I ±J ±K)/2 iE8 , jE8 , kE8
(±1 ±k ±K ±E ±I ±J)/2 kE8 , EE8 , KE8

128 of D8 half-spinors appear only in kE8
(±1 ±i ±I ±J ±I)/2
(±1 ±i ±E ±I ±J ±K)/2
(±1 ±i ±I ±J ±k ±J)/2
IE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±j ±I ±k)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8
(±1 ±i ±E ±I)/2 (±j ±k ±J ±K)/2 iE8 , EE8 , IE8
(±1 ±I ±J ±k)/2 (±i ±j ±E ±K)/2 kE8 , IE8 , JE8

128 of D8 half-spinors appear only in IE8
(±1 ±I ±j ±k)/2 (±i ±k ±E ±J)/2
(±1 ±k ±j ±I)/2 (±i ±E ±I ±J)/2
(±1 ±i ±j ±k)/2 (±i ±j ±E ±K)/2
(±1 ±I ±I ±J)/2 (±i ±j ±k ±K)/2

JE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±i ±j ±K)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , JE8
(±1 ±E ±I ±J)/2 (±i ±j ±E ±I)/2 kE8 , IE8 , JE8
(±1 ±J ±i ±K)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8

128 of D8 half-spinors appear only in JE8
(±1 ±I ±j ±k)/2 (±i ±k ±E ±J)/2
(±1 ±k ±j ±I)/2 (±i ±E ±I ±J)/2
(±1 ±i ±j ±k)/2 (±i ±j ±E ±K)/2
(±1 ±I ±I ±J)/2 (±i ±k ±E ±K)/2

KE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±i ±K ±J)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8
(±1 ±E ±k ±K)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8
(±1 ±K ±j ±I)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8

128 of D8 half-spinors appear only in KE8
(±1 ±J ±i ±k)/2 (±i ±k ±E ±J)/2
(±1 ±I ±I ±E)/2 (±i ±j ±k ±K)/2
(±1 ±I ±i ±k)/2 (±i ±j ±k ±K)/2
(±1 ±I ±j ±k)/2 (±i ±E ±I ±K)/2
"... the 240 integral Cayley numbers of norm1 ... are the vertices of 4_21

The polytope 4_21 ... has cells of two kinds ...
a seven-dimensional "cross polytope" (or octahedron-analogue) B_7
... there are ... 2160 B_7's ...
and ...
a seven-dimensional regular simplex A_7
... there are 17280 A_7's
...
the 2160 integral Cayley numbers of norm 2 are
the centers of the 2160 B_7's of a 4_21 of edge 2
...
the 17280 integral Cayley numbers of norm 4 (other than the doubles of those of norm 1) are the centers of the 17280 A_7's of a 4_21 of edge 8/3 ...

[ Using notation of \{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8\} for Octonion basis elements we have ]

**norm 1**

112 like ( +/- a_1 +/- a_2 )
[which correspond to 112 = 16 + 96 = 16 + 6x16 in each of the 7 E8 lattices]

128 like (1/2) ( - a_1 + a_2 + a_3 + ... + a_8 ) with an odd number of minus signs
[which correspond to 128 = 8x16 in each of the 7 E8 lattices]
norm 2

16 like +/- 2 a1
[which correspond to 16 fo the 112 in each of the 7 E8 lattices]

1120 like +/- a1 +/- a2 +/- a3 +/- a4
[which correspond to 70x16 = (56+14)x16 that appear in the 7 E8 lattices

with each of the 14 appearing in three of the 7 E8 lattices so that
the 14 account for (14/7)x3x16 = 6x16 = 96 in each of the 7 E8 lattices
and for 14x16 = **224 of the 1120**

and
with each of the 56 appearing in only one of the 7 E8 lattices so that
the 56 account for (56/7)x16 = 128 in each of the 7 E8 lattices
and for 56x16 = **896 = 7x128 of the 1120** ]

1024 like (1/2)( 3a1 + 3a2 + a3 + a4 + ... + a8 ) with an even number of minus signs
[which correspond to **8x128 = 8 copies of the 128-dim Mirror D8 half-spinors that** are not used in the 7 E8 lattices. ...] ...".

One of the 128-dimensional Mirror D8 half-spinors from the 1024 combines with
the 128 from the 1120 corresponding to the one of the 7 E8 lattices that corresponds
to the central norm 1 240 = 112+128
and
the result is formation of a 128+128 = 256 corresponding to the Clifford Algebra Cl(8)
so that
the norm 2 second layer contains 7 copies of 256-dimensional Cl(8)

so the 2160 norm 2 vertices can be seen as

\[ 7(128+128) + 128 + 16 + 224 = 2160 \text{ vertices}. \]
The 256 vertices of each pair 128+128 form an 8-cube with 1024 edges, 1792 square faces, 1792 cubic cells, 1120 tesseract 4-faces, 448 5-cube 5-faces, 112 6-cube 6-faces, and 16 7-cube 7-faces. The image format of African Adinkra for 256 Odu of IFA shows Cl(8) graded structure \(1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1\) of 8-cube vertices. Physically they represent **Operators in \(H^{92} \times SL(8)\)** Generalized Heisenberg Algebra that is the Maximal Contraction of E8:

**Odd-Grade Parts of Cl(8) =**

\[= 128 \text{ D8 half-spinors} \text{ of one of } iE8, jE8, kE8, EE8, IE8, JE8, KE8\]

8+56 grades-1,3 = Fermion Particle 8-Component Creation (AntiParticle Annihilation)

56+8 grades-5,7 = Fermion AntiParticle 8-Component Creation (Particle Annihilation)

**Even-Grade Subalgebra of Cl(8) = 128 Mirror D8 half-spinors** =

28 grade-2 = Gauge Boson Creation (16 for Gravity, 12 for Standard Model)

28 grade-6 = Gauge Boson Annihilation (16 for Gravity, 12 for Standard Model)

(each 28 = 24 Root Vectors + 4 of Cartan Subalgebra)

64 of grade-4 = 8-dim Position x Momentum

1+(3+3)+1 grades-0,4,8 = Primitive Idempotent:

\[(1+3) = \text{Higgs Creation}; (3+1) = \text{Higgs Annihilation}\]

\[= 112 \text{ D8 Root Vectors} + 8 \text{ of E8 Cartan Subalgebra} + 8 \text{ Higgs Operators}\]
3. The Third Grothendieck Universe is the Completion of Union of all tensor products of $\text{Cl}(16)$ Real Clifford algebra

Since the $\text{Cl}(16)$-E8 Lagrangian is Local and Classical, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global $\text{Cl}(16)$-E8 Algebraic Quantum Field Theory (AQFT).

The usual Hyperfinite II$_1$ von Neumann factor for creation and annihilation operators on Fermionic Fock Space over $\mathbb{C}^\wedge(2n)$ is constructed by completion of the union of all tensor products of 2x2 Complex Clifford algebra matrices, which have Periodicity 2, so for the $\text{Cl16}$-E8 model based on Real Clifford Algebras with Periodicity 8, whereby any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of $\text{Cl}(8)$ and of $\text{Cl}(8) \times \text{Cl}(8) = \text{Cl}(16)$, the completion of the union of all tensor products of $\text{Cl}(16) = \text{Cl}(8) \times \text{Cl}(8)$ produces a generalized Hyperfinite II$_1$ von Neumann factor that gives the $\text{Cl}(16)$-E8 model a natural Algebraic Quantum Field Theory.

The overall structure of $\text{Cl}(160)$-E8 AQFT is similar to the Many-Worlds picture described by David Deutsch in his 1997 book "The Fabric of Reality" said (pages 276-283): "… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot ... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. ... in some regions of the multiverse, and in some places in space, the snapshots of some physical objects do fall, for a period, into chains, each of whose members determines all the others to a good approximation ...".

The Real Clifford Algebra $\text{Cl}(16)$ containing E8 for the Local Lagrangian of a Region is equivalent to a "snapshot" of the Deutsch "multiverse". The completion of the union of all tensor products of all $\text{Cl}(16)$-E8 Local Lagrangian Regions forms a generalized hyperfinite II$_1$ von Neumann factor AQFT and emergently self-assembles into a structure = Deutsch multiverse.
For the Cl(16)-E8 model AQFT to be realistic, it must be consistent with EPR entanglement relations. Joy Christian in arXiv 0904.4259 said: "... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings ... The alleged non-localities ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations [such as those] among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...". Since E8 is a Lie Group and therefore parallelizable and lives in Clifford Algebra Cl(16), the Cl(16)-E8 model is consistent with EPR.

The Creation-Annihilation Operator structure of Cl(16)-E8 AQFT is given by the Maximal Contraction of E8 = semidirect product A7 x h92

where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)

The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra

\[
28 + 64 + (SL(8,R) + 1) + 64 + 28
\]

Central Even Grade 0 = SL(8,R) + 1

The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices, so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.

Odd Grades -1 and +1 = 64 + 64

Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

Even Grades -2 and +2 = 28 + 28

Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

\[
\int_{8\text{-dim SpaceTime}} \text{Gauge Gravity Standard Model} + \text{Fermion Particle-AntiParticle}
\]
4. World-Line String Bohm Quantum Potential and Quantum Consciousness

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

\[ \int \text{Gauge Gravity Standard Model} + \text{Fermion Particle-AntiParticle} \]

8-dim SpaceTime

whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes \( 8v = 8\)-dim SpaceTime Dimensions + + \( 8s+ = 8 \) Fermion Particle Types + \( 8s- = 8 \) Fermion AntiParticle Types combined in the traceless part \( J(3,O) \) of the 3x3 Octonion Hermitian Jordan Algebra

\[
\begin{align*}
& a & 8s+ & 8v \\
& 8s+* & b & 8s- \\
& 8v* & 8s-* & -a-b
\end{align*}
\]

which has total dimension \( 8v + 8s+ + 8s- + 2 = 26 \) and is the space of a 26D String Theory with Strings seen as World-Lines.

Slices of \( 8v \) SpaceTime are represented as D8 branes. Each D8 brane has Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8

Stack D8 branes to get SpaceTime with Strings = World-Lines with a and b representing ordering of D8 brane stacks and Bohm-type Quantum Potential

Let Oct16 = discrete multiplicative group \{ +/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K\}. Orbifold by Oct16 the 8s+ to get 8 Fermion Particle Types

Orbifold by Oct16 the 8s- to get 8 Fermion AntiParticle Types

Gauge Bosons from 1E8 and EE8 parts of a D8 give U(2) ElectroWeak Force
Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give SU(3) Color Force
Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give U(2,2) Conformal Gravity.

The 8x8 matrices for collective coordinates linking one D8 to the next D8 give Position x Momentum
Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986) "... For the ... closed ... bosonic string .... The first excited level ... consists of ... the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ... SO(24) ... little group of a ...[26-dim]... massless particle ... and ... a ... massless ... spin two state ...".

Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analogous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The massless spin two state is what I call the Bohmion: the carrier of the Bohm Force of the Bohm-Sarfatti Quantum Potential.

Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993) "... the total force ... from the quantum potential ... does not ... fall off with distance ... because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms. The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its $10^{18}$ Tubulin Dimers with math description in terms of a large Real Clifford Algebra.

First consider Superposition of States involving one tubulin with one electron of mass $m$ and two different position states separated by $a$.

The Superposition Separation Energy Difference is the gravitational energy

$$E_{\text{electron}} = G \frac{m^2}{a}$$

For any single given tubulin $a = 1$ nanometer = $10^{-7}$ cm so that for a single Electron

$$T = \frac{h}{E_{\text{electron}}} = \left( \frac{\text{Compton}}{\text{Schwarzschild}} \right) \left( \frac{a}{c} \right) = 10^{26} \text{ sec} = 10^{19} \text{ years}$$

Now consider the case of $N$ Tubulin Electrons in Coherent Superposition

Jack Sarfatti defines coherence length $L$ by $L^3 = N \ a^3$ so that the Superposition Energy $E_N$ of $N$ superposed Conformation Electrons is

$$E_N = G \frac{M^2}{L} = N^{(5/3)} \ E_{\text{electron}}$$

The decoherence time for the system of $N$ Tubulin Electrons is

$$T_N = \frac{h}{E_N} = \frac{h}{N^{(5/3)} \ E_{\text{electron}}} = N^{(-5/3)} \ 10^{26} \text{ sec}$$

So we have the following rough approximate Decoherence Times $T_N$

<table>
<thead>
<tr>
<th>Time</th>
<th>Number of Involved Tubulins</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{(-5)}$ sec</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>$25 \times 10^{(-3)}$ sec (40 Hz)</td>
<td>$10^{16}$</td>
</tr>
</tbody>
</table>
5. Our Universe emerged from its parent in Octonionic Inflation

As Our Parent Universe expanded to a Cold Thin State Quantum Fluctuations occurred. Most of them just appeared and disappeared as Virtual Fluctuations, but at least one Quantum Fluctuation had enough energy to produce 64 Unfoldings and reach Paola Zizzi's State of Decoherence thus making it a Real Fluctuation that became Our Universe.

As Our Universe expands to a Cold Thin State, it will probably give birth to Our Child, GrandChild, etc, Universes.

Unlike "the inflationary multiverse" described by Andrei Linde in arXiv 1402.0526 as "a scientific justification of the anthropic principle", in the Cl(16)-E8 model ALL Universes (Ours, Ancestors, Descendants) have the SAME Physics Structure as E8 Physics (viXra 1312.0036 and 1310.0182).

In the Cl(16)-E8 model, our SpaceTime remains Octonionic 8-dimensional throughout inflation.

Stephen L. Adler in his book Quaternionic Quantum Mechanics and Quantum Fields (1995) said at pages 50-52, 561: "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) | g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]... failure of unitarity in octonionic quantum mechanics ...".

The NonAssociativity and Non-Unitarity of Octonions accounts for particle creation without the need for a conventional inflaton field.
Inflation begins in Octonionic Cl(16)-E8 Physics with a Quantum Fluctuation initially containing only one Cl(16) E8 Local Lagrangian Region.

The Fermion Representation Space for a Cl(16) E8 Local Lagrangian Region is $E8 / D8 = 64+64 = 128$-dim +half-spinor space $64s++ + 64s+-+-$ of Cl(16)

- $64s++ = 8$ components of 8 Fermion Particles
- $64s+-$ = 8 components of 8 Fermion Antiparticles

By 8-Periodicity of Real Clifford Algebras Cl(16) = tensor product Cl(8) x Cl(8) where the two copies of Cl(8) can be denoted by Cl(8)G and Cl(8)SM

( in E8 Physics Cl(8)G gives Gravity with Dark Energy and Cl(8)SM gives the Standard Model )

Cl(8)G and Cl(8)SM each have 8-dim half-spinor spaces 8Gs+ 8Gs- and 8SMs+ 8SMs-

- 8Gs+ and 8SMs+ representing 8 Fermion Particles
- 8Gs- and 8SMs- representing 8 Fermion Antiparticles

so that

- $64s++ = 8Gs+ x 8SMs+$ for First Generation Particles of E8 Physics
- $64s+-$ = 8Gs+ x 8SMs- for First Generation AntiParticles of E8 Physics
- $64s+- = 8Gs- x 8SMs+$ for AntiGeneration Particles ( NOT in E8 Physics )
- $64s-- = 8Gs- x 8SMs-$ for AntiGeneration AntiParticles ( NOT in E8 Physics )

where

- +/- half-spinor of Cl(8)G determines +/- half-spinor of Cl(16)

and Generation or AntiGeneration ( only +half-spinor Generation is in E8 )

- +/- half-spinor of Cl(8)SM determines Particle or AntiParticle
E8 Physics has Representation space for 8 Fermion Particles + 8 Fermion Antiparticles on the original Cl(16) E8 Local Lagrangian Region that is $64s^{++} + 8$ of $64s^{+-} =$ 

where a Fermion Representation slot _ of the $8 + 8 = 16$ slots can be filled by Real Fermion Particles 🟢 or Real Fermion Antiparticles 🟤.

IF the Quantum Fluctuation (QF) has enough Energy to produce them as Real and IF the Cl(16) E8 Local Lagrangian Region has an Effective Path from its QF Energy to that Particular slot. (see Appendix III for Geoffrey Dixon's ideas and Effective Path of QF Energy)

Since E8 contains only the 128 +half-spinors and none of the 128 -half-spinors of Cl(16) the only Effective Path of QF Energy to E8 Fermion Representation slots goes to the only Fermion Particle slots that are also of type + that is, to the 8 Fermion Particle Representation slots 🟢 🟢 🟢 🟢 🟢 🟢 🟢 🟢 + 🟢 🟢 🟢 🟢 🟢 🟢 🟢 🟢

Next, consider the first Unfolding step of Octonionic Inflation. It is based on all $16 = 8$ Fermion Particle slots + 8 Fermion Antiparticle Representation slots whether or not they have been filled by QF Energy.

7 of the 8 Fermion Particle slots correspond to the 7 Imaginary Octonions and therefore to the 7 Independent E8 Integral Domain Lattices and therefore to 7 New Cl(16) E8 Local Lagrangian Regions. The 8th Fermion Particle slot corresponds to the 1 Real Octonion and therefore to the 8th E8 Integral Domain Lattice (not independent - see Kirmse's mistake) and therefore to the 8th New Cl(16) E8 Local Lagrangian Region.

Similarly, the 8 Fermion Antiparticle slots Unfold into 8 more New New Cl(16) E8 Local Lagrangian Regions, so that one Unfolding Step is a 16-fold multiplication of Cl(16) E8 Local Lagrangian Regions:
If the QF Energy is sufficient, the Fermion Particle content after the first Unfolding is

so it is clear that **the Octonionic Inflation Unfolding Process creates Fermion Particles with no Antiparticles, thus explaining the dominance of Matter over AntiMatter in Our Universe.**

Each Unfolding has duration of the Planck Time $T_{\text{planck}}$ and none of the components of the Unfolding Process Components are simultaneous, so that **the total duration of $N$ Unfoldings is $2^N T_{\text{planck}}$.**

**Paola Zizzi** in gr-qc/0007006 said: "... during inflation, the universe can be described as a superposed state of quantum ... [ qubits ]. the self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [ $T_{\text{decoh}} = 10^9 T_{\text{planck}} = 10^{-34} \text{ sec}$ ] ... and corresponds to a superposed state of ... [ $10^{19} = 2^{64}$ qubits ] ...".

**Why decoherence at 64 Unfoldings = $2^{64}$ qubits ?**

$2^{64}$ qubits corresponds to the Clifford algebra $\text{Cl}(64) = \text{Cl}(8x8)$. By the periodicity-8 theorem of Real Clifford algebras, $\text{Cl}(64)$ is the smallest Real Clifford algebra for which we can reflexively identify each component $\text{Cl}(8)$ with a vector in the $\text{Cl}(8)$ vector space. This reflexive identification/reduction causes our universe to decohere at $N = 2^{64} = 10^{19}$ which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.
The Real Clifford Algebra Cl(8) is the basic building block of Real Clifford Algebras due to 8-Periodicity whereby Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8)

An Octonionic basis for the Cl(8) 8-dim vector space is \{1,i,j,k,E,I,J,K\}

NonAssociativity, NonUnitarity, and Reflexivity of Octonions is exemplified by the 1-1 correspondence between Octonion Basis Elements and E8 Integral Domains

\[
\begin{align*}
1 & \iff 0E8 \\
i & \iff 1E8 \\
j & \iff 2E8 \\
k & \iff 3E8 \\
E & \iff 4E8 \\
I & \iff 5E8 \\
J & \iff 6E8 \\
K & \iff 7E8
\end{align*}
\]

where 1E8,2E8,3E8,4E8,5E8,6E8,7E8 are 7 independent Integral Domain E8 Lattices and 0E8 is an 8th E8 Lattice (Kirmse's mistake) not closed as an Integral Domain. Using that correspondence expands the basis \{1,i,j,k,E,I,J,K\} to \{0E8,1E8,2E8,3E8,4E8,5E8,6E8,7E8\}

Each of the E8 Lattices has 240 nearest neighbor vectors so the total dimension of the Expanded Space is \(240 \times 240 \times 240 \times 240 \times 240 \times 240 \times 240 \times 240\)

Everything in the Expanded Space comes directly from the original Cl(8) 8-dim space so all Quantum States in the Expanded Space can be held in Coherent Superposition. However, if further expansion is attempted, there is no direct connection to original Cl(8) space and any Quantum Superposition undergoes Decoherence.

If each 240 is embedded reflexively into the 256 elements of Cl(8) the total dimension is \(256 \times 256 \times 256 \times 256 \times 256 \times 256 \times 256 = 256^8 = 2^{8 \times 8} = 2^{64} = Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) = Cl(64)\), so the largest Clifford Algebra that can maintain Coherent Superposition is Cl(64) which is why Zizzi Quantum Inflation ends at the Cl(64) level.

**At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about \((1/2) 16^{64} = (1/2) (2^4)^{64} = 2^{255} = 6 \times 10^{76}\) Fermion Particles**

**The End of Inflation time was at about \(10^{(-34)}\) sec = \(2^{64}\) Tplanck and the size of our Universe was then about \(10^{(-24)}\) cm which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud.**

(see viXra 1311.0088)
Octonion Inflation produces Gravitational Waves that can now be observed in Polarization Patterns of the Cosmic Microwave Background.

BICEP2 in arXiv 1403.3985 said: "... Inflation predicts ... a primordial background of ... gravitational waves ...[that]... would have imprinted a unique signature upon the CMB. **Gravitational waves induce local quadrupole anisotropies** in the radiation field within the last-scattering surface, inducing polarization in the scattered light ... This polarization pattern will include a “curl” or ... inflationary gravitational wave (IGW) B-mode ... component at degree angular scales that cannot be generated primordially by density perturbations. The amplitude of this signal depends upon the **tensor-to-scalar ratio** ... $r = 0.20 ±0.07 -0.05$ ... which itself is a function of the energy scale of inflation. ...".

**In the Cl(16)-E8 model,** **Inflation is due to Non-Unitarity of Octonion Quantum Processes** that occur in 8-dim SpaceTime before freezing out of a preferred Quaternionic Frame ends Inflation and begins Ordinary Evolution in (4+4)-dim $M_4 \times CP2$ Kaluza-Klein. The unit sphere in the Euclidean version of 8-dim SpaceTime ( see viXra 1311.0088 for Schwinger's "unitary trick" to allow use of Euclidean SpaceTime ) is the 7-sphere $S_7$.

**Curl-type B-modes** (tensor) are Octonionic Quantum Processes on the surface of SpaceTime $S_7$ which is a **7-dim NonAssociative Moufang Loop Malcev Algebra**.

![B-modes look like Spirals on the Surface of S7](image from Sky and Telescope)

**Divergence-type E modes** (scalar and tensor) are Octonionic Quantum Processes from SpaceTime $S_7$ plus a spinor-type $S_7$ representing Dirac Fermions living in SpaceTime plus a 14-dim $G_2$ Octonionic Derivation Algebra connecting the two $S_7$ spheres all of which is a **28-dim D4 Lie Algebra Spin(8)**.

![E-modes look like Fermion Pair Creation either off (scalar) or on (tensor) the Surface of S7](image from Sky and Telescope)

Therefore: **for E8 Physics Octonionic Inflation the ratio** $r = 7 / 28 \approx 0.25$
End of Inflation and Low Initial Entropy in Our Universe:
Roger Penrose in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:
"... in our universe ... Entropy ... increases ... Something forced the entropy to be low in
the past. ... the low-entropy states in the past are a puzzle. ..."
The key to solving Penrose's Puzzle is given by Paola Zizzi in gr-qc/0007006:
"... The self-reduction of the superposed quantum state is ... reached at the end of
inflation ...[at]... the decoherence time ... [ Tdecoh = 10^9 Tplanck = 10(-34) sec ] ... and corresponds to a superposed state of ... [ 10^19 = 2^64 qubits ]. ... This is also the number of
superposed tubulins-qubits in our brain ... leading to a conscious event. ..."
The Zizzi Inflation phase of our universe ends with decoherence "collapse" of
the 2^64 Superposition Inflated Universe into Many Worlds of Quantum Theory,
only one of which Worlds is our World. The central white circle is the Inflation Era in
which everything is in Superposition; the boundary of the central circle marks the
decoherence/collapse at the End of Inflation; and each line radiating from the central
circle corresponds to one decohered/collapsed Universe World (of course, there are many
more lines than actually shown), only three of which are explicitly indicated in the image,
and only one of which is Our Universe World.

Since our World is only a tiny fraction of all the Worlds, it carries only a tiny
fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving
Penrose's Puzzle.
6. Quaternionic M4xCP2 Kaluza-Klein SpaceTime

At the end of Non-Unitary Octonionic Inflation Our Universe had about \((1/2) \cdot 16^{64} = (1/2) \cdot (2^4)^{64} = 2^{255} = 6 \times 10^{76}\) Fermion Particles.

The End of Inflation time was at about \(10^{(-34)}\) sec = \(2^{64}\) Tplanck and

the size of our Universe was then about \(10^{(-24)}\) cm

which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud and

the Real Clifford Algebra of 8-dim SpaceTime was \(\text{Cl}(1,7) = \text{Cl}(0,8) = M(16,R)\)

The Event that Ended Inflation was Decoherence of Zizzi Quantum Inflation that also produced decoherence of the D8 brane SpaceTime Planck-Scale Lattice superpositions of the 8 types of E8 Lattice 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8 which resulted in a decoherence choice of a particular E8 Lattice. The 240 origin-nearest-neighbor Root Vectors of such a chosen E8 Lattice can be represented as 8 circles of 30 vertices each

with \(4 \times 30 = 120\) vertices (black dots) forming a 600-cell and

the other \(4 \times 30 = 120\) vertices (white dots) forming another 600-cell at radii expanded from that of the black dots by a Golden Ratio factor. Since each 600-cell is 4-dim, the Octonionic 8-dim E8 SpaceTime is decomposed into 2 Quaternionic 4-dim parts,
giving the Post-Inflation Cl(16)-E8 model a (4+4)-dim Kaluza-Klein SpaceTime of the form M4 x CP2 where M4 is 4-dim Physical Minkowski SpaceTime on which Gravity acts and CP2 = SU(3) / U(2) is 4-dim Internal Symmetry Space for Standard Model Forces.

In the Cl(16)-E8 model, 8-dim SpaceTime,

both Octonionic

and Quarternionic

is represented by the 64-dim Adjoint D8 / D4xD4 part of E8 which is the A7 x R grade-0 part of the Maximal Contraction A7 x h92 with 5-grading

\[ 28 + 64 + (SL(8,R) + 1) + 64 + 28 \]

In the Cl(16)-E8 model Gravity is most often written as in Chapter 18 of this paper in terms of the MacDowell-Mansouri Conformal Group Spin(2,4) which is the 15-dimensional Conformal BiVector Group of the 64-dim Cl(2,4) Clifford Algebra but it can also be written in terms of 64-dim grade-0 Maximal Contraction term SL(8,R) + 1 in which case it is known as Unimodular SL(8,R) Gravity which effectively describes a generalized checkerboard of 8-dim SpaceTime HyperVolume Elements and, with respect to Cl(16) = Cl(8)xCl(8), is the tensor product of the two 8v vector spaces of the two Cl(8) factors of Cl(16). If those two Cl(8) factors are regarded as Fourier Duals, then 8v x 8v describes Position x Momentum in 8-dim SpaceTime.

Conformal Spin(2,4) = SU(2,2) Gravity and Unimodular SL(4,R) = Spin(3,3) Gravity seem to be effectively equivalent since, as Bradonjic and Stachel in arXiv 1110.2159 said: "... in ... Unimodular relativity ... the symmetry group of space-time is ... the special linear group SL(4,R) ... the metric tensor ... break[s up] ... into the conformal structure represented by a conformal metric ... with det = -1 and a four-volume element ... at each point of space-time ...[that]... may be the remnant, in the ... continuum limit, of a more fundamental discrete quantum structure of space-time itself ...". Further, Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said: "... Because of the existence of topologically nontrivial solutions, instantons, of the classical field equations associated with quantum chromodynamics (QCD), the quantized theory contains a dimensionless parameter \( \phi (0 < \phi < 2\pi) \) not explicit in the classical lagrangian. Since \( \phi \) multiplies an expression odd in CP, QCD predicts violation of that symmetry unless the phase \( \phi \) takes one of the special values ... 0 (mod \( \pi \)) ... this fine tuning is the strong CP problem ... the quantum dynamics of ... unimodular gravity ... may lead to the relaxation of \( \phi \) to \( \phi = 0 \) (mod \( \pi \)) without the need ... for a new particle ... such as the axion ...".
End of Inflation and Quaternionic Structure

In Cl(16)-E8 Physics (vixra 1405.0030) Octonionic symmetry of 8-dim spacetime is broken at the End of Non-Unitary Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2 physical spacetime x internal symmetry space.

Here are some details about that transition:

The basic local entity of Cl(16)-E8 Physics is
\[ \text{Cl}(0,16) = \text{Cl}(1,15) = \text{Cl}(4,12) = \text{Cl}(5,11) = \text{Cl}(8,8) = M(R,256) = 256 \times 256 \text{ Real Matrices} \]
which contains E8 with 8-dim Octonionic spacetime and is the tensor product \[ \text{Cl}(0,8) \times \text{Cl}(0,8) = \text{Cl}(1,7) \times \text{Cl}(1,7) \]
where \[ \text{Cl}(0,8) = \text{Cl}(1,7) = M(R,16) \text{ is the Clifford Algebra of the 8-dim spacetime.} \]

Non-Unitary Octonionic Inflation is based on Octonionic spacetime structure with superposition of E8 integral domain lattices. At the End of Inflation the superposition ends and Octonionic 8-dim structure is replaced by Quaternionic (4+4)-dim structure.

Since \[ M(R,16) = M(Q,2) \times M(Q,2) \text{ and } M(Q,2) = \text{Cl}(1,3) = \text{Cl}(0,4) \]
\[ \text{Cl}(0,8) = \text{Cl}(1,7) \text{ can be represented as } \text{Cl}(1,3) \times \text{Cl}(0,4) \]
where \[ \text{Cl}(1,3) \text{ is the Clifford Algebra for M4 physical spacetime} \]
and \[ \text{Cl}(0,4) \text{ is the Clifford Algebra for CP2 = SU}(3) / U(2) \text{ internal symmetry space} \]
thus making explicit the Quaternionic structure of (4+4)-dim M4 x CP2 Kaluza-Klein.
Cl(1,3) = Cl(0,4) = M(Q,2) has graded structure based on 1 2 1 grading of 2x2 matrices and 1 2 1 grading of the Quaternions, so that its total graded structure is

\[
\begin{array}{cccccc}
1 & 2 & 1 \\
2 & 4 & 2 \\
1 & 2 & 1 \\
1 & 4 & 6 & 4 & 1 \\
\end{array}
\]

and its Spinor structure is 2x1 Quaternion matrices

\[
\begin{array}{cccccc}
1 & 2 & 1 \\
1 & 2 & 1 \\
2 & 4 & 2 \\
= & 1 & 2 & 1 & + & 1 & 2 & 1 \\
\end{array}
\]

1 2 1 = 4-dim Shilov Boundary for Lie Sphere Spin(6) / Spin(4)xU(1) =
= half-spinors for First Generation Lepton + 3 Quarks

4s+ for Electron + 3 Up Quarks
and

4s- for Neutrino + 3 Down Quarks

One copy of Cl(1,3) only has room for Particles, no AntiParticles

Cl(1,3) vectors can represent M4 physical spacetime but

the CP2 part of M4 x CP2 Kaluza-Klein is not directly represented by Cl(1,3).
Cl(0,8) = Cl(1,7) = M(R,16) = M(Q,2) x M(Q,2) has graded structure

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<thead>
<tr>
<th>1</th>
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</table>

1 8 28 56 70 56 28 8 1

and its Spinor structure based on M(Q,2) = 16-dim is

Spinors = \sqrt{16 \times 16} = 16 = 8+8

Their Real / Octonionic M(R,16) structure is:

with 8-dim +half-spinors

and 8-dim -half-spinors

and 8-dim vectors related to each other by Triality

Their Quaternionic M(Q,2) x M(Q,2) structure is

with 2-Quaternionic +half-spinors

and 2-Quaternionic -half-spinors

and 2-Quaternionic vectors representing (4+4)-dim Kaluza-Klein M4 x CP2, related to half-spinors by Triality.

The 8-dim vectors of Cl(0,8) = Cl(1,7) correspond to B4 / D4 = OP1

Spinors = 8+8 = F4 / B4 = 52-36 = OP2

\[ F4 = 8 + 28 + (8+8) \]

8 = Shilov Boundary for Lie Sphere Spin(10) / Spin(8)xU(1) =

= half-spinors for First Generation Fermion Particles / AntiParticles

8s+ for Particles and 8s- for AntiParticles

One copy of Cl(8) only has room for one Generation, no AntiGeneration

The AntiGeneration appears for Cl(16) = Cl(8) x Cl(8)

but is not in E8 which omits the AntiGeneration half-spinors of Cl(16)
\[
\text{Cl}(0,16) = \text{Cl}(1,15) = \text{M}(R,256) = \text{M}(Q,2) \times \text{M}(Q,2) \times \text{M}(Q,2) \times \text{M}(Q,2)
\]

has graded structure

\[
\begin{array}{cccccccccc}
1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \\
8 & 64 & ... \\
28 & ... \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & 16 & 120 & ... \\
\end{array}
\]

and its Spinor structure based on \(\text{M}(Q,2) = 16\)-dim is

\[
\text{Spinors} = \sqrt{16 \times 16 \times 16 \times 16} = 16 \times 16 = 256 = 128 + 128
\]

( equivalent to \(\text{M}(R,256)\) Spinors = 256x1 Real = 256 = 128 + 128 )

\[
\text{Spinors} = 128 + 128
\]

\[
\text{E8} = 120 + 128
\]

\[
128 = \text{Cl}(16) \text{ half-spinors for One Generation Fermion Particles and AntiParticles}
\]

The other 128 is for One AntiGeneration that is not in E8
Quaternionic structure similar to that of Cl(1,3) = Cl(0,4) = M(Q,2) is seen
in

$$\text{Cl}(2,4) = \text{M}(Q,4) = 4 \times 4 \text{ Quaternion matrices with grading based on } 4 \times 4 = \begin{array}{cccc}
1 & 4 & 6 & 4 \\
1 & 2 & 1 & \\
4 & 8 & 4 & \\
6 & 12 & 6 & \\
4 & 8 & 4 & \\
1 & 2 & 1 & \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{array}$$

Conformal Gravity $\text{Spin}(2,4) = \text{SU}(2,2)$ of $\text{Cl}(2,4) = \text{M}(Q,4)$ 4x4 Quaternionic Matrices have $(4+4) \times 4 = 32$-dim spinors with

- 2-Quaternionic +half-spinors
- 2-Quaternionic -half-spinors

$\text{Cl}(2,4)$ vectors are 6-dim but $\text{Spin}(2,4) = \text{SU}(2,2)$ so the Twistor Correspondence produces 1-Quaternionic Twistors that represent the M4 part of M4xCP2 Kaluza-Klein

with the CP2 part not directly represented by $\text{Cl}(2,4)$.

Spinors = 4x1 Quaternion

$$16 = 4 \quad 8 \quad 4 \quad = \quad 2 \quad 4 \quad 2 \quad + \quad 2 \quad 4 \quad 2$$

$$2 \quad 4 \quad 2 \quad 8 \quad = \text{Lie Sphere Spin}(6) / \text{Spin}(4) \times \text{U}(1) \text{ Complex Domain has } \text{Cl}(1,3) \text{ half-spinor Shilov Boundary } \text{Cl}(2,4) \text{ is in some sense a } (1,1) \text{ Complexification of } \text{Cl}(1,3)$$
and in

\[ \text{Cl}(2,6) = \text{Cl}(3,5) = M(Q,8) = 8x8 \text{ Q-matrix grading based on } 8 \times 8 = 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \]

\[
\begin{array}{cccccc}
1 & 2 & 1 \\
6 & 12 & 6 & \\
& 15 & 30 & 15 \\
& 20 & 40 & 20 \\
& 15 & 30 & 15 \\
& 6 & 12 & 6 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \\
\end{array}
\]

Quaternionic M(Q,8) 8x8 Quaternionic Matrices have (4+4)x4 = 32-dim spinors

with 4-Quaternionic +half-spinors

and 4-Quaternionic -half-spinors

and 2-Quaternionic vectors

that represent the two 4-dim spaces of Kaluza-Klein M4 x CP2

The 8-dim vectors do not correspond to 16-dim D5 / D4xU(1) = (CxO)P1

If you were to expand the vectors to 16-dim you would go to Cl(16) = Cl(8)xCl(8)

Spinors = 8x1 Quaternion = E6 / D5xU(1) = 78-45-1 = 32

\[ E6 = 15 + 30 + 32 \]

\[ 32 = 8 \quad 16 \quad 8 = 4 \quad 8 \quad 4 + 4 \quad 8 \quad 4 \]

4 8 4 = Lie Sphere Spin(10) / Spin(8)xU(1)

The Quaternionic half-spinors in Cl(2,6) correspond to Lie Sphere Complex Domains whereas

the half-spinors in Cl(1,7) = Cl(0,8) correspond to Shilov Boundaries

and

the E6 of Cl(2,6) is in some aspects a Complexification of the F4 of Cl(1,7) = Cl(0,8).
7. Batakis Standard Model Gauge Groups and Mayer-Trautman Higgs

The Mayer-Trautman Mechanism reduces the Lagrangian integral over the 8-dim SpaceTime whose 8-Position x 8-Momentum is represented by 64-dim D8 / D4 x D4 where D8 is the Adjoint part of E8.

\[ \int \text{Gauge Gravity Standard Model} \ + \ \text{Fermion Particle-AntiParticle} \]

8-dim SpaceTime

\[ \int \text{GG SM} \ + \ \text{Fermion Particle-AntiParticle} \ + \ \text{Higgs} \]

4-dim M4

by integrating out the Lagrangian Density over the CP2 Internal Symmetry Space and so creating a new Higgs term in the Lagrangian Density integrated only over M4.

Since the D4 = U(2,2) of Gauge Gravity acts on the M4, there is no problem with it.

As to the D4 = U(4) of the Standard Model, U(4) contains as a subgroup color SU(3) which is also the global symmetry group of the CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space of M4 X CP2 Kaluza-Klein SpaceTime.

A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 said: "... In a standard Kaluza-Klein framework, M4 x CP2 allows the classical unified description of an SU(3) gauge field with gravity ... [and] the possibility of an additional SU(2) x U(1) gauge field structure is uncovered. ...".

Since the CP2 = SU(3) / U(2) has global SU(3) action, the SU(3) can be considered as a local gauge group acting on the M4, so there is no problem with it.

However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so the local action of U(2) on CP2 must be integrated out to get the desired U(2) = SU(2)xU(1) local action directly on M4.
Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4.

This is done by a mechanism due to Meinhard Mayer and A. Trautman in “A Brief Introduction to the Geometry of Gauge Fields” and “The Geometry of Symmetry Breaking in Gauge Theories”, Acta Physica Austriaca, Suppl. XXIII (1981)

where they say: "...

... We start out from ... four-dimensional M [ M4 ] ...[and]... R ...[that is]... obtained from ... G/H [ CP2 = SU(3) / U(2) ] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on M [M4] with values in H [SU(2)] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ... S_YM = Integral Tr ( F ∧ *F ) 

... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2].

We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ...

Then the integrand ... becomes

Tr( F_!! F^!! + 2 F_! F^!? + F_?? F^?? )

...
The first term .. becomes the $[SU(2)]$ Yang-Mills action for the reduced $[SU(2)]$ Yang-Mills theory

... the middle term .. becomes, symbolically,
Tr Sum $D_! \cdot PHI(?) \cdot D^! \cdot PHI(?)$
where $PHI(?)$ is the Lie-algebra-valued 0-form corresponding to the invariance of $A$ with respect to the vector field $?$, in the $G/H [CP2]$ direction

... the third term .. involves the contraction $F_{??}$ of $F$ with two vector fields lying along $G/H [CP2]$ ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]
$2 \cdot F_{??} = [PHI(?) \cdot PHI(?) \cdot PHI([?,?])$
... Thus,
the third term .. reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:
Tr $F_{??} \cdot F^{??} = (1/4) \cdot Tr \cdot (PHI \cdot PHI \cdot PHI)^2$
... special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...


**Theorem 11.7.** Assume in Theorem 11.5 that $\mathfrak{t}$ admits a subspace $m$ such that $\mathfrak{t} = j + m$ (direct sum) and $ad(J)(m) = m$, where $ad(J)$ is the adjoint representation of $J$ in $\mathfrak{t}$. Then ... The curvature form $\Omega$ of the $K$-invariant connection defined by $\Lambda_m$ satisfies the following condition:

$2\Omega_{\omega}(\vec{X}, \vec{Y}) = [\Lambda_m(X), \Lambda_m(Y)] - \Lambda_m([X, Y]_m) - \lambda([X, Y]_m)$
for $X, Y \in m$.
Along the same lines, Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): “...

... each point of ... the ... fibre bundle ... E consists of a four-dimensional spacetime point \( x \) [in \( M^4 \)] to which is attached the homogeneous space \( G/H \) [\( SU(3)/U(2) = \text{CP}2 \)] ... the components of the curvature lying in the homogeneous space \( G/H \) [\( = SU(3)/U(2) \)] could be reinterpreted as Higgs scalars (with respect to spacetime [\( M^4 \)]) ... the Yang-Mills action reduces to a Yang-Mills action for the h-components [\( U(2) \) components] of the curvature over \( M \) [\( M^4 \)] and a quartic functional for the “Higgs scalars”, which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...”.
8. 2nd and 3rd Generation Fermions

The 8 First Generation Fermion Particles can each be represented by the 8 basis elements \{1, i, j, k, E, I, J, K\} of the Octonions O

1 \(\leftrightarrow\) e-neutrino
i \(\leftrightarrow\) red down quark
j \(\leftrightarrow\) green down quark
k \(\leftrightarrow\) blue down quark

E \(\leftrightarrow\) electron
I \(\leftrightarrow\) red up quark
J \(\leftrightarrow\) green up quark
K \(\leftrightarrow\) blue up quark

with Antiparticles being represented similarly.

The Second and Third Generations can be represented by Pairs of Octonions OxO and Triples of Octonions OxOxO respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein M4 x CP2 at the End of Inflation, there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in M4, so its path can be represented by the single O;

2 - Either A or B, but not both, is in CP2, so its path must be augmented by one projection from CP2 to M4, which projection can be represented by a second O, giving a second generation OxO;

3 - Both A and B are in CP2, so its path must be augmented by two projections from CP2 to M4, which projections can be represented by a second O and a third O, giving a third generation OxOxO.

Combinatorics contributes to Fermion mass ratios. For example:

Blue Down Quark is 1 out of 8 and Blue Up Quark is 1 out of 8, so the Down Quark : Up Quark mass ratio is 1 : 1

Blue Strange Quark is 3 out of 8x8 = 64 and Blue Charm Quark is 17 out of 8x8 = 64, so the Strange Quark : Charm Quark mass ratio is 3 : 17

Blue Beauty Quark is 7 out of 8x8x8 = 512 and Blue Truth Quark is 161 out of 8x8x8 = 512, so the Beauty Quark : Truth Quark mass ratio is 7 : 161
9. Schwinger Sources with inherited Monster Group Symmetry

Kerr-Newman Black Hole structure size about $10^{(-24)}$ cm

and

Geometry of Bounded Complex Domains and Shilov boundaries

The Cl(16)-E8 model Lagrangian over 4-dim Minkowski SpaceTime M4 is

$$\int_{\text{4-dim M4}} \text{GG SM} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

Consider the Fermion Term.

In the conventional picture, the spinor fermion term is of the form $m \, S \, S^*$ where $m$ is the fermion mass and $S$ and $S^*$ represent the given fermion.

The Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picture, an ad hoc inclusion.

The Cl(16)-E8 model does not put in the mass $m$ in an ad hoc way, but constructs the Lagrangian integral such that the mass $m$ emerges naturally from the geometry of the spinor fermions by setting the spinor fermion mass term as the volume of the Schwinger Source Fermions.

Effectively the integral over the Schwinger Source spacetime region of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion gives the volume of the Schwinger Source fermion and defines its mass, which, since it is dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

The Cl(16)-E8 model constructs the Lagrangian integral such that the mass $m$ emerges as the integral over the Schwinger Source spacetime region of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion so that the volume of the Schwinger Source fermion defines its mass, which, being dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space $\text{Spin}(10) / \text{Spin}(8) \times U(1)$ which has local symmetry of the Spin(8) gauge group from which the first generation spinor fermions are formed as $\text{+half-spinor}$ and $\text{-half-spinor}$ spaces and Bounded Complex Domain D8 of type IV8 and Shilov Boundary $Q_8 = \text{RP}1 \times S7$
Consider the **GG SM term** from Gauge Gravity and Standard Model Gauge Bosons. The process of breaking Octonionic 8-dim SpaceTime down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime.

There are 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces with Quaternionic structure consistent with 4-dim Physical SpaceTime:

- **S4** \(= 4\)-sphere = \(\text{Spin}(5) / \text{Spin}(4)\) where \(\text{Spin}(5)\) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives **MacDowell-Mansouri Gravity**

- **CP2** = complex projective 2-space = \(\text{SU}(3) / \text{U}(2)\) with the **SU(3)** of the Color Force

- **S2 x S2** = \(\text{SU}(2)/\text{U}(1) \times \text{SU}(2)/\text{U}(1)\) with two copies of the **SU(2)** of the Weak Force

- **S1 x S1 x S1 x S1** = \(\text{U}(1) \times \text{U}(1) \times \text{U}(1) \times \text{U}(1)\) = 4 copies of the **U(1)** of the EM Photon (1 copy for each of the 4 covariant components of the Photon)

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of \(D4 = \text{Spin}(8)\). They "see" M4 Physical spacetime as the 4-sphere \(S4\) so that their part of the Physical Lagrangian is

\[
\int_{S4} \text{Gravity Gauge Boson Term}.
\]

an integral over SpaceTime \(S4\).

The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons. However, due to Stabilization of Condensate SpaceTime by virtual Planck Mass Gravitational Black Holes, for Gravity, the effective force strength that we see in our experiments is not just composed of the \(S4\) volume and the Spin(5) Schwinger Source volume, but is suppressed by the square of the Planck Mass.

The unsuppressed Gravity force strength is the Geometric Part of the force strength.
The Standard Model SU(3) Color Force bosons live in a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8). They "see" M4 Physical spacetime as the complex projective plane CP2 so that their part of the Physical Lagrangian is

\[ \int_{\text{CP2}} \text{SU(3) Color Force Gauge Boson Term} \]

an integral over SpaceTime CP2.

The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.

The Color Force Strength is given by the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.

Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2).

They "see" M4 Physical spacetime as two 2-spheres S2 x S2 so that their part of the Physical Lagrangian is

\[ \int_{\text{S2xS2}} \text{SU(2) Weak Force Gauge Boson Term} \]

an integral over SpaceTime S2xS2.

The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.

However, due to the action of the Higgs mechanism, for the Weak Force, the effective force strength that we see in our experiments is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume, but is suppressed by the square of the Weak Boson masses.

The unsuppressed Weak Force strength is the Geometric Part of the force strength.
The Standard Model U(1) Electromagnetic Force bosons (photons) live in a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2). They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4 (T4 = 4-torus) so that their part of the Physical Lagrangian is

\[ \int_{T^4} (U(1) \text{ Electromagnetism Gauge Boson Term}) \]

an integral over SpaceTime T4.

The Schwinger Sources for U(1) photons are the Complex Bounded Domains and Shilov Boundaries for U(1) photons. The Electromagnetic Force Strength is given by the SpaceTime T4 volume and the U(1) Schwinger Source volume.

Schwinger Sources as described above are continuous manifold structures of Bounded Complex Domains and their Shilov Boundaries but

the Cl(16)-E8 model at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines with 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about 8 x 10^53.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single Planck-scale entity because Tachyons create a cloud of particles/antiparticles. The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.

That cloud constitutes the Schwinger Source. Its structure comes from the 24-dim Leech lattice part of the Monster Group which is \(2^{(1+24)}\) times the double cover of Co1, for a total order of about \(10^{26}\).

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice) distinct Leech lattices. The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of \(10^{27}\) x Planck scale, so the Kerr-Newman Cloud should contain about \(10^{27}\) particle/antiparticle pairs and its size should be about \(10^{(27/3)} \times 1.6 \times 10^{(-33)} \text{ cm} = 10^{-24} \text{ cm.} \)
10. Fermion Mass Calculation

In the Cl(16)-E8 model, the first generation spinor fermions are seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8). Due to Triality, Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime.

Take the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:

Take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:

the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is taken in the same way ( unlike what happens for the force strength calculation ), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.

Cl(16)-E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes, so the quark mass in the Cl(16)-E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

\[ V(Q\text{fermion}) \times N(\text{Graviton}) \times N(\text{octonion}) \times \text{Sym} \]

\( V(Q\text{fermion}) \) is the volume of the part of the half-spinor fermion particle manifold \( S^7 \times RP^1 \) related to the fermion particle by photon, weak boson, or gluon interactions.

\( N(\text{Graviton}) \) is the number of types of Spin(0,5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2). 2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons.
One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore \( 6/1 = 6 \).

\( N(\text{octonion}) \) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

\( \text{Sym} \) is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.
3 Generation Fermion Combinatorics

First Generation (8)

(geometric representation of Octonions is from arXiv 1010.2979)

electron  red  green  blue  red  green  blue  neutrino
up quark  up quark  up quark  down quark  down quark  down quark

E  I  J  K  i  j  k  1

Second Generation (64)

Mu Neutrino (1)
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element).
Muon (3)
Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative
(that is, is E which is the only Colorless NonAssociative element).

Blue Strange Quark (3)
Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Charm Quark (17)
Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element (Red x Green = Blue).

( Red and Green Strange and Charm Quarks follow similar rules )
Third Generation (512)

Tau Neutrino (1)
Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element)

Tauon (7)
Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)
Blue Beauty Quark (7)
Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Truth Quark (161)
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
   and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element and the other element is Colorless (Red \times Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative and must be Blue.

( Red and Green Beauty and Truth Quarks follow similar rules )
The first generation down quark constituent mass : electron mass ratio is:

The electron, $E$, can only be taken into the tree-level-massless neutrino, $1$, by photon, weak boson, and gluon interactions. The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks. The neutrino, being massless at tree level, does not add anything to the mass formula for the electron. Since the electron cannot be related to any other massive Dirac fermion, its volume $V(Q_{\text{electron}})$ is taken to be $1$.

Next consider a red down quark $i$. By gluon interactions, $i$ can be taken into $j$ and $k$, the blue and green down quarks. By also using weak boson interactions, it can also be taken into $I$, $J$, and $K$, the red, blue, and green up quarks. Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos. Therefore the red down quark (similarly, any down quark) is related to all parts of $S^7 \times \mathbb{RP}^1$, the compact manifold corresponding to $\{ 1, i, j, k, E, I, J, K \}$ and therefore a down quark should have a spinor manifold volume factor $V(Q_{\text{down quark}})$ of the volume of $S^7 \times \mathbb{RP}^1$.

The ratio of the down quark spinor manifold volume factor to the electron spinor manifold volume factor is $\frac{V(Q_{\text{down quark}})}{V(Q_{\text{electron}})} = V(S^7 \times \mathbb{RP}^1)/1 = \pi^5 / 3$.

Since the first generation graviton factor is $6$, $\frac{m_d}{m_e} = 6 \frac{V(S^7 \times \mathbb{RP}^1)}{1} = 2 \pi^5 = 612.03937$

As the up quarks correspond to $I$, $J$, and $K$, which are the octonion transforms under $E$ of $i$, $j$, and $k$ of the down quarks, the up quarks and down quarks have the same constituent mass $m_u = m_d$.

Antiparticles have the same mass as the corresponding particles. Since the model only gives ratios of masses, the mass scale is fixed so that the electron mass $m_e = 0.5110$ MeV.

Then, the constituent mass of the down quark is $m_d = 312.75$ MeV, and the constituent mass for the up quark is $m_u = 312.75$ MeV.

These results when added up give a total mass of first generation fermion particles: $\Sigma f_1 = 1.877$ GeV
As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

\[ m_{\text{proton}} = m_u + m_u + m_d = 938.25 \text{ MeV} \]

which is close to the experimental value of 938.27 MeV.

**The third generation** fermion particles correspond to triples of octonions.
There are \( 8^3 = 512 \) such triples.

The triple \( \{1,1,1\} \) corresponds to the tau-neutrino.

The other 7 triples involving only 1 and \( E \) correspond to the tauon:

\[
\begin{align*}
\{ E, E, E \} \\
\{ E, E, 1 \} \\
\{ E, 1, E \} \\
\{ 1, E, E \} \\
\{ 1, 1, E \} \\
\{ 1, E, 1 \} \\
\{ E, 1, 1 \}
\end{align*}
\]

The symmetry of the 7 tauon triples is the same as the symmetry of the first generation tree-level-massive fermions, 3 down quarks, the 3 up quarks, and the electron, so by the Sym factor the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV, the effective tauon mass should be renormalized from the energy level of 1 GeV at which the mass is 1.88 GeV to the energy level of 2 GeV. Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent, the effective tauon mass at 2 GeV would be about 1.78 GeV. The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.
The beauty quark corresponds to 21 triples. They are triples of the same form as the 7 tauon triples involving 1 and E, but for 1 and I, 1 and J, and 1 and K, which correspond to the red, green, and blue beauty quarks, respectively.

The seven red beauty quark triples correspond to the seven tauon triples, except that the beauty quark interacts with 6 Spin(0,5) gravitons while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times the third generation graviton factor $6/2 = 3$, so the red beauty quark mass is $m_b = 5.63111$ GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a constituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV. Therefore, the calculated beauty quark mass of 5.63 GeV corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant $\alpha_s$ is known. The conventional value of $\alpha_s$ at about 5 GeV is about 0.22.

Using $\alpha_s (5 \text{ GeV}) = 0.22$, a pole mass of 5.0 GeV gives an MSbar 1-loop beauty quark mass of 4.6 GeV, and an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV, the MSbar mass decreases by about 1.3 GeV, giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV, with error bars $\pm 0.25$ (stat) $\pm 0.34$ (frag) $\pm 0.27$ (theo).
The theoretical model calculated Beauty Quark mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value of the color force strength constant $\alpha_s$ at about 5 GeV is about 0.166, while the conventional value of the color force strength constant $\alpha_s$ at about 5 GeV is about 0.216, and the theoretical model calculated value of the color force strength constant $\alpha_s$ at about 90 GeV is about 0.106, while the conventional value of the color force strength constant $\alpha_s$ at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV), and a color force strength $\alpha_s$ at 5 GeV (0.166) such that $1 + \alpha_s = 1.166$ is about 4 percent lower than the conventional value of $1 + \alpha_s = 1.216$ at 5 GeV.

Triples of the type $\{1, I, J\}$, $\{I, J, K\}$, etc., do not correspond to the beauty quark, but to the truth quark. The truth quark corresponds to those $512 - 1 - 7 - 21 = 483$ triples, so the constituent mass of the red truth quark is $161 / 7 = 23$ times the red beauty quark mass, and the red T-quark mass is $m_t = 129.5155$ GeV.

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in the Cl(16)-E8 model. The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994. The Low and High Mass States of the Truth Quark have, in my opinion, also been observed by Fermilab (see Chapter 17 of this paper) but the Fermilab and CERN establishments disagree.

All other masses than the electron mass (which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value $v = 252.514$ GeV), including the Higgs scalar mass and Truth quark mass, are calculated (not assumed) masses in the Cl(16)-E8 model. These results when added up give a total mass of third generation fermion particles:

$$\Sigma f_3 = 1,629 \text{ GeV}$$
The second generation fermion particles correspond to pairs of octonions. There are $8^2 = 64$ such pairs.

The pair $\{1,1\}$ corresponds to the mu-neutrino.

The pairs $\{1,E\}$, $\{E,1\}$, and $\{E,E\}$ correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles:

The pair $\{E,E\}$ should correspond to the E electron.

The other two muon pairs have a symmetry group S2, which is 1/3 the size of the color symmetry group S3, which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of the $\{E,E\}$ electron mass and the $\{1,E\}$, $\{E,1\}$ symmetry mass, which is 1/3 of the up or down quark mass. Therefore, $\mu = 104.76$ MeV.

According to the 1998 Review of Particle Physics of the Particle Data Group, the experimental muon mass is about 105.66 MeV which may be consistent with radiative corrections for the calculated tree-level $\mu = 104.76$ MeV as Bailin and Love, in "Introduction to Gauge Field Theory", IOP (rev ed 1993), say: "... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to $\mu^5$ the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.
The red, blue and green strange quark each corresponds to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs \( \{ 1, i \}, \{ i, 1 \}, \{ i, i \} \)
because \( i \) is the red down quark.
Its mass should be the sum of two parts:
the \( \{ i, i \} \) red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.

Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.

Therefore the graviton factor for the second and third generations is \( 6/2 = 3 \).

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and
the red strange quark constituent mass is \( m_s = 312.75 \text{ MeV} + 312.75 \text{ MeV} = 625.5 \text{ MeV} \)

The blue strange quarks correspond to the three pairs involving \( j \),
the green strange quarks correspond to the three pairs involving \( k \),
and their masses are similarly determined to also be 625.5 MeV.
The charm quark corresponds to the remaining \( 64 - 1 - 3 - 9 = 51 \) pairs.

Therefore, the mass of the red charm quark should be the sum of two parts:
the \( \{ i, i \} \), red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor \( 51 / 9 \),
which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is
\( m_c = 312.75 \text{ MeV} + 1,772.25 \text{ MeV} = 2.085 \text{ GeV} \)

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a constituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.
The pole mass can be converted to an MSbar mass if the color force strength constant \( \alpha_s \) is known. The conventional value of \( \alpha_s \) at about 2 GeV is about 0.39, which is somewhat lower than the theoretical model value. Using \( \alpha_s (2 \text{ GeV}) = 0.39 \), a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

These results when added up give a total mass of second generation fermion particles:

\[
\Sigma_{f2} = 32.9 \text{ GeV}
\]
The Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

\[ Smf1 = 7.508 \text{ GeV}, \]

and the similar sums for second-generation and third-generation fermions, denoted by

\[ Smf2 = 32.94504 \text{ GeV} \text{ and } Smf3 = 1,629.2675 \text{ GeV}. \]

The resulting KM matrix is:

<table>
<thead>
<tr>
<th></th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.975</td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>0.222</td>
<td>0.00249</td>
</tr>
<tr>
<td>c</td>
<td>-0.222</td>
<td>-0.000161i</td>
</tr>
<tr>
<td>t</td>
<td>0.00698</td>
<td>-0.00378i</td>
</tr>
</tbody>
</table>
Below the energy level of ElectroWeak Symmetry Breaking
the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current $W^\pm$ interactions couple to the ... quarks with couplings given by ...

\[
\begin{align*}
V_{ud} & \quad V_{us} & \quad V_{ub} \\
V_{cd} & \quad V_{cs} & \quad V_{cb} \\
V_{td} & \quad V_{ts} & \quad V_{tb}
\end{align*}
\]

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix. It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from 
\[
V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0,
\]
by dividing each side by the best-known one, $V_{cd} V_{cb}^*$

\[\bar{\rho} + i \bar{\eta} = -(V_{ud} V_{ub}^*)/(V_{cd} V_{cb}^*)\] is phase-convention-independent ...

... $\sin 2\beta = 0.673 \pm 0.023 \ldots \alpha = 89.0 \pm 4.4 \ldots -4.2 \text{ degrees} \ldots \gamma = 73 \pm 22 \pm 25 \text{ degrees} \ldots$

The sum of the three angles of the unitarity triangle, $\alpha + \beta + \gamma = (183 \pm 22 \pm 25) \text{ degrees}$, is ... consistent with the SM expectation. ...
The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J, which is a phase-convention-independent measure of CP violation, defined by $\text{Im} \ V_{ij} V_{kl}^{*} V_{jk}^{*} = J \sum (m,n) \varepsilon_{ikm} \varepsilon_{jln}$.

The fit results for the magnitudes of all nine KM elements are ...

<table>
<thead>
<tr>
<th>Element</th>
<th>Magnitude</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{ud}$</td>
<td>0.97428 ± 0.00015</td>
<td></td>
</tr>
<tr>
<td>$V_{us}$</td>
<td>0.2253 ± 0.0007</td>
<td></td>
</tr>
<tr>
<td>$V_{ub}$</td>
<td>0.00347 ±0.00012</td>
<td></td>
</tr>
<tr>
<td>$V_{cd}$</td>
<td>0.2252 ± 0.0007</td>
<td></td>
</tr>
<tr>
<td>$V_{cs}$</td>
<td>0.97345 ±0.00016</td>
<td></td>
</tr>
<tr>
<td>$V_{cb}$</td>
<td>0.0410 ±0.0011</td>
<td></td>
</tr>
<tr>
<td>$V_{td}$</td>
<td>0.00862 ±0.00026</td>
<td></td>
</tr>
<tr>
<td>$V_{ts}$</td>
<td>0.0403 ±0.0011</td>
<td></td>
</tr>
<tr>
<td>$V_{tb}$</td>
<td>0.999152 ±0.000030</td>
<td></td>
</tr>
</tbody>
</table>

and the Jarlskog invariant is $J = (2.91 ±0.19-0.11) \times 10^{-5}$. ...".
Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed that in the Massless Realm the mixing matrix might be democratic.
the mass matrix ... MD ... of the type ... 1/3 x m x

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}
\]

... has name... "democratic" family mixing ...
the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

\[
\begin{pmatrix}
1/\sqrt(2) & -1/\sqrt(2) & 0 \\
1/\sqrt(6) & 1/\sqrt(6) & -2/\sqrt(6) \\
1/\sqrt(3) & 1/\sqrt(3) & 1/\sqrt(3)
\end{pmatrix}
\]
as A MD At =

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & m
\end{pmatrix}
\]

...".

Up in the Massless Realm you might just say that there is no mass matrix, just a democratic mixing matrix of the form 1/3 x

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}
\]

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking then you might as a first approximation use m = 1 so that all the mass first goes to the third generation as

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

which is physically like the Higgs being a T-Tbar quark condensate.
Consider a 3-dim Euclidean space of generations:

The case of mass only going to one generation can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

If mass only goes to one other generation that can be represented by a red line extending to a second dimension forming a small blue-red-black triangle

that can be extended by reflection to form six small triangles making up a large triangle

Each of the six component triangles has 30-60-90 angle structure:
If mass goes on further to all three generations that can be represented by a green line extending to a third dimension.

If you move the blue line from the top vertex to join the green vertex you get a small blue-red-green-gray-gray-gray tetrahedron that can be extended by reflection to form 24 small tetrahedra making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds to the 12+12 = 24 elements of the Binary Tetrahedral Group.
The basic blue-red-green triangle of the basic small tetrahedron

has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module"
with lengths

\[
\begin{align*}
V1.V2 &= (1/2) \ EL = \text{Half of the regular Tetrahedron's edge length.} \\
V1.V3 &= (1 / \sqrt{3}) \ EL = 0.577350269 \ EL \\
V1.V4 &= 3 / (2 \sqrt{6}) \ EL = 0.612372436 \ EL \\
V2.V3 &= 1 / (2 \sqrt{3}) \ EL = 0.288675135 \ EL \\
V2.V4 &= 1 / (2 \sqrt{2}) \ EL = 0.353553391 \ EL \\
V3.V4 &= 1 / (2 \sqrt{6}) \ EL = 0.204124145 \ EL
\end{align*}
\]

the Unitarity Triangle angles are:

\[
\begin{align*}
\beta &= V3.V1.V4 = \arccos(2 \sqrt{2}/3) = 19.471220634 \ degrees \ so \ \sin 2\beta = 0.6285 \\
\alpha &= V1.V3.V4 = 90 \ degrees \\
\gamma &= V1.V4.V3 = \arcsin(2 \sqrt{2}/3) = 70.528779366 \ degrees
\end{align*}
\]

which is substantially consistent with the 2010 Review of Particle Properties

\[
\begin{align*}
\sin 2\beta &= 0.673 \pm 0.023 \ so \ \beta = 21.1495 \ degrees \\
\alpha &= 89.0 + 4.4 - 4.2 \ degrees \\
\gamma &= 73 + 22 - 25 \ degrees
\end{align*}
\]

and so also consistent with the Standard Model expectation.
The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):

In the Cl(16)-E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by 
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted 
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.
The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

phase angle $\theta_{13} = \gamma = 70.529$ degrees

$$\sin(\theta_{12}) = s_{12} = \frac{m_e + 3m_d + 3m_{\mu}}{\sqrt{(m_e^2 + 3m_d^2 + 3m_{\mu}^2) + (m_{\mu}^2 + 3m_s^2 + 3m_c^2)}} = 0.222198$$

$$\sin(\theta_{13}) = s_{13} = \frac{m_e + 3m_d + 3m_{\mu}}{\sqrt{(m_e^2 + 3m_d^2 + 3m_{\mu}^2) + (m_{\tau}^2 + 3m_b^2 + 3m_t^2)}} = 0.004608$$

$$\sin(\theta_{23}) = s_{23} = \frac{m_{\mu} + 3m_s + 3m_c}{\sqrt{(m_{\tau}^2 + 3m_b^2 + 3m_t^2) + (m_{\mu}^2 + 3m_s^2 + 3m_c^2)}}$$

$$\sin(\theta_{23}) = s_{23} = \sin(\theta_{23}) \sqrt{\frac{\Sigma f_2}{\Sigma f_1}} = 0.04234886$$

The factor $\sqrt{\frac{\Sigma f_2}{\Sigma f_1}}$ appears in $s_{23}$ because an $s_{23}$ transition is to the second generation and not all the way to the first generation, so that the end product of an $s_{23}$ transition has a greater available energy than $s_{12}$ or $s_{13}$ transitions by a factor of $\frac{\Sigma f_2}{\Sigma f_1}$.

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an $s_{23}$ transition has greater available energy than the $s_{12}$ or $s_{13}$ transitions by a factor of $\frac{\Sigma f_2}{\Sigma f_1}$

the effective magnitude of the $s_{23}$ terms in the KM entries is increased by the factor $\sqrt{\frac{\Sigma f_2}{\Sigma f_1}}$.

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\
0 & -\sin(\theta_{23}) & \cos(\theta_{23})
\end{bmatrix}
$$

$$
\begin{bmatrix}
\cos(\theta_{13}) & 0 & \sin(\theta_{13})\exp(-i\,d_{13}) \\
0 & 1 & 0 \\
-\sin(\theta_{13})\exp(i\,d_{13}) & 0 & \cos(\theta_{13})
\end{bmatrix}
$$

$$
\begin{bmatrix}
\cos(\theta_{12}) & \sin(\theta_{12}) & 0 \\
-\sin(\theta_{12}) & \cos(\theta_{12}) & 0 \\
0 & 0 & 1
\end{bmatrix}
$$
The resulting Kobayashi-Maskawa parameters for W+ and W- charged weak boson processes, are:

\[
\begin{array}{ccc}
  d & s & b \\
  u & 0.975 & 0.222 & 0.00249 -0.00388i \\
  c & -0.222 -0.000161i & 0.974 -0.0000365i & 0.0423 \\
  t & 0.00698 -0.00378i & -0.0418 -0.00086i & 0.999 \\
\end{array}
\]

The matrix is labelled by either \((u \ c \ t)\) input and \((d \ s \ b)\) output, or, as above, \((d \ s \ b)\) input and \((u \ c \ t)\) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either \((u \ c \ t)\) input and \((u'c't')\) output, or, as below, \((d \ s \ b)\) input and \((d's'b')\) output:

\[
\begin{array}{ccc}
  d & s & b \\
  d' & 0.975 & 0.222 & 0.00249 -0.00388i \\
  s' & -0.222 -0.000161i & 0.974 -0.0000365i & 0.0423 \\
  b' & 0.00698 -0.00378i & -0.0418 -0.00086i & 0.999 \\
\end{array}
\]

Since neutrinos of all three generations are massless at tree level, the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model, the only source of CP violation is the Kobayashi-Maskawa (KM) phase ... The study of CP violation is, at last, experiment driven. ... The CKM matrix provides a consistent picture of all the measured flavor and CP violating processes. ... There is no signal of new flavor physics. ... Very likely, the KM mechanism is the dominant source of CP violation in flavor changing processes. ... The result is consistent with the SM predictions. ..."."
12. Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev, does not often swap places with a 2.09 Gev charm sea quark, but a 313 Mev down valence quark can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force constituent mass of the down valence quark is heavier by about

\[(m_s - m_d) \left( \frac{m_d}{m_s} \right)^2 a(w) |V_{ds}| = 312 \times 0.25 \times 0.253 \times 0.22 \text{ Mev} = 4.3 \text{ Mev},\]

(where \(a(w) = 0.253\) is the geometric part of the weak force strength and \(|V_{ds}| = 0.22\) is the magnitude of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass \(Q_{md}\) of the down quark is

\[Q_{md} = 312.75 + 4.3 = 317.05 \text{ MeV}.\]

Similarly, the up quark Quantum color force mass increase is about

\[(m_c - m_u) \left( \frac{m_u}{m_c} \right)^2 a(w) |V_{uc}| = 1777 \times 0.022 \times 0.253 \times 0.22 \text{ Mev} = 2.2 \text{ Mev},\]

(where \(|V_{uc}| = 0.22\) is the magnitude of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass \(Q_{mu}\) of the up quark is

\[Q_{mu} = 312.75 + 2.2 = 314.95 \text{ MeV}.\]

Therefore, the Quantum color force Neutron-Proton mass difference is

\[m_N - m_P = Q_{md} - Q_{mu} = 317.05 \text{ Mev} - 314.95 \text{ Mev} = 2.1 \text{ Mev}.\]

Since the electromagnetic Neutron-Proton mass difference is roughly

\[m_N - m_P = -1 \text{ MeV}\]

the total theoretical Neutron-Proton mass difference is

\[m_N - m_P = 2.1 \text{ Mev} - 1 \text{ Mev} = 1.1 \text{ Mev},\]

an estimate that is comparable to the experimental value of 1.3 Mev.
13. Pion as Sine-Gordon Breather

The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Schwinger Source Kerr-Newman Black Hole with constituent mass M 312 MeV.

The antiquark is also a Schwinger Source Kerr-Newman Black Hole, with constituent mass M 312 MeV.

According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):
"... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...
Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ...

... toroidal stage just after merger ...

At merger, the two pincers join to form a single ... toroidal black hole.
The inner hole of the torus subsequently begins to close up (superluminally) ... If the closing proceeds to completion, it produces first a peanut shaped black hole and finally a spherical black hole. ..."

In the physical case of quark and antiquark forming a pion, the toroidal black hole remains a torus. The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):
"... The surface $r = r^+$ is ... the event horizon ... and is a null surface ...

... On the surface $r = r^+$ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ..."
A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather. The soliton and antisoliton of a Sine-Gordon Breather correspond to the quark and antiquark that make up the pion, analoguous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985), where he writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

\[ L = \frac{1}{B^2} \left( \frac{1}{2} (df)^2 + A (\cos(f) - 1) \right) \]

Coleman says: "... We see that, in classical physics, B is an irrelevant parameter: if we can solve the sine-Gordon equation for any non-zero B, we can solve it for any other B. The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [eq. 4.4]"

\[ L / \hbar = \frac{1}{(B^2 \hbar)} \left( \frac{1}{2} (df)^2 + A (\cos(f) - 1) \right) \]

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ... the classical limit, vanishing \( \hbar \), is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set \( \hbar \) equal to one. ..."...

... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

\[ f(x, t) = \frac{4}{B} \arctan \left( \frac{n \sin(w t)}{\cosh(n w x)} \right) \]

where [eq. 4.60] \( n = \sqrt{A - w^2} / w \) and \( w \) ranges from 0 to A. This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As \( \sin(w t) \) increases, the soliton and antisoliton move farther apart from each other. When \( \sin(w t) \) passes through one, they turn around and begin to approach one another. As \( \sin(w t) \) comes down to zero ... the soliton and antisoliton are on top of each other ... when \( \sin(w t) \) becomes negative .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]"

\[ E = 2M \sqrt{1 - \left( \frac{w^2}{A} \right)} \]

where [eq. 4.65] \( M = 8 \sqrt{A} / B^2 \) is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass, as we would expect from a soliton-antisoliton pair. ...

...[ found that ]... there is only a single series of bound states, labeled by the integer N ...
The energies ... are ... [ eq. 4.82 ]

$$E_N = 2 M \sin \left( \frac{B'^2 N}{16} \right)$$

where \( N = 0, 1, 2 \ldots < \frac{8 \pi}{B'^2} \) , [ eq. 4.83 ]

\( B'^2 = B^2 / \left( 1 - \frac{B^2}{8 \pi} \right) \) and \( M \) is the soliton mass.

\( M \) is not given by Eq. (4.65), but is the soliton mass corrected by the DHN formula, or, equivalently, by the first-order weak coupling expansion. ...

I have written the equation in this form .. to eliminate \( A \), and thus avoid worries about renormalization conventions.

Note that the DHN formula is identical to the Bohr-Sommerfeld formula, except that \( B \) is replaced by \( B' \). ...

Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter family of periodic motions, labeled by the period, \( T \),
then an energy eigenstate occurs whenever [ eq. 4.66 ]

$$\left[ \int_0^T dt \, p \, qdot = 2 \pi N, \right.$$

where \( N \) is an integer. ... Eq.( 4.66 ) is cruder than the WKB formula, but it is much more general;
it is always the leading approximation for any dynamical system ...

Dashen et al speculate that Eq. (4.82) is exact. ...

the sine-Gordon equation is equivalent ... to the massive Thirring model.

This is surprising, because the massive Thirring model is a canonical field theory whose Hamiltonian is expressed in terms of fundamental Fermi fields only.

Even more surprising, when \( B'^2 = 4 \pi \), that sine-Gordon equation is equivalent to a free massive Dirac theory, in one spatial dimension. ...

Furthermore, we can identify the mass term in the Thirring model with the sine-Gordon interaction, [ eq. 5.13 ]

$$M = - \left( \frac{A}{B^2} \right) N_m \cos (B \, f)$$

.. to do this consistently ... we must say [ eq. 5.14 ]

$$B^2 / (4 \pi) = 1 / \left( 1 + g / \pi \right)$$

...[where]... \( g \) is a free parameter, the coupling constant [ for the Thirring model ]...

Note that if \( B^2 = 4 \pi \), \( g = 0 \),

and the sine-Gordon equation is the theory of a free massive Dirac field. ...

It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.

Certainly this could not happen in three dimensions, where it would be forbidden by the spin-statistics theorem.

However, there is no spin-statistics theorem in one dimension, for the excellent reason that there is no spin. ...

the lowest fermion-antifermion bound state of the massive Thirring model is an obvious candidate for the fundamental meson of sine-Gordon theory. ...

equation (4.82) predicts that all the doublet bound states disappear when \( B'^2 \) exceeds \( 4 \pi \).
This is precisely the point where
the Thirring model interaction switches from attractive to repulsive. ... 
these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ... 
define identical physics. ... 
I have computed the predictions of ...[various]... approximation methods 
for the ration of the soliton mass to the meson mass for three values of B^2 : 
4 pi (where the qualitative picture of the soliton as a lump totally breaks down), 
2 pi, and pi . At 4 pi we know the exact answer ... 
I happen to know the exact answer for 2 pi, so I have included this in the table. ... 

<table>
<thead>
<tr>
<th>Method</th>
<th>B^2 = pi</th>
<th>B^2 = 2 pi</th>
<th>B^2 = 4 pi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zeroth-order weak coupling expansion eq2.13b</td>
<td>2.55</td>
<td>1.27</td>
<td>0.64</td>
</tr>
<tr>
<td>Coherent-state variation</td>
<td>2.55</td>
<td>1.27</td>
<td>0.64</td>
</tr>
<tr>
<td>First-order weak coupling expansion</td>
<td>2.23</td>
<td>0.95</td>
<td>0.32</td>
</tr>
<tr>
<td>Bohr-Sommerfeld eq4.64</td>
<td>2.56</td>
<td>1.31</td>
<td>0.71</td>
</tr>
<tr>
<td>DHN formula eq4.82</td>
<td>2.25</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>Exact</td>
<td>?</td>
<td>1.00</td>
<td>0.50</td>
</tr>
</tbody>
</table>

...[eq. 2.13b ] 
E = 8 sqrt(A) / B^2 
...[ is the ]... energy of the lump ... of sine-Gordon theory ... 
frequently called 'soliton...' in the literature ... 
[ Zeroth-order is the classical case, or classical limit. ] ... 
... Coherent-state variation always gives 
the same result as the ... Zeroth-order weak coupling expansion ... . 
The ... First-order weak-coupling expansion ... 
explicit formula ... is ( 8 / B^2 ) - ( 1 / pi ). ...".

Using the Cl(16)-E8 model constituent mass of the Up and Down quarks and 
antiququarks, about 312.75 MeV, as the soliton and antisoliton masses, 
and setting B^2 = pi and using the DHN formula, 
the mass of the charged pion is calculated to be ( 312.75 / 2.25 ) MeV = 139 MeV 
which is close to the experimental value of about 139.57 MeV.

**Why is the value B^2 = pi the special value that gives the pion mass ?**
(or, using Coleman's eq. ( 5.14 ), the Thirring coupling constant g = 3 pi )
**Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the ( probably exact ) DHN formula.** In other words, 
The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.
14. Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos:
\( \nu_e \) (electron neutrino); \( \nu_m \) (muon neutrino); \( \nu_t \)
and three neutrino mass states: \( \nu_1 \); \( \nu_2 \); \( \nu_3 \)
and the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state \( \nu_3 \) corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry
space, lying entirely therein. According to the Cl(16)–E8 model
the mass of \( \nu_3 \) is zero at tree-level
but it picks up a first-order correction
propagating entirely through internal symmetry space by merging
with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at beginning and ending points
so
the first-order corrected mass of \( \nu_3 \) is given by
\[
M_{\nu_3} \times (1/\sqrt{2}) = M_e \times GW(m_{\text{proton}}^2) \times \alpha_E
\]
where the factor \((1/\sqrt{2})\) comes from the \(U_3\) component
of the neutrino mixing matrix
so that
\[
M_{\nu_3} = \sqrt{2} \times M_e \times GW(m_{\text{proton}}^2) \times \alpha_E
= 1.4 \times 5 \times 10^5 \times 1.05 \times 10^{-5} \times (1/137) \text{ eV} =
= 7.35 / 137 = 5.4 \times 10^{-2} \text{ eV}.
\]

The neutrino-plus-electron loop can be anchored by weak force
action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are \(6 \times 6 = 36\) different possible anchorings.
The intermediate mass state $\nu_2$ corresponds to a neutrino whose propagation begins or ends in CP2 internal symmetry space and ends or begins in M4 physical Minkowski spacetime, thus having only one point (either beginning or ending) lying in CP2 internal symmetry space where it can act not merely as a point but as a point plus an electron loop.

According to the $\mathrm{Cl}(16)$-E8 model the mass of $\nu_2$ is zero at tree-level but it picks up a first-order correction at only one (but not both) of the beginning or ending points so that so that there are 6 different possible anchorings for $\nu_2$ first-order corrections, as opposed to the 36 different possible anchorings for $\nu_3$ first-order corrections, so that the first-order corrected mass of $\nu_2$ is less than the first-order corrected mass of $\nu_3$ by a factor of 6, so

the first-order corrected mass of \(\nu_2\) is
\[
M_{\nu_2} = \frac{M_{\nu_3}}{\mathrm{Vol}(\mathrm{CP2})} = \frac{5.4 \times 10^{-2}}{6} = 9 \times 10^{-3}\text{eV}.
\]

The low mass state $\nu_1$ corresponds to a neutrino whose propagation begins and ends in physical Minkowski spacetime, thus having only one anchoring to CP2 internal symmetry space.

According to the $\mathrm{Cl}(16)$-E8 model the mass of $\nu_1$ is zero at tree-level but it has only 1 possible anchoring to CP2 as opposed to the 36 different possible anchorings for $\nu_3$ first-order corrections or the 6 different possible anchorings for $\nu_2$ first-order corrections so that the first-order corrected mass of $\nu_1$ is less than the first-order corrected mass of $\nu_2$ by a factor of 6, so

the first-order corrected mass of $\nu_1$ is
\[
M_{\nu_1} = \frac{M_{\nu_2}}{\mathrm{Vol}(\mathrm{CP2})} = \frac{9 \times 10^{-3}}{6} = 1.5 \times 10^{-3}\text{eV}.
\]
Therefore:

the mass-squared difference \( D(M_{23}^2) = M_{\nu_3}^2 - M_{\nu_2}^2 = \)
\[
= (2916 - 81) \times 10^{-6} \text{ eV}^2 = \\
= 2.8 \times 10^{-3} \text{ eV}^2
\]

and

the mass-squared difference \( D(M_{12}^2) = M_{\nu_2}^2 - M_{\nu_1}^2 = \)
\[
= (81 - 2) \times 10^{-6} \text{ eV}^2 = \\
= 7.9 \times 10^{-5} \text{ eV}^2
\]

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix \( U \)

\[
\begin{array}{ccc}
\text{nu}_1 & \text{nu}_2 & \text{nu}_3 \\
\text{nu}_e & U_{e1} & U_{e2} & U_{e3} \\
\text{nu}_m & U_{m1} & U_{m2} & U_{m3} \\
\text{nu}_t & U_{t1} & U_{t2} & U_{t3} \\
\end{array}
\]

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

\[
U = c_{12} c_{13} - s_{12} c_{23} s_{13} \text{eid} \\
\quad - c_{12} s_{23} s_{13} \text{eid} \\
\quad - c_{12} c_{23} s_{13} \text{eid} \\
\quad s_{12} s_{23} \\
\quad c_{12} c_{23} - s_{12} s_{23} s_{13} \text{eid} \\
\quad - c_{12} s_{23} - c_{12} c_{23} s_{13} \text{eid} \\
\quad c_{23} c_{13}
\]

where \( c_{ij} = \cos(\theta_{ij}) \), \( s_{ij} = \sin(\theta_{ij}) \)
The angles are

\[ \theta_{23} = \pi/4 = 45 \text{ degrees} \]

because
nu_3 has equal components of nu_m and nu_t so
that \( U_{m3} = U_{t3} = 1/\sqrt{2} \) or, in conventional
notation, mixing angle \( \theta_{23} = \pi/4 \)
so that \( \cos(\theta_{23}) = 0.707 = \sqrt{2}/2 = \sin(\theta_{23}) \)

\[ \theta_{13} = 9.594 \text{ degrees} = \arcsin(1/6) \]
and \( \cos(\theta_{13}) = 0.986 \)
because \( \sin(\theta_{13}) = 1/6 = 0.167 = |U_{e3}| = \text{fraction of } \nu_3 \text{ that is } \nu_e \)

\[ \theta_{12} = \pi/6 = 30 \text{ degrees} \]
because
\( \sin(\theta_{12}) = 0.5 = 1/2 = U_{e2} = \text{fraction of } \nu_2 \text{ begin/end points} \)
that are in the physical spacetime where massless \( \nu_e \) lives
so that \( \cos(\theta_{12}) = 0.866 = \sqrt{3}/2 \)

\[ d = 70.529 \text{ degrees is the Dirac CP violation phase} \]
\[ e^{i d} = \cos(70.529) + i \sin(70.529) = 0.333 + 0.943 i \]
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
\[ \beta = V_{31} V_{31} V_{41} = \arccos( 2 \sqrt{2} / 3 ) \approx 19.471 \ 220 \ 634 \text{ degrees so } \sin 2\beta = 0.6285 \]
\[ \alpha = V_{13} V_{31} V_{41} = 90 \text{ degrees} \]
\[ \gamma = V_{14} V_{41} V_{31} = \arcsin( 2 \sqrt{2} / 3 ) \approx 70.528 \ 779 \ 366 \text{ degrees} \]

The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):
Then we have for the neutrino mixing matrix:

\[
\begin{array}{ccc}
\text{nu}_1 & \text{nu}_2 & \text{nu}_3 \\
\text{nu}_e & 0.866 \times 0.986 & 0.50 \times 0.986 & 0.167 \times \text{e-id} \\
\text{nu}_m & -0.5 \times 0.707 & 0.866 \times 0.707 & 0.707 \times 0.986 \\
& -0.866 \times 0.707 \times 0.167 \times \text{e-id} & -0.5 \times 0.707 \times 0.167 \times \text{e-id} \\
\text{nu}_t & 0.5 \times 0.707 & -0.866 \times 0.707 & 0.707 \times 0.986 \\
& -0.866 \times 0.707 \times 0.167 \times \text{e-id} & -0.5 \times 0.707 \times 0.167 \times \text{e-id} \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{nu}_1 & \text{nu}_2 & \text{nu}_3 \\
\text{nu}_e & 0.853 & 0.493 & 0.167 \times \text{e-id} \\
\text{nu}_m & -0.354 & 0.612 & 0.697 \\
& -0.034 - 0.096 \times \text{i} & -0.020 - 0.056 \times \text{i} \\
\text{nu}_t & 0.354 & -0.612 & 0.697 \\
& -0.034 - 0.096 \times \text{i} & -0.020 - 0.056 \times \text{i} \\
\end{array}
\]

Since \( \text{ei}(70.529) = \cos(70.529) + \text{i} \sin(70.529) = 0.333 + 0.943 \times \text{i} \)
and \( 0.333 \times \text{e-i}(70.529) = \cos(70.529) - \text{i} \sin(70.529) = 0.333 - 0.943 \times \text{i} \)

\[
\begin{array}{ccc}
\text{nu}_1 & \text{nu}_2 & \text{nu}_3 \\
\text{nu}_e & 0.853 & 0.493 & 0.056 - 0.157 \times \text{i} \\
\text{nu}_m & -0.354 & 0.612 & 0.697 \\
& -0.034 - 0.096 \times \text{i} & -0.020 - 0.056 \times \text{i} \\
\text{nu}_t & 0.354 & -0.612 & 0.697 \\
& -0.034 - 0.096 \times \text{i} & -0.020 - 0.056 \times \text{i} \\
\end{array}
\]

for a result of

\[
\begin{array}{ccc}
\text{nu}_1 & \text{nu}_2 & \text{nu}_3 \\
\text{nu}_e & 0.853 & 0.493 & 0.056 - 0.157 \times \text{i} \\
\text{nu}_m & -0.388 - 0.096 \times \text{i} & 0.592 - 0.056 \times \text{i} & 0.697 \\
\text{nu}_t & 0.320 - 0.096 \times \text{i} & 0.632 - 0.056 \times \text{i} & 0.697 \\
\end{array}
\]

which is consistent with the approximate experimental values of mixing angles
shown in the Michaelmas Term 2010 Particle Physics handout
of Prof Mark Thomson if the matrix is modified by taking into account
the March 2012 results from Daya Bay
observing non-zero theta_13 = 9.54 degrees.
15. Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex.

Once a Planck-mass black hole is formed, it is stable in the E8 model. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles for a total of 64 particle-antiparticle pairs. Of the 64 particle-antiparticle pairs, 12 are bosonic pions.

A typical combination should have about 6 pions so it should have a mass of about $0.14 \times 6 \text{ GeV} = 0.84 \text{ GeV}$.

Just as the pion mass of $0.14$ GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as $0.1$ GeV, and if the typical combination has one such pair and 4 other pions, then the typical combination could have a mass in the range of $0.66$ GeV.

Summing over all $2^{64}$ combinations, the total mass of a one-vertex universe should give a Planck mass roughly around $0.66 \times 2^{64} = 1.217 \times 10^{19}$ GeV.

The value for the Planck mass given in by the 1998 Particle Data Group is $1.221 \times 10^{19}$ GeV.
16. Force Strength and Boson Mass Calculation

Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the Cl(16)-E8 model Lagrangian (as the D4xD4 subalgebra of the D8 subalgebra of E8)

\[ \int \text{GG SM} + \text{Fermion Particle-AntiParticle} + \text{Higgs} \]

4-dim M4

with the Higgs term coming from integrating over the CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein by the Mayer-Trautman Mechanism

This shows that the Force Strength is made up of two parts:

- the relevant spacetime manifold of gauge group global action
- the relevant symmetric space manifold of gauge group local action.

The 4-dim spacetime Lagrangian GG SM gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally
of the gauge force term of the gauge boson acting locally
for the gauge bosons of each of the four forces:

- U(1) for electromagnetism
- SU(2) for weak force
- SU(3) for color force
- Spin(5) - compact version of antiDeSitter Spin(2,3) subgroup of Conformal Spin(2,4) for gravity by the MacDowell-Mansouri mechanism.

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit ( like g |F|^2 ) or an implicit ( incorporated into the |F|^2 ) form.
Either way, the conventional picture is that the force strength g is an ad hoc inclusion.

The Cl(16)-E8 model does not put in force strength g ad hoc,
but constructs the integral such that
the force strength emerges naturally from the geometry of each gauge force.
To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:
   - the U(1) photon sees 4-dim spacetime as \( T^4 = S_1 \times S_1 \times S_1 \times S_1 \)
   - the SU(2) weak boson sees 4-dim spacetime as \( S_2 \times S_2 \)
   - the SU(3) weak boson sees 4-dim spacetime as CP2
   - the Spin(5) of gravity sees 4-dim spacetime as S4

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson. The nontrivial Shilov boundaries are:
   - for SU(2) Shilov = RP^1xS^2
   - for SU(3) Shilov = S^5
   - for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space
- S1 for U(1)
- \( S_2 = SU(2)/U(1) = Spin(3)/Spin(2) \) for SU(2)
- CP2 = \( SU(3)/SU(2) \times U(1) \) for SU(3)
- \( S_4 = Spin(5)/Spin(4) \) for Spin(5)

Each gauge group is the local symmetry of a symmetric space
- U(1) for itself
- SU(2) for Spin(5) / SU(2)xU(1)
- SU(3) for SU(4) / SU(3)xU(1)
- Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains
- SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
- SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
- Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries
- SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
- SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
- Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4
Very roughly, think of the force strength as integral over global symmetry space of physical (i.e., Shilov Boundary) volume = strength of the force.

That is:
The geometric strength of the force is given by the product of the volume of a 4-dimensional thing with global symmetry of the force and the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff), you see that roughly:

- Volume product for gravity is the largest volume
- So since (as Feynman says) force strength = probability to emit a gauge boson means that the highest force strength or probability should be 1
- The gravity volume product is normalized to be 1, and so (approximately):
  
  \[
  \begin{align*}
  \text{Volume product for gravity} &= 1 \\
  \text{Volume product for color} &= 2/3 \\
  \text{Volume product for weak} &= 1/4 \\
  \text{Volume product for electromagnetism} &= 1/137
  \end{align*}
  \]

There are two further main components of a force strength:

1. For massive gauge bosons, a suppression by a factor of 1/M^2
2. Renormalization running (important for color force)

Consider Massive Gauge Bosons:

Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes, so that the geometric strength of gravity should be reduced by 1/Mp^2

The weak force is carried by weak bosons, so that the geometric strength of the weak force should be reduced by 1/MW^2

That gives the result (approximate):

\[
\begin{align*}
\text{gravity strength} &= G \ (\text{Newton’s } G) \\
\text{color strength} &= 2/3 \\
\text{weak strength} &= G_F \ (\text{Fermi’s weak force } G) \\
\text{electromagnetism} &= 1/137
\end{align*}
\]
Consider Renormalization Running for the Color Force:: That gives the result:

\[
\begin{align*}
\text{gravity strength} &= G \ (\text{Newton's } G) \\
\text{color strength} &= 1/10 \ \text{at weak boson mass scale} \\
\text{weak strength} &= G_F \ (\text{Fermi's weak force } G) \\
\text{electromagnetism} &= 1/137
\end{align*}
\]

The use of compact volumes is itself a calculational device, because it would be more nearly correct, instead of the integral over the compact global symmetry space of the compact physical (ie Shilov Boundary) volume=strength of the force to use the integral over the hyperbolic spacetime global symmetry space of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios, and the compact volumes (finite and easy to look up in the book by Hua) have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices, and that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.
Here are less approximate more detailed force strength calculations:

The force strength of a given force is

$$\alpha_{\text{force}} = \left( \frac{1}{\text{M}_{\text{force}}^2} \right) \left( \frac{\text{Vol} (\text{MIS}_{\text{force}})}{\text{Vol} (\text{Q}_{\text{force}})} \right) \left( \frac{\text{Vol} (\text{D}_{\text{force}})}{\text{Vol} (\text{D}_{\text{force}})^{\frac{1}{m_{\text{force}}}}} \right)$$

where:

- $\alpha_{\text{force}}$ represents the force strength;
- $\text{M}_{\text{force}}$ represents the effective mass;
- $\text{MIS}_{\text{force}}$ represents the relevant part of the target Internal Symmetry Space;
- $\text{Vol} (\text{MIS}_{\text{force}})$ stands for volume of $\text{MIS}_{\text{force}}$ and is sometimes also denoted by $\text{Vol} (\text{M})$;
- $\text{Q}_{\text{force}}$ represents the link from the origin to the relevant target for the gauge boson;
- $\text{Vol} (\text{Q}_{\text{force}})$ stands for volume of $\text{Q}_{\text{force}}$;
- $\text{D}_{\text{force}}$ represents the complex bounded homogeneous domain of which $\text{Q}_{\text{force}}$ is the Shilov boundary;
- $m_{\text{force}}$ is the dimensionality of $\text{Q}_{\text{force}}$, which is
  - 4 for Gravity and the Color force,
  - 2 for the Weak force (which therefore is considered to have two copies of $\text{Q}_{\text{W}}$ for SpaceTime),
  - 1 for Electromagnetism (which therefore is considered to have four copies of $\text{Q}_{\text{E}}$ for SpaceTime);
- $\text{Vol} (\text{D}_{\text{force}})^{\frac{1}{m_{\text{force}}}}$ stands for a dimensional normalization factor (to reconcile the dimensionality of the Internal Symmetry Space of the target vertex with the dimensionality of the link from the origin to the target vertex).

The $\text{Q}_{\text{force}}$, Hermitian symmetric space, and $\text{D}_{\text{force}}$ manifolds for the four forces are:

<table>
<thead>
<tr>
<th>Force Force</th>
<th>Manifold</th>
<th>Symbol</th>
<th>Dimensionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin(5)</td>
<td>Spin(7) / Spin(5)xU(1)</td>
<td>IV5</td>
<td>4</td>
</tr>
<tr>
<td>SU(3)</td>
<td>SU(4) / SU(3)xU(1)</td>
<td>B^6(ball)</td>
<td>4</td>
</tr>
<tr>
<td>SU(2)</td>
<td>Spin(5) / SU(2)xU(1)</td>
<td>IV3</td>
<td>2</td>
</tr>
<tr>
<td>U(1)</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>
The geometric volumes needed for the calculations are mostly taken from the book *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains* (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

<table>
<thead>
<tr>
<th>Force</th>
<th>M</th>
<th>Vol(M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity</td>
<td>$S^4$</td>
<td>$8\pi^2/3$ - $S^4$ is 4-dimensional</td>
</tr>
<tr>
<td>color</td>
<td>$CP^2$</td>
<td>$8\pi^2/3$ - $CP^2$ is 4-dimensional</td>
</tr>
<tr>
<td>weak</td>
<td>$S^2 \times S^2$</td>
<td>$2 \times 4\pi$ - $S^2$ is a 2-dim boundary of 3-dim ball</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-dim $S^2 \times S^2 = $ topological boundary of 6-dim 2-polyball</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shilov Boundary of 6-dim 2-polyball = $S^2 + S^2 =$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 2-dim surface frame of 4-dim $S^2 \times S^2$</td>
</tr>
<tr>
<td>e-mag</td>
<td>$T^4$</td>
<td>$4 \times 2\pi$ - $S^1$ is 1-dim boundary of 2-dim disk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-dim $T^4 = S^1 \times S^1 \times S^1 \times S^1 = $ topological boundary of 8-dim 4-polydisk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shilov Boundary of 8-dim 4-polydisk = $S^1 + S^1 + S^1 + S^1 =$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 1-dim wire frame of 4-dim $T^4$</td>
</tr>
</tbody>
</table>

Note (thanks to Carlos Castro for noticing this) also that the volume listed for $CP^2$ is unconventional, but physically justified by noting that $S^4$ and $CP^2$ can be seen as having the same physical volume, with the only difference being structure at infinity. Note that for $U(1)$ electromagnetism, whose photon carries no charge, the factors $Vol(Q)$ and $Vol(D)$ do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral $U(1)$ photons of Electromagnetism, so we take $QE$ and $DE$ to be equal to unity.

<table>
<thead>
<tr>
<th>Force</th>
<th>M</th>
<th>Vol(M)</th>
<th>Q</th>
<th>Vol(Q)</th>
<th>D</th>
<th>Vol(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity</td>
<td>$S^4$</td>
<td>$8\pi^2/3$</td>
<td>$RP^1 \times S^4$</td>
<td>$8\pi^3/3$</td>
<td>$IV5$</td>
<td>$\pi^5/2^4$ 5!</td>
</tr>
<tr>
<td>color</td>
<td>$CP^2$</td>
<td>$8\pi^2/3$</td>
<td>$S^5$</td>
<td>$4\pi^3$</td>
<td>$B^6(ball)$</td>
<td>$\pi^3/6$</td>
</tr>
<tr>
<td>Weak</td>
<td>$S^2 \times S^2$</td>
<td>$2 \times 4\pi$</td>
<td>$RP^1 \times S^2$</td>
<td>$4\pi^2$</td>
<td>$IV3$</td>
<td>$\pi^3/24$</td>
</tr>
<tr>
<td>e-mag</td>
<td>$T^4$</td>
<td>$4 \times 2\pi$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note (thanks to Carlos Castro for noticing this) that the volume listed for $S^5$ is for a squashed $S^5$, a Shilov boundary of the complex domain corresponding to the symmetric space $SU(4) / SU(3) \times U(1)$.
Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

<table>
<thead>
<tr>
<th>Force Group</th>
<th>Force Type</th>
<th>Characteristic Energy Level</th>
<th>Force Strength</th>
<th>(G G m_{\text{proton}}^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin(5)</td>
<td>gravity</td>
<td>approx 10^{19} GeV</td>
<td>1</td>
<td>(5 \times 10^{-39})</td>
</tr>
<tr>
<td>SU(3)</td>
<td>color</td>
<td>approx 245 MeV</td>
<td>0.6286</td>
<td>0.6286</td>
</tr>
<tr>
<td>SU(2)</td>
<td>weak</td>
<td>approx 100 GeV</td>
<td>0.2535</td>
<td>(1.05 \times 10^{-5})</td>
</tr>
<tr>
<td>U(1)</td>
<td>e-mag</td>
<td>approx 4 KeV</td>
<td>(1/137.03608)</td>
<td>(1/137.03608)</td>
</tr>
</tbody>
</table>

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels. The effect is particularly pronounced with the color force. The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

<table>
<thead>
<tr>
<th>Energy Level</th>
<th>Color Force Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>245 MeV</td>
<td>0.6286</td>
</tr>
<tr>
<td>5.3 GeV</td>
<td>0.166</td>
</tr>
<tr>
<td>34 GeV</td>
<td>0.121</td>
</tr>
<tr>
<td>91 GeV</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV.
Higgs:

As with forces strengths, the calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.

In the Cl(16)-E8 model, the value of the fundamental mass scale vacuum expectation value $v = \langle \Phi \rangle$ of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, $W^+$, $W^-$, and $Z^0$, whose tree-level masses will then be shown by ratio calculations to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively, and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and $v$ is given by the Ginzburg-Landau term from the Mayer Mechanism as

$$(1/4) \, \text{Tr} \left( [ \Phi, \Phi ] - \Phi \right)^2$$

or, in the notation of quant-ph/9806009 by Guang-jiong Ni

$$(1/4!) \, \lambda \, \Phi^4 - (1/2) \, \sigma \, \Phi^2$$

where the Higgs mass $M_H = \sqrt{2 \, \sigma}$

Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:

$$\lambda = 3 \left( \frac{M_H}{\Phi} \right)^2$$

which remains unchanged irrespective of the order ...".

Since $\langle \Phi \rangle^2 = v^2$, and assuming that $\lambda = ( \cos( \pi / 6 ) )^2 = 0.866^2$ (a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165) we have

$$\frac{M_H^2}{v^2} = \frac{( \cos( \pi / 6 ) )^2}{3}$$

In the Cl(16)-E8 model, the fundamental mass scale vacuum expectation value $v$ of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and $v$ is set to be 252.514 GeV so that

$$M_H = v \, \cos( \pi / 6 ) / \sqrt{1 / 3} = 126.257 \, \text{GeV}$$

This is the value of the Low Mass State of the Higgs observed by the LHC. Middle and High Mass States come from a Higgs-Tquark Condensate System. The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.
A Non-Condensate Higgs is represented by a Higgs at a point in M4 that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass:

$$\text{Higgs} \quad \text{in CP2 Internal Symmetry Space}$$

$$\text{mass} = 145 \quad \text{Non-Condensate Higgs Mass} = 145$$

$$\text{Higgs} \quad \text{in M4 spacetime}$$

and the value of lambda is $1 = 1^2$

so that the Higgs mass would be $M_H = v / \sqrt{3} = 145.789 \text{ GeV}$

However, in the Cl(16)-E8 model, the Higgs has structure of a Tquark condensate:

$$\text{mass} = 145$$

$$T \quad ----- \quad \overline{T} \quad \text{Effective Higgs in CP2 Internal Symmetry Space}$$

$$\text{mass} = 145 \quad \text{Higgs Effective Mass} = 145 \times \cos(\pi/6) = 145 \times 0.866 = 126$$

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle composed of 2 right triangles (one from the CP2 origin to the T and to the M4 Higgs and another from the CP2 origin to the Tbar and to the M4 Higgs).

In the T-quark condensate picture

$$\lambda(T) = 1^2$$

$$\lambda(H) = (\cos(\pi/6))^2$$

and

Therefore the Effective Higgs mass observed by LHC is:

$$\text{Higgs Mass} = 145.789 \times \cos(\pi/6) = 126.257 \text{ GeV}. $$
To get W-boson masses, denote the 3 SU(2) high-energy weak bosons (massless at energies higher than the electroweak unification) by $W^+$, $W^-$, and $W_0$, corresponding to the massive physical weak bosons $W^+$, $W^-$, and $Z_0$.

The triplet $\{ W^+, W^-, W_0 \}$ couples directly with the $T$ - $\bar{T}$ quark-antiquark pair, so that the total mass of the triplet $\{ W^+, W^-, W_0 \}$ at the electroweak unification is equal to the total mass of a $T$ - $\bar{T}$ pair, 259.031 GeV.

The triplet $\{ W^+, W^-, Z_0 \}$ couples directly with the Higgs scalar, which carries the Higgs mechanism by which the $W_0$ becomes the physical $Z_0$, so that the total mass of the triplet $\{ W^+, W^-, Z_0 \}$ is equal to the vacuum expectation value $\nu$ of the Higgs scalar field, $\nu = 252.514$ GeV.

What are individual masses of members of the triplet $\{ W^+, W^-, Z_0 \}$?

First, look at the triplet $\{ W^+, W^-, W_0 \}$ which can be represented by the 3-sphere $S^3$. The Hopf fibration of $S^3$ as $S^1 \rightarrow S^3 \rightarrow S^2$ gives a decomposition of the $W$ bosons into the neutral $W_0$ corresponding to $S^1$ and the charged pair $W^+$ and $W^-$ corresponding to $S^2$.

The mass ratio of the sum of the masses of $W^+$ and $W^-$ to the mass of $W_0$ should be the volume ratio of the $S^2$ in $S^3$ to the $S^1$ in $S^3$. The unit sphere $S^3$ in $R^4$ is normalized by $1 / 2$. The unit sphere $S^2$ in $R^3$ is normalized by $1 / \sqrt{3}$. The unit sphere $S^1$ in $R^2$ is normalized by $1 / \sqrt{2}$. The ratio of the sum of the $W^+$ and $W^-$ masses to the $W_0$ mass should then be $(2 / \sqrt{3}) \pi(S^2) / (2 / \sqrt{2}) \pi(S^1) = 1.632993$

Since the total mass of the triplet $\{ W^+, W^-, W_0 \}$ is 259.031 GeV, the total mass of a $T$ - $\bar{T}$ pair, and the charged weak bosons have equal mass, we have

$$M_{W^+} = M_{W^-} = 80.326 \text{ GeV} \text{ and } M_{W_0} = 98.379 \text{ GeV}.$$  

The charged $W^+/-$ neutrino-electron interchange must be symmetric with the electron-neutrino interchange, so that the tree-level absence of right-handed neutrino particles requires that the charged $W^+/-$ SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector, so that the charged $W^+/-$ SU(2) weak bosons act only on left-handed fermion particles of all types.
The neutral $W^0$ weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral $W^0$ weak bosons are related to the charged $W^{\pm/-}$ weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral $W^0$ must be equal to the left-handed (entire) component of the charged $W^{\pm/-}$.

Since the mass of the $W^0$ is greater than the mass of the $W^{\pm/-}$, there remains for the $W^0$ a component acting on both types of fermions.

Therefore the full $W^0$ neutral weak boson interaction is proportional to $(M_{W^{\pm/-}}^2 / M_{W^0}^2)$ acting on left-handed fermions and $(1 - (M_{W^{\pm/-}}^2 / M_{W^0}^2))$ acting on both types of fermions.

If $(1 - (M_{W^{\pm/-}}^2 / M_{W^0}^2))$ is defined to be $\sin^2(\theta_W)$ and denoted by $K$, and if the strength of the $W^{\pm/-}$ charged weak force (and of the custodial SU(2) symmetry) is denoted by $T$, then the $W^0$ neutral weak interaction can be written as $W^0L = T + K$ and $W^0LR = K$.

Since the $W^0$ acts as $W^0L$ with respect to the parity violating SU(2) weak force and as $W^0LR$ with respect to the parity conserving U(1) electromagnetic force, the $W^0$ mass $m_{W^0}$ has two components: the parity violating SU(2) part $m_{W^0L}$ that is equal to $M_{W^{\pm/-}}$ and the parity conserving part $m_{W^0LR}$ that acts like a heavy photon.

As $M_{W^0} = 98.379$ GeV = $M_{W^0L} + M_{W^0LR}$, and as $M_{W^0L} = M_{W^{\pm/-}} = 80.326$ GeV, we have $M_{W^0LR} = 18.053$ GeV.

Denote by $\alpha_E = e^2$ the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength $\alpha_E = e^2 = 1 / 137.03608$ was calculated above using the volume $V(S^1)$ of an $S^1$ in $R^2$, normalized by $1 / \sqrt{2}$.

The $\alpha_E$ force is part of the SU(2) weak force whose strength $\alpha_W = w^2$ was calculated above using the volume $V(S^2)$ of an $S^2 \subset R^3$, normalized by $1 / \sqrt{3}$.

Also, the electromagnetic force strength $\alpha_E = e^2$ was calculated above using a 4-dimensional spacetime with global structure of the 4-torus $T^4$ made up of four $S^1$ 1-spheres, while the SU(2) weak force strength $\alpha_W = w^2$ was calculated above using two 2-spheres $S^2 \times S^2$, each of which contains one 1-sphere of the $\alpha_E$ force.
Therefore
\[ *\alpha_E = \alpha_E \left( \frac{\sqrt{2}}{\sqrt{3}} \right) \left( \frac{2}{4} \right) = \alpha_E / \sqrt{6}, \]
\[ *e = e / (4th \ root \ of \ 6) = e / 1.565, \]
and
the mass \( m_{W0LR} \) must be reduced to an effective value
\[ M_{W0LR_{\text{eff}}} = M_{W0LR} / 1.565 = 18.053/1.565 = 11.536 \text{ GeV} \]
for the \( \alpha_E \) force to act like an electromagnetic force in the E8 model:
\[ *e \ M_{W0LR} = e \ (1/5.65) \ M_{W0LR} = e \ M_{Z0}, \]
where the physical effective neutral weak boson is denoted by \( Z0 \).

Therefore, the correct CI(16)-E8 model values for weak boson masses and the Weinberg angle \( \theta_w \) are:
\[ M_{W^+} = M_{W^-} = 80.326 \text{ GeV}; \]
\[ M_{Z0} = 80.326 + 11.536 = 91.862 \text{ GeV}; \]
\[ \sin(\theta_w)^2 = 1 - \left( \frac{M_{W^+/-}}{M_{Z0}} \right)^2 = 1 - \left( \frac{6452.2663}{8438.6270} \right) = 0.235. \]

Radiative corrections are not taken into account here, and may change these tree-level values somewhat.
17. Higgs - Truth Quark Condensate System with 3 Mass States

The Magenta Dot is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model. That high-mass Higgs is around 250 GeV in the range of the Higgs Vacuum Instability Boundary which range includes the Higgs VEV.

The Gold Line leading down from the Critical Point roughly along the Triviality Boundary line is based on Renormalization Group calculations with the result that $MH / MT = 1.1$ as described by Koichi Yamawaki in hep-ph/9603293.

The Cyan Dot where the Gold Line leaves the Triviality Boundary to go into our Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they show that for 8-dimensional Kaluza-Klein spacetime with the Higgs as a Truth Quark condensate, $172 < MT < 175$ GeV and $178 < MH < 188$ GeV. That mid-mass Higgs is around the 200 GeV range of the Higgs Triviality Boundary at which the composite nature of the Higgs as T-Tbar condensate in (4+4)-dim Kaluza-Klein becomes manifest.

The Green Dot where the Gold Line terminates in our Ordinary Phase is the low-mass state of a 130 GeV Truth Quark and a 126 GeV Higgs.
The Cl(16)-E8 model view is that there are 3 Mass States for Higgs and Truth Quark.

Opposed to the Cl(16)-E8 view is the Fermilab / CERN / Establishment view that there is only one Higgs Mass State (Low Mass around 126 GeV) and only one Truth Quark Mass State (Middle Mass around 174 GeV). Their view is represented on the Mh - Mt diagram by a Horizontal Red Line for their Higgs Mass State and a Vertical Red Line for their Truth Quark Mass State so the Fermilab / CERN / Establishment view is that the State of Our Universe is given by the Intersection of the two Red Lines which is at or in the Region of Vacuum Instability for Our Universe.

Therefore, the Fermilab / CERN / Establishment view has problems with Vacuum Instability (see arXiv 1307.3536) while the Cl(16)-E8 model view has no such problem, as it does not go beyond the Critical Triple Point (Magenta Dot) of High Mass States.

Theoretical support for the Cl(16)-E8 model view comes by identifying the Higgs with Primitive Idempotents of the Cl(8) real Clifford algebra, whereby the Higgs is not seen as a simple-minded single fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate with which it interacts to make the fermions appear massive.
The conventional Standard Model has structure:
spacetime is a base manifold
particles are representations of gauge groups
gauge bosons are in the adjoint representation
fermions are in other representations (analogous to spinor)
Higgs boson is in scalar representation

The Cl(16)-E8 model has structure
(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):
spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)
gauge bosons are in the adjoint D8 part of E8 (28+28 = 56 of the 120 D8 adjoints)
fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors.

There is no room for a fundamental Higgs directly appearing in the E8, rather, it emerges from the Mayer-Trautman Mechanism with formation of Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime. To see how that Higgs works in terms of the Cl(16) = Cl(8)xCl(8) Clifford Algebra, embed 248-dim E8 into the 256-dim real Clifford algebra Cl(8):

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
\text{Cl}(8) & & & & & & & & & \\
\text{Primitive} & 256 & = & 1 & + & 8 & + & 28 & + & 56 & + & 70 & + & 56 & + & 28 & + & 8 & + & 1 \\
\text{Idempotent} & 16 & = & 1 & + & 6 & + & 1 & + & 8 \\
\text{E8 Root Vectors} & 240 & = & 8 & + & 28 & + & 56 & + & 56 & + & 56 & + & 28 & + & 8 \\
\text{E8} & 248 & = & 8 & + & 28 & + & 56 & + & 64 & + & 56 & + & 28 & + & 8 \\
\end{array}
\]

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed into two 8-dimensional half-spinor parts each of which is related by Triality to 8-dimensional spacetime and has Octonionic structure.

In that decomposition: the 1+6+1 = (1+3)+(3+1) is related to two copies of a 4-dimensional Associative Quaternionic subspace of the Octonionic structure and the 8 = 4+4 is related to two copies of a 4-dimensional Co-Associative subspace of the Octonionic structure (see the book "Spinors and Calibrations" by F. Reese Harvey).

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent when combined with the 240 E8 Root Vectors forms the full 248-dimensional E8. It represents a Cartan subalgebra of the E8 Lie algebra.

The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent corresponds to the Higgs of the Cl(16)-E8 model.
The half-spinors generated by the Higgs part of the Cl(8) Primitive Idempotent represent neutrino; red, green, blue down quarks; red, green, blue up quarks; electron so the E8 Higgs effectively creates/annihilates the fundamental fermions and

**the E8 Higgs is effectively a condensate of fundamental fermions.**

In the Cl(16)-E8 model the high-energy 8-dimensional Octonionic spacetime reduces, by freezing out a preferred 4-dim Associative Quaternionic subspace, to a 4+4 -dimensional Batakis Kaluza-Klein of the form M4 x CP2 with 4-dim M4 physical spacetime.

The (1+3)+(3+1) part of the Cl(8) Primitive Idempotent includes

the 1 of Cl(8) grade-0 scalar ( that determines M4 transformation properties )
and 3+3 = 6 of the Cl(8) grade-4
and the 1 of Cl(8) grade-8

so the Cl(16)-E8 model Higgs transforms as a scalar with respect to 4-dim M4 Physical SpaceTime
and is consistent with LHC observations ( see arXiv 1307.1432 ).

Not only does the Cl(16)-E8 model Higgs fermion condensate transform with respect to 4-dim physical spacetime like the Standard Model Higgs
but the geometry of the reduction from 8-dim Octonionic spacetime
to (4+4)-dimensional Batakis Kaluza-Klein,
by the Mayer-Trautman Mechanism,
gives the Cl(16)-E8 Higgs ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically from the reduction from 8-dim to 4+4 -dim Kaluza-Klein,
they are also created/annihilated by the Primitive Idempotent Cl(16)-E8 Higgs
and are present in the fermion condensate.

**Since the Truth Quark is so much more massive that the other fermions, the Cl(16)-E8 model Higgs is effectively a Truth Quark condensate.**

When Triviality and Vacuum Stability are taken into account,
the Cl(16)-E8 model Higgs and Truth Quark system has 3 mass states.
As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176:

"... The Higgs quartic coupling has a complicated scale dependence. It evolves according to 
\[ \frac{d \lambda}{dt} = \frac{1}{16 \pi^2} \beta_\lambda \]
where the one loop contribution is given by
\[ \beta_\lambda = 12 \lambda^2 - ... - 4 H ... \]
The value of \( \lambda \) at low energies is related to the physical value of the Higgs mass according to the tree level formula
\[ m_H = v \sqrt{2 \lambda} \]
while the vacuum value is determined by the Fermi constant

... for a fixed vacuum value \( v \), let us assume that the Higgs mass and therefore \( \lambda \) is large. In that case, \( \beta_\lambda \) is dominated by the \( \lambda^2 \) term, which drives the coupling towards its Landau pole at higher energies. Hence the higher the Higgs mass, the higher \( \lambda \) is and the closer the Landau pole to experimentally accessible regions. This means that for a given (large) Higgs mass, we expect the standard model to enter a strong coupling regime at relatively low energies, losing in the process our ability to calculate. This does not necessarily mean that the theory is incomplete, only that we can no longer handle it...

it is natural to think that this effect is caused by new strong interactions, and that the Higgs actually is a composite ...

The resulting bound on \( \lambda \) is sometimes called the triviality bound. The reason for this unfortunate name (the theory is anything but trivial) stems from lattice studies where the coupling is assumed to be finite everywhere; in that case the coupling is driven to zero, yielding in fact a trivial theory.

In the standard model \( \lambda \) is certainly not zero. ..."

Composite Higgs as Tquark condensate studies by Yamawaki et al have produced realistic models that are consistent with the Cl(16)-E8 model with a 3-State System:

1 - The basic Cl(16)-E8 model state
with Tquark mass = 130 GeV and Higgs mass = 126 GeV

2 - Triviality boundary 8-dim Kaluza-Klein state described by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:

"... "... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in \( D(=6,8,10,...) \) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for \( D=8 \) ... We predict masses of the top (m_\( t \)) and the Higgs (m_H) ...
based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[ Kaluza-Klein type ]... dimension... D=8 ... 

\[ m_t = 172-175 \text{ GeV and } m_H=176-188 \text{ GeV } \].

3 - Critical point BHL state
with \textbf{Tquark mass = 218 +/- 3 GeV and Higgs mass = 239 +/- 3 GeV}

As Yamawaki said in hep-ph/9603293: "... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ... BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale \( \Lambda = 10^{19} \text{ GeV} \)... there must be a certain matching scale \( \Lambda_{\text{Matching}} \) such that the perturbative picture (BHL) is valid for \( \mu < \Lambda_{\text{Matching}} \), while only the nonperturbative picture (MTY) becomes consistent for \( \mu > \Lambda_{\text{Matching}} \) ... However, thanks to the presence of a quasi-infrared fixed point, BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect \[ \text{mt} = \text{mt}(\text{BHL}) = \ldots = 1/(\sqrt{2}) \text{ybart v} \] within 1-2\%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ... the one-loop RG equation ...

The composite Higgs loop changes ybart^2 by roughly the factor \( N_c/(N_c +3/2) = 2/3 \) compared with the MTY value, i.e., \( 250 \text{ GeV} \to 250 \times \sqrt{2/3} = 204 \text{ GeV} \), while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by \( \text{mt} = 218 +/- 3 \text{ GeV}, \text{at} \Lambda = 10^{19} \text{ GeV} \).

The Higgs boson was predicted as a tbar-t bound state with a mass \( MH = 2mt \) based on the pure NJL model calculation. Its mass was also calculated by BHL through the full RG equation ... the result being \( \text{MH}/\text{mt} = 1.1 \) at \( \Lambda = 10^{19} \text{ GeV} \) ...

... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate. The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositeness condition. BHL essentially incorporates 1/Nc sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at 1/Nc-leading order. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".
8-dim Kaluza-Klein spacetime physics as required by Hashimoto, Tanabashi, and Yamawaki for the Middle State of the 3-State System was described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 in terms a M4xCP2 structure similar to that of the Cl(16)-E8 model. Although spacetime and Standard Model gauge bosons worked well for Batakis, he became discouraged by difficulties with fermions, perhaps because he did not use Clifford Algebras with natural spinor structures for fermions.

Calculations of the Low-Mass State of Higgs and Truth Quark have been given in Chapters 10 and 16 of this paper. Here are similar details for Middle and High Mass:

**Middle Mass State:**

In the Cl(16)-E8 model, the Middle-Mass Higgs has structure that is not restricted to Effective M4 Spacetime as is the case with the Low-Mass Higgs Ground State but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

\[ T \quad \quad Tbar \quad \quad \text{in CP2 Internal Symmetry Space} \]

\[ \quad \quad \quad \quad \quad \quad \quad \text{Higgs} \quad \quad \quad \text{in M4 Physical SpaceTime} \]

Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.

The T and Tbar form a Pion-like state.
Since Tquark Mid-Mass State is 174 GeV
the Middle-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein has mass \((174+174) \times (135 / (312+312)) = 75 \text{ GeV} \).

The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.
So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein with mass \(75+125 = 200 \text{ GeV} \).
This is consistent with the Mid-Mass States of the Higgs and Tquark being on the Triviality Boundary of the Higgs - Tquark System and with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.
As to the cross-section of the Middle-Mass Higgs

consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime (left white circle)
while the Middle-Mass Higgs cross-section lives in full $4+4 = 8$-dim Kaluza-Klein (right circle with red area only in CP2 ISS and white area partly in CP2 ISS with only green area effectively living in 4-dim M4 spacetime)
so that
our 4-dim M4 Physical Spacetime experiments only see for the Middle-Mass Higgs a cross-section that is 25% of the full Ground State cross-section.
The 25% may also be visualized in terms of 8-dim coordinates \(\{1,i,j,k,E,I,J,K\}\)
in which \(\{1,i,j,k\}\) represent M4 and \(\{E,I,J,K\}\) represent CP2.
High Mass State:

In the Cl(16)-E8 model, the the High-Mass Higgs State is at the Critical Point of the Higgs-Tquark System

where the Triviality Boundary intersects the Vacuum Instability Boundary which is also at the Higgs Vacuum Expectation Value VEV around 250 GeV.

As with the Mid-dleMass Higgs, the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions and so has a cross-section that is about 25% of the Higgs Ground State cross-section.
In 1994 a semileptonic histogram from CDF seems to me to show all three states of the T-quark.
In 1997 a semileptonic histogram from D0 also seems to me to show all three states of the T-quark. The fact that the low (green) state showed up in both independent detectors indicates a significance of 4 sigma. Some object that the low (green) state peak should be as wide as the peak for the middle (cyan) state, but my opinion is that the middle (cyan) state should be wide because it is on the Triviality boundary where the composite nature of the Higgs as T-Tbar condensate becomes manifest and the low (cyan) state should be narrow because it is in the usual non-trivial region where the T-quark acts more nearly as a single individual particle.
In 1998 a dilepton histogram from CDF (from hep-ex/9802017) seems to me to show both the low (green) state and the middle (cyan) T-quark state.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF (from hep-ex/9801014) showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.
In 1997 the Ph.D. thesis of Erich Ward Varnes (Varnes-fermilab-thesis-1997-28) at page 159 said:

"... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...

![Graph]

..." (colored bars added by me)

The event for all 3 jets (solid curve) sees to me to correspond to decay of a middle (cyan) T-quark state with one of the 3 jets corresponding to decay from the Triviality boundary down to the low (green) T-quark state, whose immediately subsequent decay is corresponds to the 2-jet (dashed curve) event at the low (green) energy level.

After 1998 Fermilab and CERN have focussed attention on detailed analysis of the middle (cyan) T-quark state, getting much valuable detailed information about it but not producing much information about the low or high Tquark states.
In the 25/fb of data collected through the run ending with the long shutdown at the end of 2012, the LHC has observed a 126 GeV state of the Standard Model Higgs boson.

Here are some details about the LHC observation at 126 GeV and related results shown at Moriond 2013:

The digamma histogram for ATLAS shows only one peak below 160 GeV and it is around 126 GeV.
CMS shows the cross sections for Higgs at 125.8 GeV to be substantially consistent with the Standard Model for the WW and ZZ channels, a bit low for tau-tau and bb channels (but that is likely due to very low statistics there), and a bit high for the digamma channel (but that may be due to phenomena related to the Higgs as a Tquark condensate).
A CMS histogram (some colors added by me) for the Golden Channel Higgs to ZZ to 4l shows the peak around 126 GeV (green dots - lowHiggs mass state). The CMS histogram also indicates other excesses around 200 GeV (cyan dots - midHiggs mass state) and around 250 GeV (magenta dots - highHiggs mass state). An image of one of the events is shown below the histogram.
An ATLAS ZZ to 4l histogram (some colors added by me) show the peak around 126 GeV (green dots - low Higgs mass state). The ATLAS histogram also indicates other excesses around 200 GeV (cyan dots - middle Higgs mass state) and around 250 GeV (magenta dots - high Higgs mass state). An image of one of the events is shown below the histogram.
CMS showed a Brazil Band Plot for the High Mass Higgs to ZZ to 4l/2l2tau channel where
the top red line represents the expected cross section of a single Standard Model Higgs
and the lower red line represents about 20% of the expected Higgs SM cross section.

The green dot peak is at the 126 GeV Low Mass Higgs state with expected Standard Model cross section.
The cyan dot peak is around the 200 GeV Mid Mass Higgs state expected to have about 25% of the SM cross section.
The magenta dot peak is around the 250 (+/- 20 or so) GeV High Mass Higgs state expected to have about 25% of the SM cross section.

The (?) peak is around 320 GeV where I would not expect a Higgs Mass state and I note that in fact it seems to have gone away in the full ATLAS ZZ to 4l histogram shown above because between 300 and 350 GeV the two sort-of-high excess bins are adjacent to deficient bins.
It will probably be no earlier than 2016 (after the long shutdown) that the LHC will produce substantially more data than the 25/fb available at Moriond 2013 and therefore no earlier than 2016 for the green and yellow Brazil Bands to be pushed down (throughout the 170 GeV to 500 GeV region) below 10 per cent (the 10^{-1} line) of the SM cross section as is needed to show whether or not the cyan dot, magenta dot, and/or (?) peaks are real or statistical fluctuations.
My guess (based on the Cl(16)-E8 model) is that the cyan dot and magenta dot peaks will prove to be real and that the (?) peak will go away as a statistical fluctuation.
Sagittarius A* (Sgr A*) is a very massive black hole in the center of our Galaxy into which large amounts of Hydrogen fall. As the Hydrogen approaches Sgr A* it increases in energy, ionizing into protons and electrons, and eventually producing a fairly dense cloud of infalling energetic protons whose collisions with ambient protons are at energies similar to the proton-proton collisions at the LHC.

Andrea Albert at The Fermi Symposium 11/2/2012 said: "... gamma rays detectable by the Fermi Large Area Telescope [ FLAT ] ...

... Line-like Feature near 135 GeV ... localized in the galactic center ...

In addition to the Galactic Center observations, Fermi LAT looked at gamma rays from Cosmic Rays hitting Earth's atmosphere by looking at the Earth Limb.
Andrea Albert at The Fermi Symposium 11/2/2012 also said: "... Earth Limb is a bright gamma-ray source ... From cosmic-ray interactions in the atmosphere ...

... Line-like feature ... at 135 GeV .. Appears when LAT is pointing at the Limb ...

Since 90% of high-energy Cosmic Rays are Protons and since their collisions with Protons and other nuclei in Earth's atmosphere produce gamma rays, the 135 GeV Earth Limb Line seen by Fermi LAT is also likely to be the Higgs produced by collisions analogous to those at the LHC.

Olivier K. in a comment in Jester's blog on 10 November 2012 said: "... Could the 135 GeV bump be related ... to current Higgs ... properties ? ... The coincidence between GeV figures ...[ for LHC ] Higgs mass and this [ Fermi LAT ] bump is thrilling for an amateur like me...

Jester in his resonances blog on 17 April 2012 said, about Fermi LAT: "... the plot shows the energy of *single* photons as measured by Fermi, not the invariant mass of photon pairs ...

Since the LHC 125 GeV peak is for "invariant mass of photon pairs" and the Fermi LAT 135 GeV peak is for "single" photons how could both correspond to a Higgs mass state around 130 GeV ?
The LHC sees collisions of high-energy protons (red arrows) forming Higgs (blue dot) with the Higgs at rest decaying into a photon pair (green arrows) giving the observed Higgs peak (around 130 GeV) as invariant mass of photon pairs.

Fermi LAT at Galactic Center and Earth Limb sees collisions of one high-energy proton with a low-energy (relatively at rest) proton forming Higgs with Higgs moving fast from momentum inherited from the high-energy proton decaying into two photons: one with low energy not observed by Fermi LAT and the other being observed by Fermi LAT as a high-energy gamma ray carrying almost all of the Higgs decay energy (around 130 GeV) as a "single" photon.

Therefore, the coincidence noted by Olivier K. is probably a realistic phenomenon.
18. Segal-type Conformal gravity with conformal generator structure giving
Dark Energy, Dark Matter, and Ordinary Matter ratio

MacDowell-Mansouri Gravity is described by Rabindra Mohapatra in section 14.6 of his book “Unification and Supersymmetry”:

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators $X_A$ of a group

$$[X_A, X_B] = f^C_{AB} X_C,$$

(14.6.1)

where $f^C_{AB}$ are structure constants of the group. We can then introduce a gauge field connection $h^A_a$ as follows:

$$h^A_a = h^A_{,a} X_A.$$

(14.6.2)

Let us denote the parameter associated with $X_A$ by $e^A$. The gauge transformations on the fields $h^A_a$ are given as follows:

$$\delta h^A_a = \partial_a e^A + h^B_{,b} f^A_{CB} = (D_a e)^A.$$

(14.6.3)

We can then define a covariant curvature

$$R^A_{\mu \nu} = \partial_{\nu} h^A_a - \partial_{\mu} h^A_a + h^B_{,\mu} f^A_{CB}.$$

(14.6.4)

Under a gauge transformation

$$\delta_{\text{gauge}} R^A_{\mu \nu} = R^B_{\mu \nu} f^A_{CB}.$$

(14.6.5)

We can then write the general gauge invariant action as follows:

$$S = \int d^4 x \, Q^{\mu \nu \rho \sigma} R^A_{\mu \nu} R^A_{\rho \sigma}.$$

(14.6.6)

Let us now apply this formalism to conformal gravity. In this case

$$h^A_a = P_{\mu e} e^m_{,a} + M_{\mu \nu} \omega^m_{\nu a} + K_{\mu f^m_{,a}} + D_{b f^m_{,a}}.$$

(14.6.7)

The various $R_{\mu \nu}$ are

$$R_{\mu \nu}(P) = \partial_{\mu} e^m_{,a} - \partial_{\nu} e^m_{,a} + \omega^m_{\mu a} e^m_{,a} - \omega^m_{,a e^m_{,a}} - b_{,a} e^m_{,a} + b_{e^m_{,a}}.$$

(14.6.8)

$$R_{\mu \nu}(M) = \partial_{\mu} \omega^m_{\nu a} - \partial_{\nu} \omega^m_{,a} - \omega^m_{\mu a} \omega^m_{\nu b} - \omega^m_{\nu a} \omega^m_{\mu b} - \omega^m_{\mu b} \omega^m_{\nu a} - 4(e^m_{,\mu} f^m_{,a} - e^m_{,\nu} f^m_{,a}).$$

(14.6.9)

$$R_{\mu \nu}(K) = \partial_{\mu} f^m_{,a} - \partial_{\nu} f^m_{,a} - b_{,a} f^m_{,a} + b_{,a} f^m_{,a} + \omega^m_{\mu a} f^m_{,a} - \omega^m_{\nu a} f^m_{,a}.$$

(14.6.10)

$$R_{\mu \nu}(D) = \partial_{\mu} b_{,a} - \partial_{\nu} b_{,a} + 2 e^m_{,\mu} f^m_{,a} - 2 e^m_{,\nu} f^m_{,a}.$$

(14.6.11)

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4 x \, \epsilon_{\mu \nu \rho \sigma} R^m_{\mu \nu}(M) R^m_{\rho \sigma}(M).$$

(14.6.12)

We also impose the constraint that

$$R_{\mu \nu}(P) = 0,$$

(14.6.13)
which expresses \( \omega^m_\alpha \) as a function of \((e, b)\). The reason for imposing this constraint has to do with the fact that \( P_\alpha \) transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein \( e^a_\alpha \). Under coordinate transformations

\[
\delta G (\xi) e^m_\alpha = \delta \xi^\mu e^m_\alpha + \xi^\nu \partial_\nu e^m_\alpha . \tag{14.6.14}
\]

Using eqn. (14.6.8) we can rewrite

\[
\delta G (\xi) e^m_\alpha = \delta \mu (\xi e^\nu) e^m_\alpha + \delta \mu (\xi \omega^m_\nu) e^m_\alpha + \delta \mu (\xi b^m_\nu) e^m_\alpha + \xi^\nu R^m_\nu (P),
\]

where

\[
\delta \mu (\xi e^\nu) e^m_\alpha = \partial_\mu \xi^\nu + \xi^\mu \omega^m_\nu + \xi^\nu b^m_\mu . \tag{14.6.15}
\]

If \( R^m_\nu (P) = 0 \), the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries \( P_\alpha = \partial_\mu - i X_\mu X_\nu \); now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator \( \bar{P} \) such that

\[
\delta \bar{P} = \delta G (\xi) - \sum A^\alpha \delta \alpha (\xi^\alpha b^\alpha_\mu), \tag{14.6.16}
\]

where \( A^\alpha \) goes over all gauge transformations excluding translation. The rule is

\[
\delta \bar{P} (\xi^\alpha) \phi = \xi^\alpha D^\alpha \phi. \tag{14.6.17}
\]

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to \( D^\alpha \) and the orbital parts do not play any role.

Coming back to the constraints we can then vary the action with respect to \( f^m_\alpha \) to get an expression for it, i.e.,

\[
f^m_\alpha f^m_{\beta \alpha} = -\frac{1}{2} [e^\mu_\alpha e^\nu_\mu R^m_{\nu \beta} - \delta^m_{\nu \beta} \epsilon]. \tag{14.6.18}
\]

where \( f^m_\alpha \) has been set to zero in \( R \) written in the right-hand side.

This eliminates (from the theory the degrees of freedom) \( \omega^m_\alpha \) and \( f^m_\alpha \) and we are left with \( e^m_\alpha \) and \( b^m_\mu \). Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field \( \phi \). It has conformal weight \( \lambda = 1 \). So we can write a covariant derivative for it, eqn. (14.6.17)

\[
D^\alpha \phi = \partial_\mu \phi - \phi \partial^\alpha . \tag{14.6.19}
\]

We note that the conformal charge of \( \phi \) can be assumed to be zero since \( K_\mu = x^\mu \phi \) and is the dimension of inverse mass. In order to calculate \( \Box ^\alpha \phi \) we
After the scale and conformal gauges have been fixed, the conformal Lagrangian becomes a de Sitter Lagrangian. Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian, as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739). (Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri ... approach to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ... S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ..., and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977); also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".)
The minimal group required to produce Gravity, and therefore the group that is used in calculating Force Strengths, is the [anti] de Sitter group, as is described by Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) (chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields \( w^{ab\_u} \) for the Lorentz group and a further set \( e^{a\_u} \) for the translations, ...

Everybody knows though, that Einstein’s theory contains but one spin two field, originally chosen by Einstein as \( g_{uv} = e^{a\_u} e^{b\_v} n_{ab} \) (\( n_{ab} = \) Minkowski metric).

What happened to the \( w^{ab\_u} \)?

The field equations obtained from the Hilbert-Einstein action by varying the \( w^{ab\_u} \) are algebraic in the \( w^{ab\_u} \) ... permitting us to express the \( w^{ab\_u} \) in terms of the \( e^{a\_u} \) ... The \( w \) do not propagate ...

We start from the four-dimensional de-Sitter algebra ... so(3,2).

Technically this is the anti-de-Sitter algebra ...

We envision space-time as a four-dimensional manifold \( M \).

At each point of \( M \) we have a copy of SO(3,2) (a fibre ...) ...

and we introduce the gauge potentials (the connection) \( h^{A\_mu}(x) \)

\( A = 1,\ldots, 10\), \( \mu = 1,\ldots, 4 \). Here \( x \) are local coordinates on \( M \).

From these potentials \( h^{A\_mu} \) we calculate the field-strengths (curvature components) [let \( \partial \) denote partial derivative]

\[ R^{A\_munu} = \partial\_\mu h^{A\_nu} - \partial\_\nu h^{A\_mu} + f^{A\_BC} h^{B\_mu} h^{C\_nu} \]

[where]... the structure constants \( f^{C\_AB} \) ...
[are for]... the anti-de-Sitter algebra ....

We now wish to write down the action \( S \) as an integral over the four-manifold \( M \) ...

\[ S(Q) = \text{INTEGRAL}_M R^A \wedge R^B Q_{AB} \]

where \( Q_{AB} \) are constants ... to be chosen ... we require ...

... the invariance of \( S(Q) \) under local Lorentz transformations

... the invariance of \( S(Q) \) under space inversions ...

...[ AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE ]...

we shall see ...[that]... the action becomes invariant under all local [anti]de-Sitter transformations ...[and]... we recognize ... the familiar Hilbert-Einstein action with cosmological term in vierbein notation ...

Variation of the vierbein leads to the Einstein equations with cosmological term ...

Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.

So at this level full sp(4) invariance has been checked.

... Were it not for the assumed space-inversion invariance ...

we could have had a parity violating gravity. ...

Unlike Einstein's theory ...[MacDowell-Mansouri]... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.

Parity violation and torsion go hand-in-hand.

Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether the universe as a whole is in a space-inversion symmetric configuration. ...".
According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by
the de Sitter instead of the Poincare group, the P-symmetry of the weak
cosmological-constant limit and the Q-symmetry of the strong cosmological constant
limit can be considered as limiting cases of the fundamental symmetry. ... 
... N ...[ is the space ]... whose geometry is gravitationally related to an infinite
cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and
whose group of motion is Q. Analogously to the Minkowski case, N is also a
homogeneous space, but now under the kinematical group Q, that is, N = Q/L
[ where L is the Lorentz Group of Rotations and Boosts ]. In other words, the
point-set of N is the point-set of the special conformal transformations.
Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as
base space and L as the typical fiber. The kinematical group Q, like the Poincare
group, has the Lorentz group L as the subgroup accounting for both the isotropy
and the equivalence of inertial frames in this space. However, the special
conformal transformations introduce a new kind of homogeneity. Instead of
ordinary translations, all the points of N are equivalent through special conformal
transformations. ...

... Minkowski and the cone-space can be considered as dual to each other, in the
sense that their geometries are determined respectively by a vanishing and an
infinite cosmological constants. The same can be said of their kinematical group of
motions: P is associated to a vanishing cosmological constant and Q to an infinite
cosmological constant.
The dual transformation connecting these two geometries is the spacetime
inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group
P is transformed into the group Q, and the Minkowski space M becomes the conespace
N. The points at infinity of M are concentrated in the vertex of the conespace
N, and those on the light-cone of M becomes the infinity of N. It is
concepts of space isotropy and equivalence between inertial frames in the conespace
N are those of special relativity. The difference lies in the concept of
uniformity as it is the special conformal transformations, and not ordinary
translations, which act transitively on N. ..."
Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

- 3 Rotations
- 3 Boosts
- 4 Translations
- 4 Special Conformal transformations
- 1 Dilatation

The Cosmological Constant / Dark Energy comes from the 10 Rotation, Boost, and Special Conformal generators of the Conformal Group Spin(2,4) = SU(2,2), so the fractional part of our Universe of the Cosmological Constant should be about 10 / 15 = 67% for tree level.

Black Holes, including Dark Matter Primordial Black Holes, are curvature singularities in our 4-dimensional physical spacetime, and since Einstein-Hilbert curvature comes from the 4 Translations of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2) through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate), the fractional part of our Universe of Dark Matter Primordial Black Holes should be about 4 / 15 = 27% at tree level.

Since Ordinary Matter gets mass from the Higgs mechanism which is related to the 1 Scale Dilatation of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2), the fractional part of our universe of Ordinary Matter should be about 1 / 15 = 6% at tree level.

However, as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates, so that the differences in evolution must be taken into account from the initial End of Inflation to the Present Time.

Without taking into account any evolutionary changes with time, our Flat Expanding Universe should have roughly:

- 67% Cosmological Constant
- 27% Dark Matter - possibly primordial stable Planck mass black holes
- 6% Ordinary Matter
As Dennnis Marks pointed out to me, since density $\rho$ is proportional to $(1+z)^3(1+w)$ for red-shift factor $z$ and a constant equation of state $w$:

- $w = -1$ for $\Lambda$ and the average overall density of $\Lambda$ Dark Energy remains constant with time and the expansion of our Universe;
- and
- $w = 0$ for nonrelativistic matter so that the overall average density of Ordinary Matter declines as $1/R^3$ as our Universe expands;
- and
- $w = 0$ for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as $1/R^3$ as our Universe expands;

so that the ratio of their overall average densities must vary with time, or scale factor $R$ of our Universe, as it expands. Therefore, the above calculated ratio $0.67 : 0.27 : 0.06$ is valid only for a particular time, or scale factor, of our Universe.

When is that time? Further, what is the value of the ratio now?

Since WMAP observes Ordinary Matter at 4% NOW, the time when Ordinary Matter was 6% would be at redshift $z$ such that

$$\frac{1}{(1+z)^3} = \frac{0.04}{0.06} = \frac{2}{3} ,$$

or

$$\frac{1+z}{1} = \frac{1.5}{1} ,$$

or

$$1+z = 1.145 ,$$

or

$$z = 0.145 .$$

To translate redshift into time, in billions of years before present, or Gy BP, use this chart

![Redshift vs time chart](https://www.supernova.lbl.gov/)

from a [www.supernova.lbl.gov](http://www.supernova.lbl.gov) file SNAPoverview.pdf to see that the time when Ordinary Matter was 6% would have been a bit over 2 billion years ago, or 2 Gy BP.
In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "...
reveals changes in the rate of expansion since the universe's birth 15 billion years
ago. The more shallow the curve, the faster the rate of expansion. The curve
changes noticeably about 7.5 billion years ago, when objects in the universe began
flying apart at a faster rate. ...").
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the
Supernova Cosmology Project ... and his team have studied altogether some 80
high red-shift type Ia supernovae. Their results imply that the universe was
decelerating for the first half of its existence, and then began accelerating
approximately 7 billion years ago. ...").
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7
provides direct evidence for an early phase of slowing expansion if the dark energy
is a cosmological constant ...").
3 - the Last Intersection of the Accelerating Expansion of our Universe of Linear Expansion (green line) with the Third Intersection (at red vertical line at \( z = 0.145 \) and about 2 Gy BP), which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, \( \text{Fe}_2\text{O}_3 \) Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.
Those four Special Times define four Special Epochs:
The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe ( see gr-qc/0007006 ).
The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.
The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.
The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant \( \Lambda \) is more prominent than it would be under the "standard norm" conditions of Linear Expansion.
Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy \( \Lambda \) and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

\( \Lambda \) and CDM would be effectively created during expansion in their natural ratio \( 67 : 27 = 2.48 = 5 / 2 \), each having proportionate fraction \( 5 / 7 \) and \( 2 / 7 \), respectively; CDM Black Hole decay would be ignored; and pre-existing CDM Black Hole density would decline by the same \( 1 / R^3 \) factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.
The Ordinary Matter excess $0.06 - 0.04 = 0.02$ plus the first-order CDM excess $0.27 - 0.18 = 0.09$ should be summed to get a total first-order excess of $0.11$, which in turn should be distributed to the $\Lambda$ and CDM factors in their natural ratio $67 : 27$, producing, for NOW after 2 Gy of expansion:

$$\text{CDM Black Hole factor} = 0.18 + 0.11 \times \frac{2}{7} = 0.18 + 0.03 = 0.21$$

for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

$$0.75 : 0.21 : 0.04$$

so that the present ratio of $0.73 : 0.23 : 0.04$ observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

2013 Planck Data ( arxiv 1303.5062 ) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

Now the Cl(16)-E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05

- Planck: DE : DM : OM = 69 : 26 : 05

Since uncertainties are substantial, I think that there is reasonable consistency.
19. Dark Energy explanations for Pioneer Anomaly and Uranus spin-axis tilt

After the Inflation Era and our Universe began its current phase of expansion, some regions of our Universe become Gravitationally Bound Domains (such as, for example, Galaxies) in which the 4 Conformal GraviPhoton generators are frozen out, forming domains within our Universe like IceBergs in an Ocean of Water. On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.

Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.

In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about 68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood.... [the above image is] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager

... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...
The last communication from Pioneer 11 was received in November 1995, when
the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should
pass close to the nearest star in the constellation Aquila in about 4 million years ...
Calculations of the motion of a spacecraft are made on the basis of the range
time-delay and/or the Doppler shift in the signals. This type of data was used to
determine the positions, the velocities, and the magnitudes of the orientation
maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this
study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ...
analyses of radio Doppler ... data ... indicated that an apparent anomalous
acceleration is acting on Pioneer 10 and 11 ... The data implied an anomalous,
constant acceleration with a magnitude a_P = 8 x 10^-8 cm/cm/s^2, directed
towards the Sun ...
the size of the anomalous acceleration is of the order c H, where H is the
Hubble constant ...
Without using the apparent acceleration, CHASMP shows a steady frequency
drift of about -6 x 10^-9 Hz / s, or 1.5 Hz over 8 years (one-way only). ... This
equates to a clock acceleration, -a_t, of -2.8 x 10^-18 s / s^2 . The identity with
the apparent Pioneer acceleration is a_P = a_t c. ...
Having noted the relationships
a_P = c a_t
and that of ...
a_H = c H -> 8 x 10^-8 cm / s^2
if H = 82 km / s / Mpc ...
we were motivated to try to think of any ... "time" distortions that might ... fit the
CHASMP Pioneer results ... In other words ...
Is there any evidence that some kind of "time acceleration" is being seen?
In particular we considered ... Quadratic Time Augmentation. This model adds a
quadratic-in-time augmentation to the TAI-ET ( International Atomic Time -
Ephemeris Time ) time transformation, as follows
ET -> ET + (1/2) a_ET ET^2
The model fits Doppler fairly well ...

There was one [other] model of the ...[time acceleration]... type that was
especially fascinating. This model adds a quadratic in time term to the light time as
seen by the DSN station:
delta_TAI = TAI_received - TAI_sent ->
-> delta_TAI + (1/2) a_quad (TAI_received^2 - TAI_sent^2 )
It mimics a line of sight acceleration of the spacecraft, and could be thought of as
an expanding space model.
Note that a_quad affects only the data. This is in contrast to the a_t ... that affects
both the data and the trajectory. ... This model fit both Doppler and range very
well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant
and a_P ... remains an interesting conjecture. ...".
In his book “Mathematical Cosmology and Extragalactic Astronomy” (Academic Press 1976) (pages 61-62 and 72), Irving Ezra Segal says:
"... Temporal evolution in ... Minkowski space ... is
H -> H + s I
... unispace temporal evolution ... is ...
H -> ( H + 2 tan(a/2) ) / ( 1 - (1/2) H tan(a/2) ) = H + a I + (1/4) a H^2 + O(s^2)
...".

Therefore,
the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:
"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates; its constant value is ... H c ...
This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.
The Rosales and Sanchez-Gomez paper very nicely unites:

the physical 2-phase (bounded and unbounded orbits) view;
the Foucault pendulum idea; and the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez, can be summarized as a 2-phase model based on Segal's work which has two phases with different metrics:
a metric for outside the inner solar system, a dark energy phase in which gravity is
described in which all 15 generators of the conformal group are effective, some of
which are related to the dark energy by which our universe expands;
and
a metric for where we are, in regions dominated by ordinary matter, in which the 4
special conformal and 1 dilation degrees of freedom of the conformal group are
suppressed and the remaining 10 generators (anti-deSitter or Poincare, etc) are
effective, thus describing ordinary matter phenomena.

If you look closely at the difference between the metrics in those two regions, you
see that the full conformal dark energy region gives an "extra acceleration" that
acts as a "quadratic in time term" that has been considered as an explanation of the
Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S.
Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar
anomalous acceleration in regions where \( c^2 \) dark energy / dark matter is
effectively present. If there is a phase transition (around Uranus at 20 AU)
whereby ordinary matter dominates inside that distance from the sun
and exotic dark energy / dark matter appears at greater distances,
then Jack's model could also explain the Pioneer anomaly
and it may be that Jack's model with ordinary and exotic phases
and my model with deSitter/Poincare and Conformal phases
may be two ways of looking at the same thing.
As to what might be the physical mechanism of the phase transition, Jack says
"... Rest masses of [ordinary matter] particles ... require the smooth non-random
Higgs Ocean ... which soaks up the choppy random troublesome zero point
energy ...".
In other words in a region in which ordinary matter is dominant, such as the Sun
and our solar system, the mass-giving action of the Higgs mechanism "soaks up"
the Dark Energy zero point conformal degrees of freedom that are dominant in
low-ordinary mass regions of our universe (which are roughly the intergalactic
voids that occupy most of the volume of our universe).
That physical interpretation is consistent with my view.
Transition at Orbit of Uranus:
It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than nuclear energy.

In gr-qc/0104064 Anderson et al say:
"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,

which lies almost in its orbital plane.
The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.
Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now. However, such an effect may have been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.

In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.

In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.

Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from “Universe”, 4th ed, by William Kaufmann, Freeman 1994).
"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [ or 2 x 10^11 cm ] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... [C]omets ... suffer ... from outgassing ... [ and their nuclei are hard to locate precisely ] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".
20. Dark Energy experiment by BSCCO Josephson Junctions and geometry of 600-cell

I. E. Segal proposed a Minkowski-Conformal 2-phase Universe and Beck and Mackey proposed 2 Photon-GraviPhoton phases: Minkowski/Photon phase locally Minkowski with ordinary Photons and Gravity weakened by 1 / (M_Planck)^2 = 5 x 10^(-39). so that we see Dark Energy as only 3.9 GeV/m^3

Conformal/GraviPhoton phase with GraviPhotons and Conformal symmetry (like the massless phase of energies above Higgs EW symmetry breaking) With massless Planck the 1 / M_Planck^2 Gravity weakening goes away and the Gravity Force Strength becomes the strongest possible = 1 so Conformal Gravity Dark Energy should be enhanced by M_Planck^2 from the Minkowski/Photon phase value of 3.9 GeV/m^3.

The Energy Gap of our Universe as superconductor condensate spacetime is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length). Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses = critical temperature Tc of BSCCO superconducting crystals. Neutrino masses are involved because their mass is zero at tree level and their masses that we observe come from virtual graviphotons becoming virtual neutrino-antineutrino pairs.

BSCCO superconducting crystals are by their structure natural Josephson Junctions. Dark Energy accumulates (through graviphotons) in the superconducting layers of BSCCO. Josephson Junction control voltage acts as a valve for access to the BSCCO Dark Energy, an idea due to Jack Sarfatti.

Christian Beck and Michael C. Mackey in astro-ph/0703364 said: "... Electromagnetic dark energy .... is based on a Ginzburg-Landau ... phase transition for the gravitational activity of virtual photons ... in two different phases: gravitationally active [GraviPhotons] ... and gravitationally inactive [Photons] ...

Let |P|^2 be the number density of gravitationally active photons ... start from a Ginzburg-Landau free energy density ...

F = a |P|^2 + (1/2) b |P|^4

... The equilibrium state Peq is ... a minimum of F ... for T > Tc ...

Peq = 0 [and] Feq = 0

... for T< Tc

|Peq|^2 = - a / b [and] Fdeq = -(1/2) a^2 / b

... temperature T [of] virtual photons underlying dark energy ... is ..

h v = ln3 k T
dark energy density ...[is]...
\[\rho_{\text{dark}} = \frac{1}{2} \left( \frac{\pi h}{c^3} \right) (v_c)^4\]
The currently observed dark energy density in the universe of about 3.9 GeV/m^3 implies that the critical frequency \( v_c \) is ...
\[v_c = 2.01 \text{ THz}\]

BCS Theory yields ... for Fermi energy ... in copper ... 7.0 eV and the critical temperature of ... YBCO ... around 90 K ...
\[h v_c = 8 \times 10^{-3} \text{ eV}\]

Solar neutrino measurements provide evidence for a neutrino mass of about \( m_v c^2 = 9 \times 10^{-3} \text{ eV} \)

[ the Cl(16)-E8 model has first-order masses for the 3 generations of neutrinos as 1 \times 10^{-3} and 9 \times 10^{-3} and 5.4 \times 10^{-2} \text{ eV }]

... in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor ... (i.e. the energy obtained when a Cooper pair forms out of two electrons) ...

for [graviphotons] ... at low temperatures (frequencies) Cooper-pair like states [of neutrino-antineutrino pairs] can form in the vacuum ... the energy gap would be of the order of typical neutrino mass differences ...

Clovis Jacinto de Matos and Christian Beck in arXiv 0707.1797 said: "...

Tajmar's experiments ... at Austrian Research Centers Gmbh-ARC ... with ... rotating superconducting rings ... demonstrated ...
a clear azimuthal acceleration ... directly proportional to the superconductive ring angular acceleration, and
an angular velocity orthogonal to the ring's equatorial plane ...

In 1989 Cabrera and Tate, through the measurement of the London moment magnetic trapped flux, reported an anomalous Cooper pair mass excess in thin rotating Niobium superconductive rings ...

A non-vanishing cosmological constant (CC) \( \Lambda \) can be interpreted in terms of a non-vanishing vacuum energy density
\[\rho_{\text{vac}} = \left( \frac{c^4}{8 \pi G} \right) \Lambda\]
which corresponds to dark energy with equation of state \( w = -1 \).
The ... astronomically observed value [is]... \( \Lambda = 1.29 \times 10^{\Lambda(-52)} \) [1/m^2] ...

Graviphotons can form weakly bounded states with Cooper pairs, increasing their mass slightly from \( m \) to \( m' \).
The binding energy is \( E_c = u c^2 \):
\[m' = m + my - u\]

... Since the graviphotons are bounded to the Cooper pairs, their zeropoint energies form a condensate capable of the gravitoelectrodynamic properties of superconductive cavities ...

Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff \( v_c \) of vacuum fluctuations:
\[\rho^* = \frac{1}{2} \left( \frac{\pi h}{c^3} \right) (v_c)^4\]
in vacuum one may put $\rho^{*} = \rho_{\text{vac}}$ from which the cosmological cutoff frequency $v_{cc}$ is estimated as

$$v_{cc} = 2.01 \text{ THz}$$

The corresponding "cosmological" quantum of energy is:

$$E_{cc} = h v_{cc} = 8.32 \text{ MeV}$$

... In the interior of superconductors ... the effective cutoff frequency can be different ... $h \nu = \ln 3 \ k \ T$ ... we find the cosmological critical temperature $T_{cc}$

$$T_{cc} = 87.49 \text{ K}$$

This temperature is characteristic of the BSCCO High-Tc superconductor.

"...

Xiao Hu and Shi-Zeng Lin in arXiv 0911.5371 said: "... The Josephson effect is a phenomenon of current flow across two weakly linked superconductors separated by a thin barrier, i.e. Josephson junction, associated with coherent quantum tunneling of Cooper pairs. ... The Josephson effect also provides a unique way to generate high-frequency electromagnetic (EM) radiation by dc bias voltage ... The discovery of cuprate high-Tc superconductors accelerated the effort to develop novel source of EM waves based on a stack of atomically dense-packed intrinsic Josephson junctions (IJJs), since the large superconductivity gap covers the whole terahertz (THz) frequency band. Very recently, strong and coherent THz radiations have been successfully generated from a mesa structure of Bi$_2$Sr$_2$CaCu$_2$O$_{8+d}$ single crystal ...[ BSCCO image from Wikipedia

which works both as the source of energy gain and as the cavity for resonance. This experimental breakthrough posed a challenge to theoretical study on the phase dynamics of stacked IJJs, since the phenomenon cannot be explained by the known solutions of the sine-Gordon equation so far. It is then found theoretically that, due to huge inductive coupling of IJJs produced by the nanometer junction separation and the large London penetration depth ... of the material, a novel dynamic state is stabilized in the coupled sine-Gordon system, in which $\pm \pi$ kinks in phase differences are developed responding to the standing wave of Josephson plasma and are
stacked alternately in the c-axis. This novel solution of the inductively coupled sine-Gordon equations captures the important features of experimental observations. The theory predicts an optimal radiation power larger than the one observed in recent experiments by orders of magnitude ...".

**What are some interesting BSCCO JJ Array configurations?**

Christian Beck and Michael C. Mackey in astro-ph/0605418 describe "... the AC Josephson effect ...

a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory each superconductor is described by a complex wave function whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions ... by P(t).

... at zero external voltage a superconductive current given by \( I_s = I_c \sin(P) \) flows between the two superconducting electrodes ... \( I_c \) is the maximum superconducting current the junction can support.

... if a voltage difference \( V \) is maintained across the junction, then the phase difference \( P \) evolves according to \[ \frac{dP}{dt} = \frac{2e}{\hbar} V \]
i.e. \( P \) becomes an oscillating curent with amplitude \( I_c \) and frequency \( v = \frac{2e}{\hbar} V \)

This frequency is the ... Josephson frequency ... The quantum energy \( h v \) ... can be interpreted as the energy change of a Cooper pair that is transferred across the junction ...".

Xiao Hu and Shi-Zeng Lin in arXiv 1206.516 said:
"... to enhance the radiation power in terahertz band based on the intrinsic Josephson Junctions of Bi2Sr2CaCu2O8+d single crystal ...

we focus on the case that the Josephson plasma is uniform along a long crystal as established by the cavity formed by the dielectric material. ...

A ... \( \pi \) kink state ... is characterized by static +/- \( \pi \) phase kinks in the lateral directions of the mesa, which align themselves alternatingly along the c -axis. The \( \pi \) phase kinks provide a strong coupling between the uniform dc current and the cavity modes, which permits large supercurrent flow into the system at the cavity resonances, thus enhances the plasma oscillation and radiates strong EM wave ...

The maximal radiation power ... is achieved when the length of BSCCO single crystal at c-axis equals the EM wave length. ...".
Each long BSCCO single crystal looks geometrically like a line so configure the JJ Array using BSCCO crystals as edges.

The simplest polytope, the **Tetrahedron**, is made of 6 edges:

Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 said:

“... Superconducting tetrahedral quantum bits ...”

![Diagram of a tetrahedral superconducting qubit](image)

**Fig. 1:** (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling $E_J$ and charging energy $E_C$); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases $\phi_i$, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages $v_i$ and bias currents $i_i$. In order to measure the qubit’s state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the ‘connected’ tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_m \gg E_J$ on the outer ring which are driven by external currents $I_m$ (schematic, see Fig. 6 for details); the large coupling $E_m$ effectively binds the ring segments into one island.

... tetrahedral qubit design ... emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit. Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin; the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space. ...“.
**42 edges make an Icosahedron plus its center**  
(image from Physical Review B 72 (2005) 115421 by Rogan et al)

with 30 exterior edges and 12 edges from center to vertices. It has 20 cells which are approximate Tetrahedra in flat 3-space but become exact regular Tetrahedra in curved 3-space.

Could an approximate-20Tetrahedra-Icosahedron configuration of 42 BSCCO JJ tap into Dark Energy so that the Dark Energy might regularize the configuration to exact Tetrahedra and so curve/warp spacetime from flat 3-space to curved 3-space ?
720 edges make a 4-dimensional 600-cell
(image from Wikipedia)

At each vertex 20 Tetrahedral faces meet forming an Icosahedron
which is exact because the 600-cell lives on a curved 3-shere in 4-space.
It has 600 Tetrahedral 3-dim faces and 120 vertices

Could a 600 approximate-Tetrahedra configuration of 720 BSCCO JJ
approximating projection of a 600-cell into 3-space
tap into Dark Energy so that the Dark Energy might regularize
the configuration to exact Tetrahedra and an exact 600-cell
and so curve/warp spacetime from flat 3-space to curved 3-space?
The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell as follows: Consider 3-dim models of 600-cell such as metal sculpture from Bathsheba Grossman who says: "... for it I used an orthogonal projection rather than the Schlegel diagrams of the other polytopes I build. ... In this projection all cells are identical, as there is no perspective distortion. ...".

For the Dark Energy experiment each of the 720 lines would be made of a single BSCCO crystal

whose layers act naturally to make the BSCCO crystal an intrinsic Josephson Junction. ( see Wikipedia and arXiv 0911.5371 )
Each of the 600 tetrahedral cells of the 600-cell has 6 BSCCO crystal JJ edges.

Since the 600-cell is in flat 3D space the tetrahedra are distorted.

According to the ideas of Beck and Mackey (astro-ph/0703364) and of Clovis Jacinto de Matos (arXiv 0707.1797) the superconducting Josephson Junction layers of the 720 BSCCO crystals will bond with Dark Energy GraviPhotons that are pushing our Universe to expand.

My idea is that the Dark Energy GraviPhotons will not like being configured as edges of tetrahedra that are distorted in our flat 3D space and they will use their Dark Energy to make all 600 tetrahedra to be exact and regular by curving our flat space (and space-time).

My view is that the Dark Energy GraviPhotons will have enough strength to do that because their strength will NOT be weakened by the \((1 / M_{\text{Planck}})^2\) factor that makes ordinary gravity so weak.

It seems to me to be a clearly designed experiment that will either

1- not work and show my ideas to be wrong or
2 - work and open the door for humans to work with Dark Energy.

Consider BSCCO JJ 600-cells in this configuration:
First put 12 of the BSCCO JJ 600-cells at the vertices of a cuboctahedron shown here as a 3D stereo pair:

Cuboctahedra do not tile 3D flat space without interstitial octahedra

but BSCCO JJ 600-cell cuboctahedra can be put together square-face-to-square-face in flat 3D configurations including flat sheets.
As Buckminster Fuller described, the 8 triangle faces of a cuboctahedron give it an inherently 4D structure consistent with the green cuboctahedron central figure of a 24-cell (3D stereo 4thD blue-green-red color) that tiles flat Euclidean 4D space.

So, cuboctahedral BSCCO JJ 600-cell structure likes flat 3D and 4D space but if BSCCO JJ Dark Energy act to transform flat space into curved space like a 720-edge 600-cell with 600 regular tetrahedra then Dark Energy should transform cuboctahedral BSCCO JJ 600-cell structure into a 720-edge BSCCO JJ 600-cell structure that likes curved space.
There is a direct Jitterbug transformation of the 12-vertex cuboctahedron to the 12-vertex icosahedron whereby the 12 cuboctahedron vertices as midpoints of octahedral edges are mapped to 12 icosahedron vertices as Golden Ratio points of octahedral edges. There are two ways to map a midpoint to a Golden Ratio point. For the Dark Energy experiment the same choice of mapping should be made consistently throughout the BSCCO JJ 600-cell structure.

The result of the Jitterbug mapping is that each cuboctahedron in the BSCCO JJ 600-cell structure with its 12 little BSCCO JJ 600-cells at its 12 vertices is mapped to an icosahedron with 12 little BSCCO JJ 600-cells at its 2 vertices.
and the overall cuboctahedral BSCCO JJ 600-cell structure is transformed into an overall icosahedral BSCCO JJ 600-cell structure does not fit in flat 3D space in a naturally characteristic way (This is why icosahedral QuasiCrystal structures do not extend as simply throughout flat 3D space as do cuboctahedral structures).

However, the BSCCO JJ 600-cell structure Jitterbug icosahedra do live happily in 3-sphere curved space within the icosahedral 120-cell
which has the same 720-edge arrangement as the 600-cell (see Wikipedia).
The icosahedral 120-cell is constructed by 5 icosahedra around each edge.
It has:

- cells - 120 \{3,5\}
- faces - 1200 \{3\}
- edges - 720
- vertices - 120
- vertex figure - \{5,5/2\}
- symmetry group H4,[3,3,5]
- dual - small stellated 120-cell

In summary,

**Jitterbug transformations and BSCCO Josephson Junctions may be the Geometric Key to controlling Dark Energy**

( as were Chain Reactions for Nuclear Fission and Ellipsoidal Focussing for H-Bombs )

The Energy Gap of our Universe as superconductor condensate spacetime is from \(3 \times 10^{-18}\) Hz (radius of universe) to \(3 \times 10^{43}\) Hz (Planck length).
Its RMS amplitude is \(10^{13}\) Hz = 10 THz = energy of neutrino masses = critical temperature \(T_c\) of BSCCO superconducting crystals.

BSCCO superconducting crystals are natural Josephson Junctions. Dark Energy accumulates in the superconducting layers of BSCCO. The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell each of whose 720 edge-lines would be made of a single BSCCO crystal. It may be useful to use a Jitterbug-type transformation between a 600-cell configuration and a configuration based on icosahedral 120-cells which also have 720 edge-lines: